Deuteron and pion form factors at high momentum transfer and JLab experiments

A.F. Krutov
Samara State University, Samara, Russia

V.E. Troitsky
D.V.Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia

N.A. Tsirova
Samara State University, Samara, Russia

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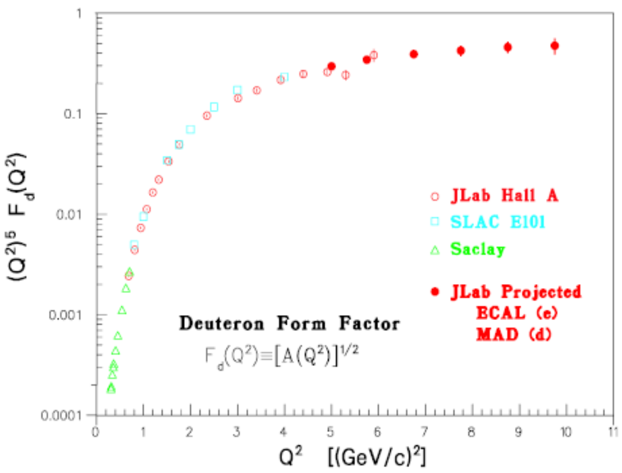
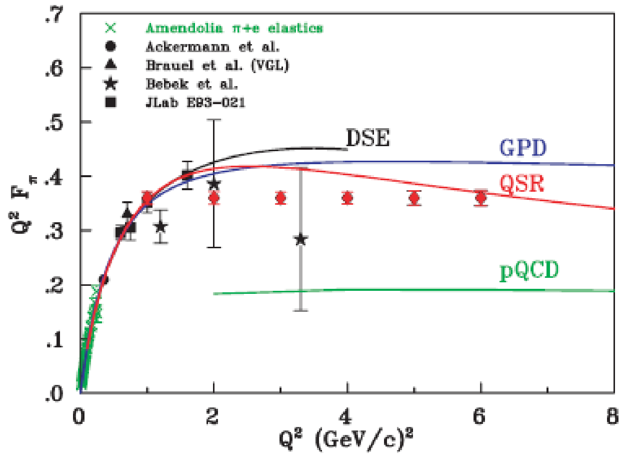
Outline

- Motivation: future JLab experiments
- Our RQM model and form factors
- Deuteron form factors asymptotics
- Pion form factors asymptotics
- Conclusions

Motivation

Possible manifestation of pQCD regime in the future JLab experiments:





Deuteron and pion form factors in the range of future JLab experiments. pQCD predictions (red dots) are expected

[J. Arrington, R.J. Holt, P.E. Reimer et al. Hall A 12 GeV Upgrade (Pre-Conceptual Design Report), Jefferson Lab. 2005]

We investigate the range of high momentum transfer, where asymptotical predictions are valid

Our RQM model

We work in the instant form of RQM developed by A.F. Krutov and V.E. Troitsky. It is based on a canonical parametrization of the electromagnetic current operator matrix element [Cheshkov A.A., Shirokov Yu.M. JETP **44** (1963) 1982]. The current operator is constructed with algebra of Poincare group conservation.

Features

- The matrix element of the electromagnetic current satisfies the relativistic covariance conditions and conservation laws
- The impulse approximation is formulated in the relativistic invariant manner
- The correspondence principle is fulfilled, i.e. there is a correct non-relativistic limit
- Electromagnetic form factors are defined unambiguously, without any “good” or “bad” current components
- Composite quark (pion) and nucleon (deuteron) systems are described well

Details are presented in papers

- A.F. Krutov and V.E. Troitsky, Eur. Phys. J. A **16** (2003) 285
- A.F. Krutov and V.E. Troitsky, Phys. Rev. C **65** (2002) 045501
- A.F. Krutov and V.E. Troitsky, Phys. Rev. C **68** (2003) 018501
- A.F. Krutov and V.E. Troitsky, Teor. Mat. Fiz. **143** (2005) 258 [English translation: Theor. Math. Phys. **143** (2005) 704]

Electromagnetic deuteron form factors

$$G_C^R(Q^2) = \sum_{l,l'} \int d\sqrt{s}\, d\sqrt{s'}\, \varphi_l(s)\, g_{0C}^{ll'}(s\,, Q^2\,, s')\, \varphi_{l'}(s')\,,$$

$$G_Q^R(Q^2) = \frac{2\,M_d^2}{Q^2} \sum_{l,l'} \int d\sqrt{s}\, d\sqrt{s'}\, \varphi_l(s)\, g_{0Q}^{ll'}(s\,, Q^2\,, s')\, \varphi_{l'}(s')\,,$$

$$G_M^R(Q^2) = -\,M_d \sum_{l,l'} \int d\sqrt{s}\, d\sqrt{s'}\, \varphi_l(s)\, g_{0M}^{ll'}(s\,, Q^2\,, s')\, \varphi_{l'}(s')$$

Deuteron wave functions in sense of RQM

$$\hat{M}_d^2\,|\psi\rangle = M_d^2\,|\psi\rangle$$

$$\varphi_l(k) = \sqrt[4]{s}\, k\, u_l(k)\,, \quad s = 4(k^2 + M^2) \qquad \sum_{l=0,2} \int_0^\infty \varphi_l^2(k)\, \frac{dk}{2\sqrt{k^2 + M^2}} = 1$$

where $$u_0(k) = \sqrt{\frac{2}{\pi}} \sum_j \frac{C_j}{(k^2 + m_j^2)}, \quad u_2(k) = \sqrt{\frac{2}{\pi}} \sum_j \frac{D_j}{(k^2 + m_j^2)}$$

Conditions on the coefficients

$$\sum_j C_j = 0\,, \quad \sum_j D_j = \sum_j D_j m_j^2 = \sum_j \frac{D_j}{m_j^2} = 0 \longrightarrow u_0(r) \sim r\,, \quad u_2(r) \sim r^3$$

Our problem is to investigate form factors behavior in the case of high momentum transfer. So we come to the problem of asymptotic decomposition of double integrals of some special kind, which is not considered in classical literature

Asymptotic expansion of n-tuple integrals

- Classical Laplas method $$F(\lambda) = \int\limits_{\Omega} f(x) e^{\lambda S(x)} dx$$
- We consider $$F(\lambda) = \int\limits_{\Omega} f(\lambda, x) e^{S(\lambda, x)} dx$$

$$\lambda \to \infty\,,\quad x = (x_1, \ldots, x_n)\,,\quad x^0 \in \partial\Omega$$

Conditions:

$S(\lambda, x^0) - S(\lambda, x)$ increases with increasing λ

$$\frac{\partial S(\lambda, x)}{\partial \lambda} \leqslant \frac{\mathrm{d}M(\lambda)}{\mathrm{d}\lambda}, \quad M(\lambda) = \sup_{x \in [\Omega]} S(\lambda, x)$$

mean, that the vicinity of the point x^0 makes the main contribution to the asymptotics

$$\frac{\partial S(\lambda, x^0)}{\partial n} \neq 0,$$

$$\left\| \frac{\partial^2 S(\lambda, x^0)}{\partial \xi_i \partial \xi_j} \right\|_{i,j=1}^{n-1} = B(\lambda)$$ is a negatively defined matrix

$\xi_1, \ldots, \xi_{n-1}$ is an orthonormal basis in the tangential to the $\partial\Omega$ plane in the point x^0

mean, that the point of maximal value x^0 is not a point of extremum in the normal to the $\partial\Omega$ direction and the point of extremum in the tangential to the $\partial\Omega$ plane

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These qualitative statements we formulated as a theorem in our paper [A.F.Krutov, V.E.Troitsky, N.A.Tsirova. J. Phys. A: Math. Theor. 41 (2008) 255401]. We obtain asymptotic series. The main asymptotic term:

$$F(\lambda) \sim -(2\pi)^{\frac{n-1}{2}} \exp[S(\lambda, x^0)] \left(\frac{\partial S(\lambda, x^0)}{\partial n} \right)^{-1} |\det B(\lambda)|^{-\frac{1}{2}} f(\lambda, x^0)$$

Asymptotics of the deuteron form factors

- Relativistic asymptotics

$$A^R(Q^2) \sim \frac{1}{Q^{16}} \frac{2^{18}(\mu_p + \mu_n)^2 a_d^8}{3\pi r_0^6 M^8} \left[\sum_j C_j m_j^2\right]^4$$

$$F_d^R(Q^2) \sim (Q^2)^{-4}$$

- Experimental data fit [A.F.Krutov, V.E.Troitsky, N.A.Tsirova. arXiv:0801.2868 [nucl-th]]

$$F_d^{exp}(Q^2) \sim (Q^2)^{-(3.76 \pm 0.41)}$$

- Conclusion: modern experiment has achieved asymptotical regime of the relativistic nucleon model

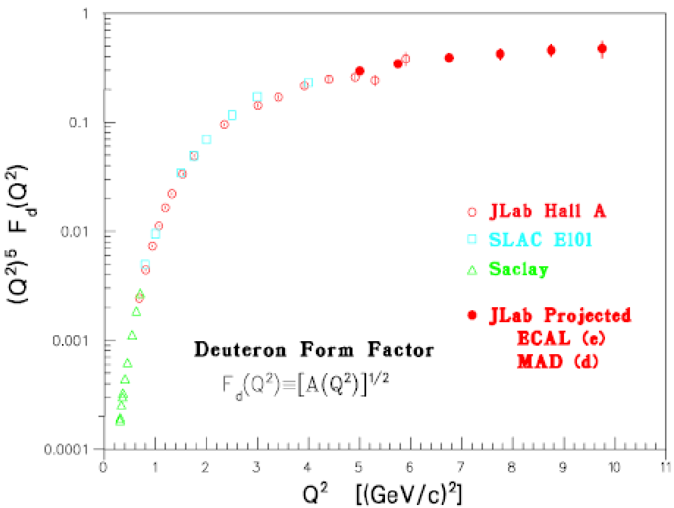
- Nonrelativistic asymptotics

$$F_d^{NR}(Q^2) \sim (Q^2)^{-5.5}$$

- Conclusion: relativistic corrections slow down the asymptotic decreasing of form factors. Independently of the interaction details, if the wave function is a solution of Schrödinger equation with standard conditions, nonrelativistic calculations are not coinciding with experimental data.

Physics of the future experiments

Predicted in the future JLab experiments deuteron form factors behavior is coinciding with pQCD prediction



pQCD: $F_d(Q^2) \sim (Q^2)^{-5}$

Is it really a pQCD manifestation?

This result could be achieved in our nucleon approach, if the wave function behavior at small distances will be

$$u_0(r) \sim r + a\,r^3\ ,\ \ u_0''(0) = 0$$

Under this the conditions on the wave functions coefficients:

$$\sum_j C_j m_j^2 = 0$$

Asymptotics of the pion form factor

Structureless quarks

Introducing quark structure (taking into account form factor $f_q(Q^2) = \frac{1}{1+\ln(1+\langle r_q^2\rangle Q^2/6)}$)

[U.Vogl and W. Weise, Progr. in Part. and Nucl. Phys. **27** 195 (1991)]

[A.F. Krutov and V.E. Troitsky, Eur. Phys. J. C **20**, 71 (2001)]

<u>The main asymptotic term:</u>

$$F_\pi(Q^2) \sim \frac{2^{5/2}M_q}{Q} e^{-\frac{M_q Q}{4b^2}}$$

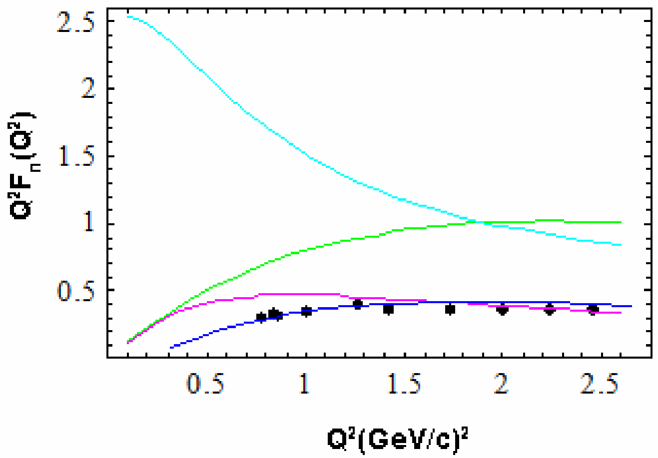
$$F_\pi(Q^2) \sim \frac{2^{5/2}M_q}{Q} e^{-\frac{M_q Q}{4b^2}} (G_E^u + G_E^{\bar{d}})$$

<u>Small quark masses limit:</u>

$$M_q \to 0\,, \quad F_\pi(Q^2) \sim 7 \cdot 2^{3/2} \frac{b^2}{Q^2}$$

$$M_q \to 0, \quad F_\pi(Q^2) \sim \frac{2^{3/2} b^2}{Q^2} f_q(Q^2)$$

Our asymptotic calculations with $M_q \to 0$ coincide with pQCD results and JLab predictions



- experimental (obtained and predicted) dots
- calculation without asymptotic decomposition
- asymptotics (structureless quarks)
- asymptotics with quark form factor
- asymptotics with quark form factor with $M_q \to 0$

All calculations are done with quark masses $M_q = 0.21\ GeV$

Pion form factor at high momentum transfer

$$F_\pi(Q^2) = \int d\sqrt{s}d\sqrt{s'}\varphi(k)g_0(s,Q^2,s')\varphi(k'),$$

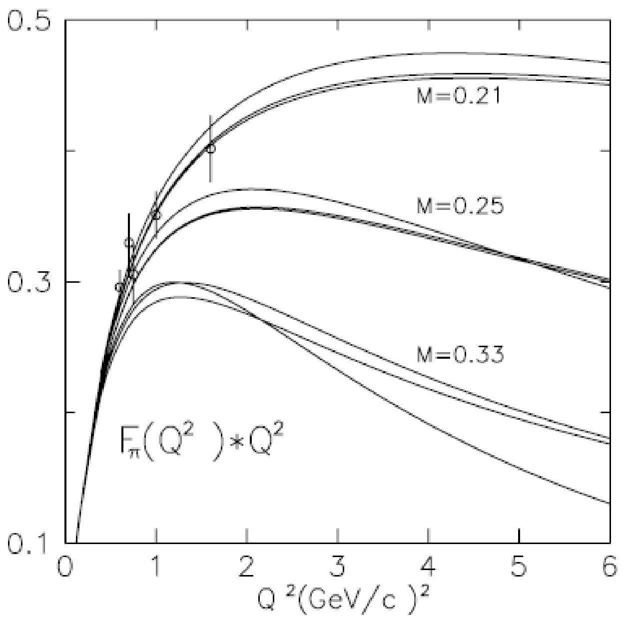
Pion wave functions

$$u(k) = N_{HO}\exp\left(-k^2/2b^2\right)$$ [P.L. Chung, F. Coester, W.N. Polyzou, Phys. Lett. B **205**, 545 (1988)]

$$u(k) = N_{PL}\left(k^2/b^2+1\right)^{-n}, \quad n = 2\,,3$$ [F. Schlumpf, Phys. Rev. D **50**, 6895 (1994)

$$u(r) = N_T\,\exp(-\alpha r^{3/2} - \beta r)$$ [H. Tezuka, J. Phys. A Math. Gen. **24**, 5267 (1991)]

Calculations with these wave functions with different quark masses



Calculations depend on the quark mass much stronger, then on the wave function type. The restriction on the constituent quark model parameters is obtained:

at high momentum transfers small quark masses should be used

$$M_q \sim 0.21\ GeV$$

Conclusions

1. Theorem on the asymptotic decomposition of n-tuple integrals of some special kind is proven

2. Asymptotics of the deuteron form factors is obtained in the relativistic impulse approximation. Comparison to the obtained experimental data fit at the highest achieved today momentum transfer leads us to the fact that modern experiment has achieved asymptotical regime of the relativistic nucleon model. Relativistic corrections slow down asymptotic decrease of form factors. Nonrelativistic asymptotics are not in agreement with experimental data

3. The behavior of the deuteron wave function (and corresponding condition on the wave function coefficients) that results in asymptotics coinciding with pQCD and JLab predictions is calculated

4. Asymptotics of the pion form factor is calculated. The necessity of taking into account quark form factors is shown

5. Restrictions on the parameters of the constituent quark model are obtained from calculations of the pion form factor at high momentum transfer: at high momentum transfer small quark masses ($M_q \sim 0.21\ GeV$) should be used