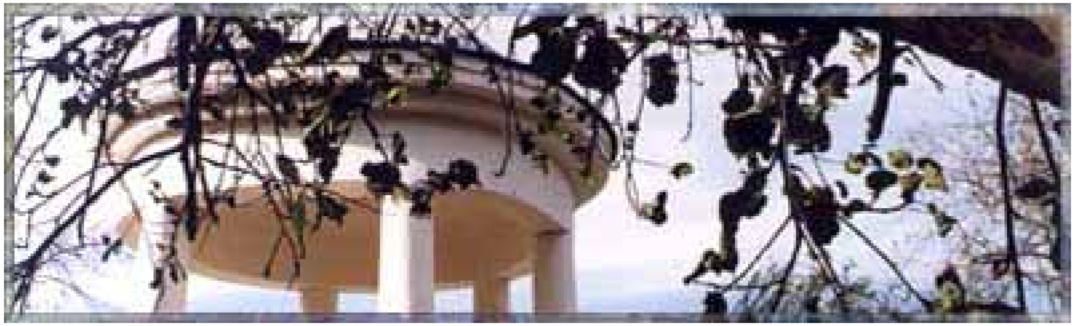
POLARIZATION PHENOMENA in ANNIHILATION and SCATTERING



Dubna, October 1, 2008

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in collaboration with G.I. Gakh and C. Adamuscin

Motivation

$$e^+ + e^- \rightarrow \text{hadrons}$$

Studied since the 70's (Novosibirsk, Frascati)

- *Interaction of light quarks*
- *Spectroscopy of their bound state*
- *Strong interaction contribution to g-2 and a(M_z^2)*
- *Tests of standard model (conservation of vector current) related to hadronic lepton decays*
- *Determination of QCD parameters, QCD sum rules*

RR Akhmetshin etal, CMD-2 at VEPP-2M

Motivation

$$e^{+} + e^{-} \rightarrow \text{hadrons}$$

..and recently at BaBar

$$e^{+} + e^{-} \rightarrow \text{multi } \pi$$

$>>$

$$e^{+} + e^{-} \rightarrow \pi^{+} + \pi^{-}$$

$\sim$

$$e^{+} + e^{-} \rightarrow \mu^{+} + \mu^{-}$$

Motivation

$$e^+ + e^- \rightarrow 4\pi$$

Present interest due to
* high luminosity,
* large solid angle
(4π) detectors

$$e^+ + e^- \rightarrow \omega + \pi$$

→ $$e^+ + e^- \rightarrow \pi + a_1(1260)$$

$$e^+ + e^- \rightarrow \pi + h_1(1170)$$

$$e^+ + e^- \rightarrow \rho + f_0(400\text{–}1200)$$

$$e^+ + e^- \rightarrow \pi + a_2(1320)$$

→ $$e^+ + e^- \rightarrow \rho^+ + \rho^-$$

$$e^+ + e^- \rightarrow \pi + \pi(1300)$$

allow to discriminate different possible intermediate states

ELSEVIER

4 November 1999

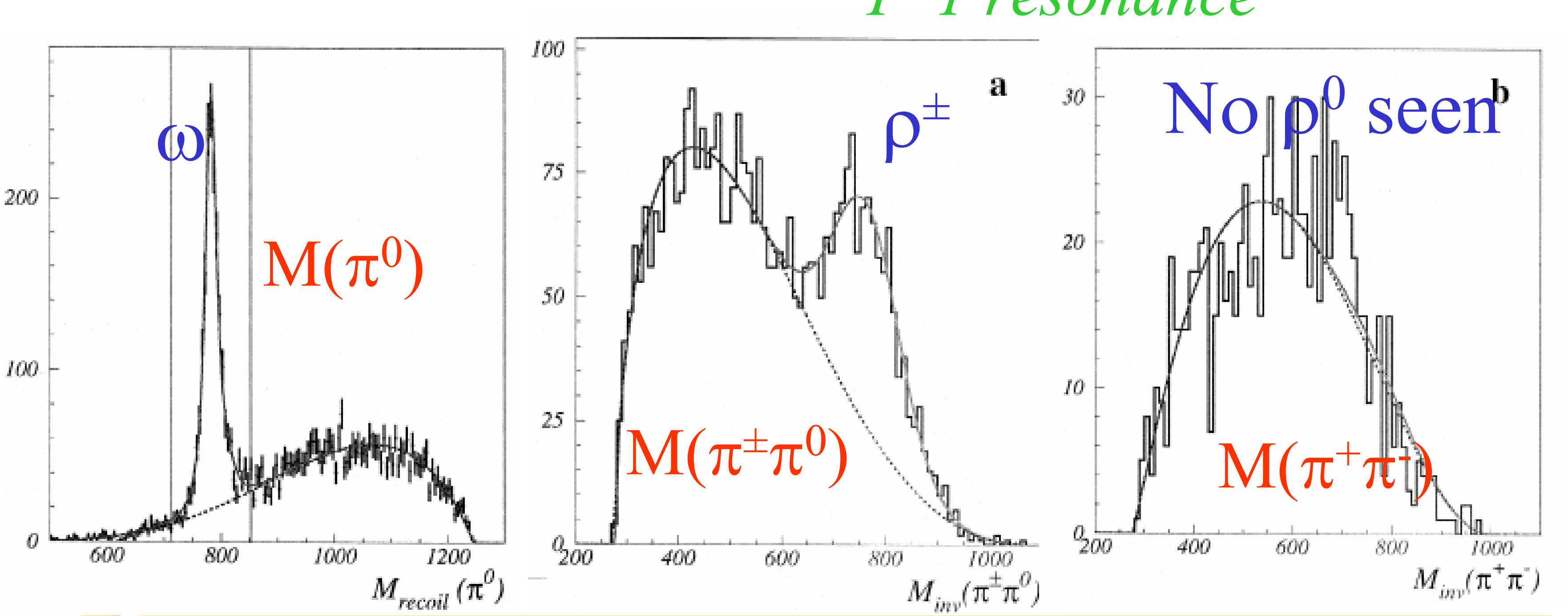
PHYSICS LETTERS B

Physics Letters B 466 (1999) 392–402

$a_1(1260)\pi$ dominance in the process $e^+e^- \to 4\pi$ at energies 1.05–1.38 GeV

R.R. Akhmetshin [a], E.V. Anashkin [a], M. Arpagaus [a], V.M. Aulchenko [a,b],

I=1 resonance

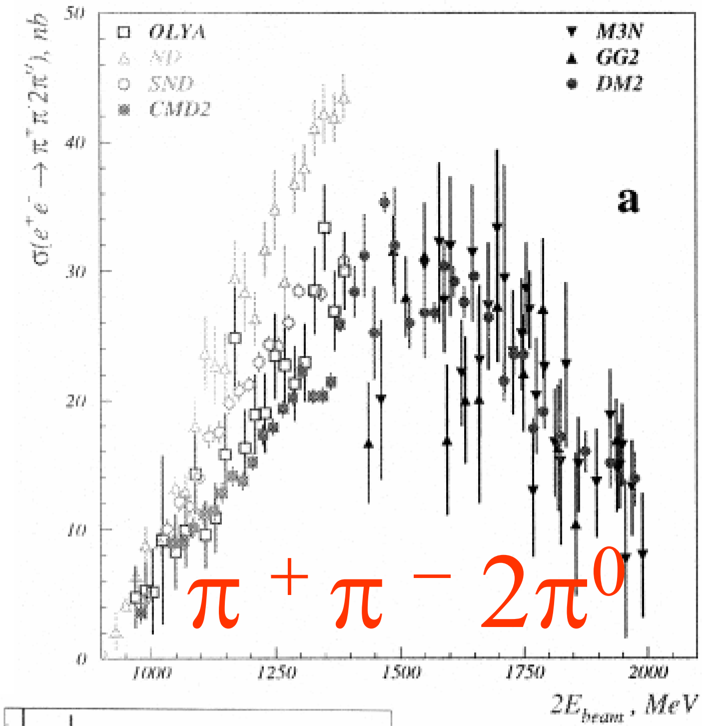


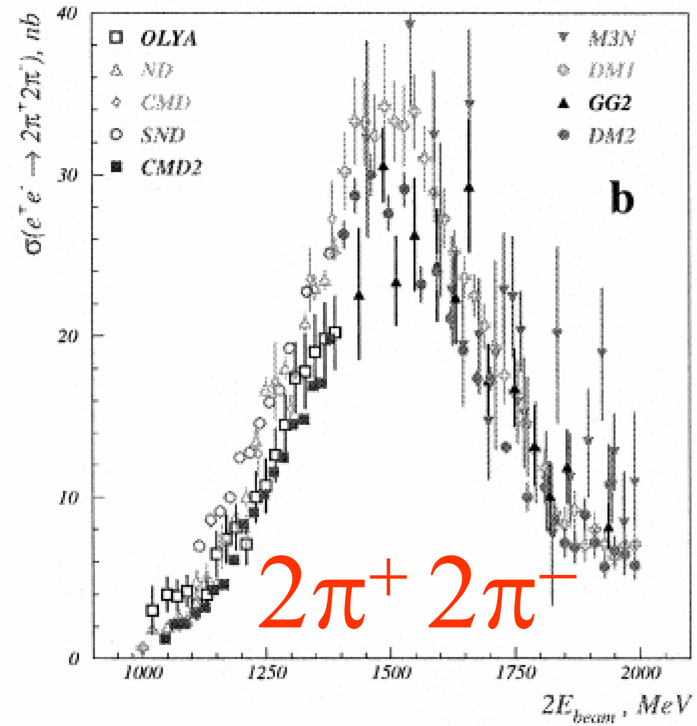
Dominance of $a_1\pi$

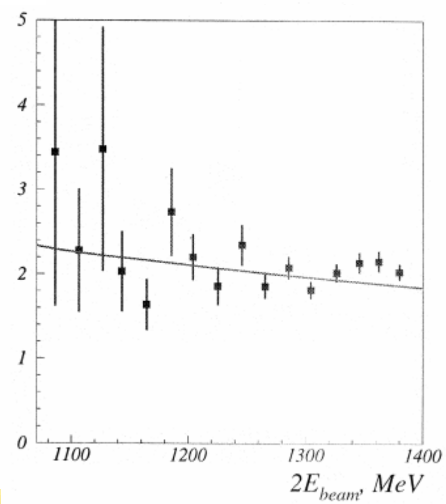
- Detector CMD-2 at VEPP-2M:
  - Measure energy and angles of charged and neutral particles
  - Analysis with kinematical fits, combinatorial, event topology
- Model
  - assumes quasi-two particle intermediate state
  - takes into account
    - identity of final pions
    - interference of all possible amplitudes

CMD2 Collaboration (R.R. Akhmetshin *et al.*)
Phys.Lett.B466:392-402,1999.

Energy dependence



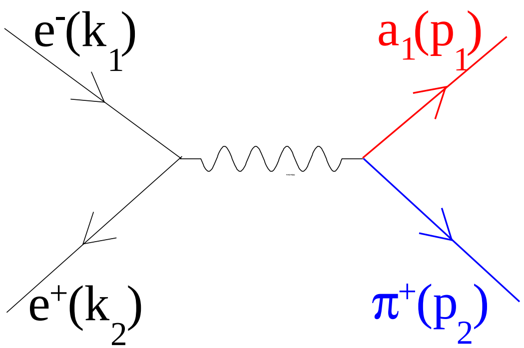




$$\frac{2\pi^+2\pi^-}{\pi^+\pi^-2\pi^0} \equiv \frac{a_1(\rho\pi)\,\pi}{\omega\pi^0}$$

CMD2 Collaboration (R.R. Akhmetshin *et al.*)

a_1 production



$$e^-(k_1) + e^+(k_2) \longrightarrow a_1(p_1) + \pi(p_2)$$

$a_1(1260)$ $I^G(J^{PC}) = 1^-(1^{++})$

The a_1 density matrix (spin 1 as deuteron):

$$\rho_{\mu\nu} = -\left(g_{\mu\nu} - \frac{p_{1\mu}p_{1\nu}}{M^2}\right) + \frac{i}{2M}\varepsilon_{\mu\nu\rho\sigma}s_\rho p_{1\sigma} + 3Q_{\mu\nu}.$$

Vector and tensor polarization:

$$s^2 = -1,\ sp_1 = 0,\ Q_{\mu\nu} = Q_{\nu\mu},\ \ Q_{\mu\mu} = 0,\ p_{1\mu}Q_{\mu\nu} = 0\ .$$

$$e^-(k_1) + e^+(k_2) \rightarrow a_1(p_1) + \pi(p_2)$$

Irfu cea saclay

The EM current for $\gamma^* \rightarrow a_1\pi$ transition

$$J_\mu = f_1(q^2)(q^2 U^*_\mu - q \cdot U^* q_\mu) + f_2(q^2)(q \cdot p_2 U^*_\mu - q \cdot U^* p_{2\mu}),$$

Form factors (complex functions)

VMD Parametrization

$$Дf_i = \frac{C_{v,i} M_v}{M_v^2 - q^2 + iM_v \Gamma}, i = 1,2$$

Normalization:

$$f_1(0) = 0$$

$$\Gamma(a_1 \rightarrow ρг) = \frac{1}{p} \frac{щ}{24\, m_{a1}^2} |M|$$

$$M = ee_М J_М$$

$$f_2^2(0) = 12 \frac{\Gamma . m_{a1}^3}{б(m_{a1}^2 - m^2)^3}$$

Relation between $\gamma^*\, a_1\pi$ and $a_1\rho\pi$

$$e^-(k_1) + e^+(k_2) \to a_1(p_1) + \pi(p_2)$$

The leptonic current (longitudinally polarized)

$$L_{\mu\nu} = -q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}) + 2i\lambda\varepsilon_{\mu\nu\sigma\rho}k_{1\sigma}k_{2\rho}$$

The differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{q^6}\frac{p}{2W}L_{\mu\nu}W_{\mu\nu},$$

The hadronic tensor

$$W_{\mu\nu} = J_\mu J_\nu^*.$$

can be splitted in three terms

$$W_{\mu\nu} = W_{\mu\nu}(0) + W_{\mu\nu}(V) + W_{\mu\nu}(T),$$

The unpolarized cross section

The differential cross section:

$$\frac{d\sigma^{un}}{d\Omega} = \frac{\alpha^2}{2q^4}\frac{p}{W}(A + B\sin^2\theta),$$

$$A = |q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2|^2, B = 2\tau p^2[q^2|f_1 + f_2|^2 - M^2|f_2|^2],$$

The total cross section:

$$\sigma_{tot}(e^+e^- \to \pi a_1) = \frac{2\pi\alpha^2}{3q^4}\frac{p}{W}\left[3|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2|^2 + \right.$$
$$\left. +4\tau p^2[q^2|f_1 + f_2|^2 - M^2|f_2|^2]\right].$$

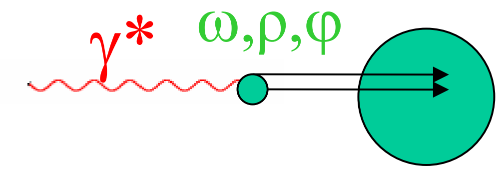
The angular asymmetry:

$$\frac{d\sigma^{un}}{d\Omega} = \sigma_{\pi/2}(1 + R\cos^2\theta), R = -B/(A+B).$$

S=1 form factors: examples

Iachello, Jakson and Landé (1973)

Isoscalar and isovector FFs



$$F_1^s(Q^2) = \frac{g(Q^2)}{2}\left[(1-\beta_\omega-\beta_\phi)+\beta_\omega\frac{\mu_\omega^2}{\mu_\omega^2+Q^2}+\beta_\phi\frac{\mu_\phi^2}{\mu_\phi^2+Q^2}\right],$$

$$F_1^v(Q^2) = \frac{g(Q^2)}{2}\left[(1-\beta_\rho)+\beta_\rho\frac{\mu_\rho^2+8\Gamma_\rho\mu_\pi/\pi}{(\mu_\rho^2+Q^2)+(4\mu_\pi^2+Q^2)\Gamma_\rho\alpha(Q^2)/\mu_\pi}\right],$$

$$F_2^s(Q^2) = \frac{g(Q^2)}{2}\left[(\mu_p+\mu_n-1-\alpha_\phi)\frac{\mu_\omega^2}{\mu_\omega^2+Q^2}+\alpha_\phi\frac{\mu_\phi^2}{\mu_\phi^2+Q^2}\right],$$

$$F_2^v(Q^2) = \frac{g(Q^2)}{2}\left[(\mu_p-\mu_n-1)\frac{\mu_\rho^2+8\Gamma_\rho\mu_\pi/\pi}{(\mu_\rho^2+Q^2)+(4\mu_\pi^2+Q^2)\Gamma_\rho\alpha(Q^2)/\mu_\pi}\right],$$

$$g(Q^2) = \frac{1}{(1+\gamma e^{i\theta}Q^2)^2}$$

$$\alpha(Q^2) = \frac{2}{\pi}\sqrt{\frac{Q^2+4\mu_\pi^2}{Q^2}}\, ln\left[\frac{\sqrt{(Q^2+4\mu_\pi^2)}+\sqrt{Q^2}}{2\mu_\pi}\right]$$

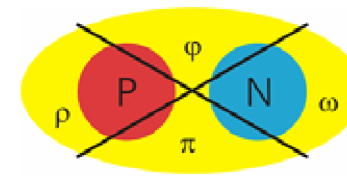
$$2F_i^p = F_i^s + F_i^v,$$

$$2F_i^n = F_i^s - F_i^v.$$

Deuteron VMD

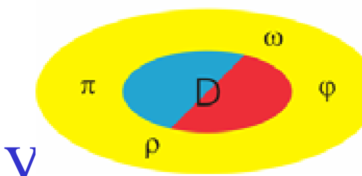
C. Adamuscin, G.I. Gakh and E.T-G. PRC ***73****, 045204 (2006)*

$$G_i(Q^2) = N_i g_i(Q^2) F_i(Q^2), \quad i = c, q, m$$



Intrinsic term

$$g_i(Q^2) = 1/[1 + \gamma_i Q^2]^{\delta_i},$$



Meson cloud: isoscalar vector meson only

$$F_i(Q^2) = 1 - \alpha_i - \beta_i + \alpha_i \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_i \frac{m_\phi^2}{m_\phi^2 + Q^2},$$

Normalization

$$N_c = G_c(0) = 1,$$

$$N_q = G_q(0) = M^2 Q_d = 25.83,$$

$$N_m = G_m(0) = \frac{M}{m} \mu_d = 1.714,$$

Results

From 12 to 6 parameters fit

1) Constrains on the nodes:

Q^2_{0C}=1.7 GeV^2, Q^2_{0M}=2 GeV^2

$$\alpha_i = \frac{m_\omega^2 + Q_{0i}^2}{Q_{0i}^2} - \beta_i \frac{m_\omega^2 + Q_{0i}^2}{m_\phi^2 + Q_{0i}^2}.$$

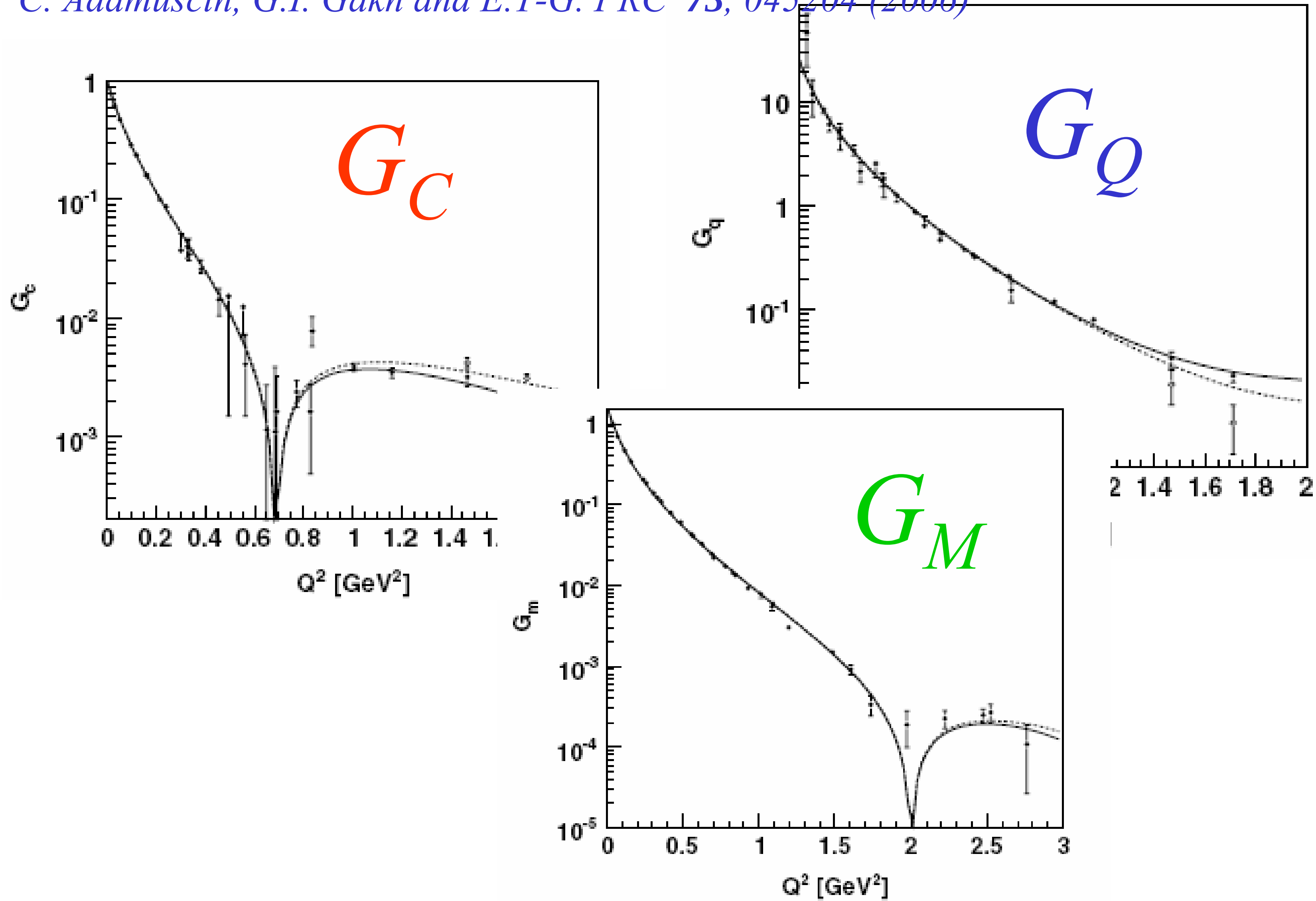
2) Intrinsic part common to the 3 FFs:

δ=1.04 ±0.03, γ=12.1 ± 0.5

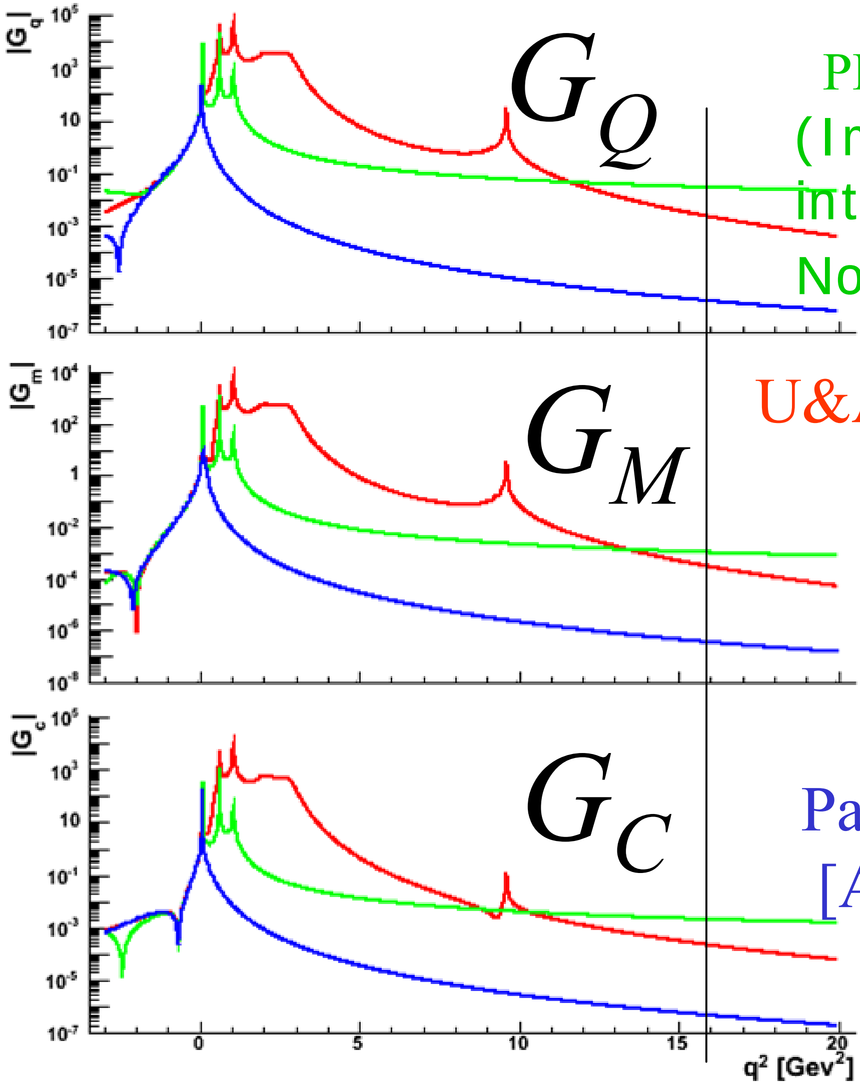
	α	β	$\chi 2/ndf$
G_c (I)	5.75 ± 0.07	-5.11 ± 0.09	0.9
G_c (II)	5.50 ± 0.06	-4.78 ± 0.08	1.3
G_q(I)	4.21 ± 0.05	-3.41 ± 0.07	0.9
G_q(II)	4.08 ± 0.07	-3.25 ± 0.09	1.6
$G_m(I)$	3.77 ± 0.04	-2.86 ± 0.05	1.6
$G_m(II)$	3.74 ± 0.04	-2.83 ± 0.05	1.7

Results

*C. Adamuscin, G.I. Gakh and E.T-G. PRC **73**, 045204 (2006)*



$e^+ + e^- \to d + \bar{d}$



PRC **74**, 025202 (2006)
(Imaginary part from the intrinsic term
No finite width for ρ,ω mesons)

U&A (Dubnicka)

Parametrization I (real part)
[Abbott, EPJA 2000]

ρ - form factors : Space-like

Parametrization

$$G_C(t) = \frac{A}{1-\frac{t}{m_{C1}^2}} + \frac{B}{1-\frac{t}{m_{C2}^2}}$$

$$G_M(t) = \frac{G_M(0)}{(1-\frac{t}{m_M^2})^2}$$

$$G_Q(t) = \frac{G_Q(0)}{(1-\frac{t}{m_Q^2})^2},$$

Node for t_0=-3 GeV^2

$$G_C(0) = A + B = 1 \; ; \; 0 = \frac{A}{1-\frac{t_0}{m_{C1}^2}} + \frac{B}{1-\frac{t_0}{m_{C2}^2}}$$

$$B = 1 - A \; ; \; A = \frac{\frac{1}{m_{C1}^2} - \frac{1}{t_0}}{\frac{1}{m_{C1}^2} - \frac{1}{m_{C2}^2}}$$

Fit on light front calculation de Melo and Federico, PRC55,2043 (1997)

A	B	m_{C1} [GeV]	m_{C2} [GeV]	m_M [GeV]	m_Q [GeV]
1.41	−0.41	0.88	2.70	1.42	1.51

ρ form factors : Time-like

Analytical extension (imaginary part from the width):

$$m_{C1} \to m_{C1} - i\frac{\Gamma_{C1}}{2} \;;\; m_{C2} \to m_{C2} - i\frac{\Gamma_{C2}}{2}$$
$$m_M \to m_M - i\frac{\Gamma_M}{2} \;;\; m_Q \to m_Q - i\frac{\Gamma_Q}{2},$$

TL-parametrization:

$$G_C(t) = \frac{A}{1 - \frac{t}{(m_{C1} - i\Gamma_{C1}/2)^2}} + \frac{B}{1 - \frac{t}{(m_{C2} - i\Gamma_{C2}/2)^2}}$$
$$G_M(t) = \frac{G_M(0)}{\left(1 - \frac{t}{(m_M - i\Gamma_M/2)^2}\right)^2}$$
$$G_Q(t) = \frac{G_Q(0)}{\left(1 - \frac{t}{(m_Q - i\Gamma_Q/2)^2}\right)^2}.$$

...no experimental constrains, no new parameter

Form factors

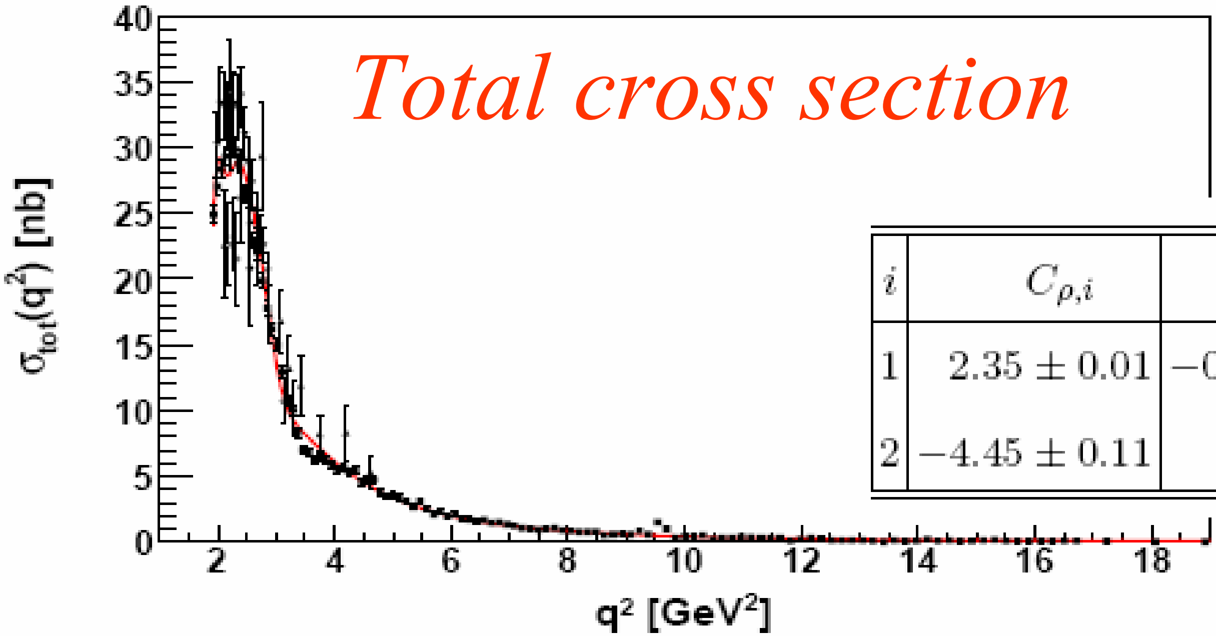
$$e^-(k_1) + e^+(k_2) \to a_1(p_1) + \pi(p_2)$$

Six parameters, fitted on cross section data

Irfu cea saclay

$$f_i = \frac{C_{\rho,i} M_\rho}{M_\rho^2 - q^2 + iM_\rho \Gamma_\rho} + \frac{C_{\rho',i} M_{\rho'}}{M_{\rho'}^2 - q^2 + iM_{\rho'} \Gamma_{\rho'}} + \frac{C_{\rho",i} M_{\rho"}}{M_{\rho"}^2 - q^2 + iM_{\rho"} \Gamma_{\rho"}}, \quad i = 1, 2.$$

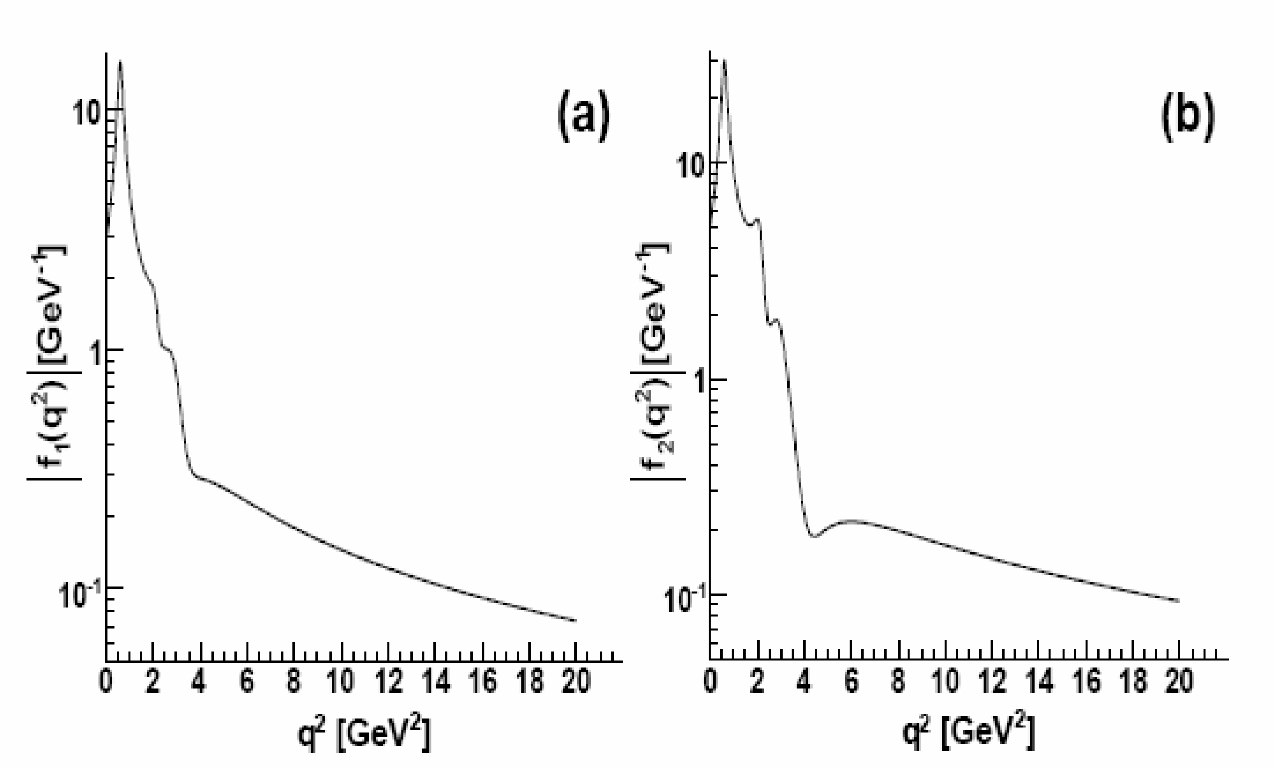
ρ(770) *ρ '(1450)* *ρ''(1700)*



i	$C_{\rho,i}$	$C_{\rho',i}$	$C_{\rho",i}$
1	2.35 ± 0.01	-0.089 ± 0.008	-0.131 ± 0.009
2	-4.45 ± 0.11	0.56 ± 0.05	0.38 ± 0.03

Form factors

$$e^-(k_1) + e^+(k_2) \to a_1(p_1) + \pi(p_2)$$



Six parameters, fitted on cross section data

$$e^-(k_1) + e^+(k_2) \rightarrow a_1(p_1) + \pi(p_2)$$

Predictions for the observables

Vector polarization observables

$$e^-(k_1) + e^+(k_2) \to a_1(p_1) + \pi(p_2)$$

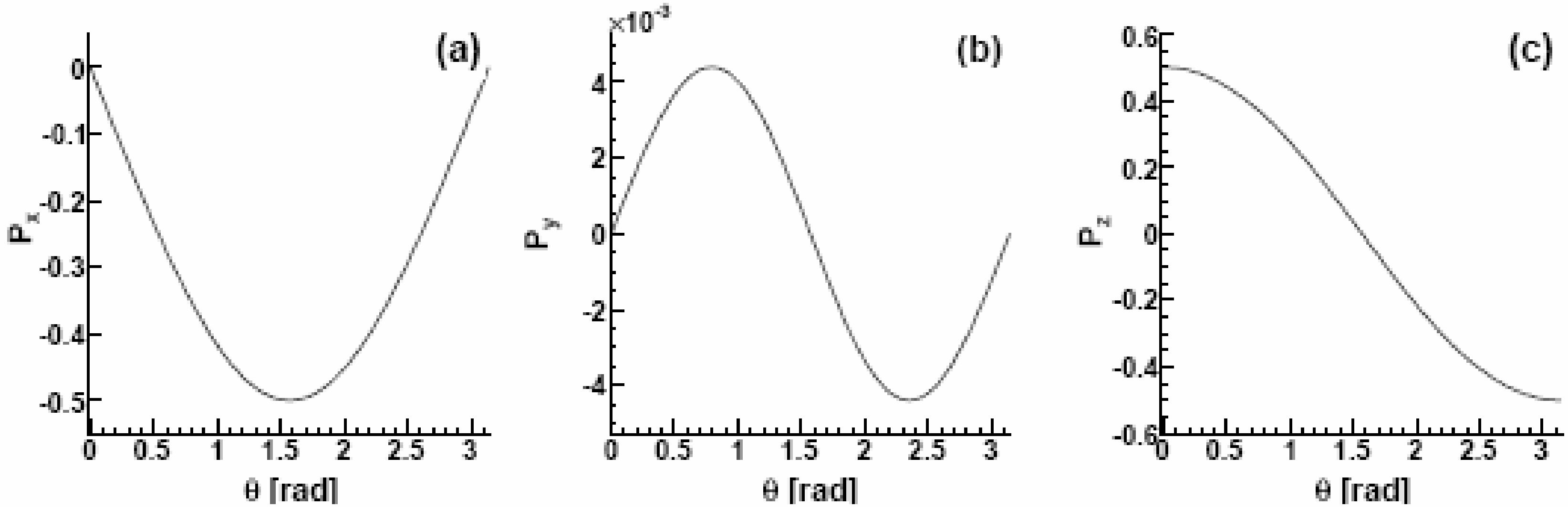
With unpolarized beams:

$$P_y = \frac{1}{8}\frac{\sqrt{\tau}}{\sigma_0}\left[(q^2+M^2-m^2)^2 - 4M^2q^2\right]\sin(2\theta)\mathrm{Im} f_1 f_2^*,$$

With longitudinally polarized electron:

$$P_x = -\frac{1}{4}\frac{\sqrt{\tau}}{\sigma_0}\sin\theta\left\{2q^2(q^2+M^2-m^2)|f_1|^2 + \left[(q^2-M^2)^2-m^4\right]|f_2|^2 + \right.$$
$$\left. + \left[q^2(3q^2-2M^2-2m^2)-(M^2-m^2)^2\right]\mathrm{Re} f_1 f_2^*\right\},$$
$$P_z = \frac{1}{2}\frac{1}{\sigma_0}\cos\theta\left|q^2 f_1 + \frac{1}{2}(q^2-M^2+m^2)f_2\right|^2.$$

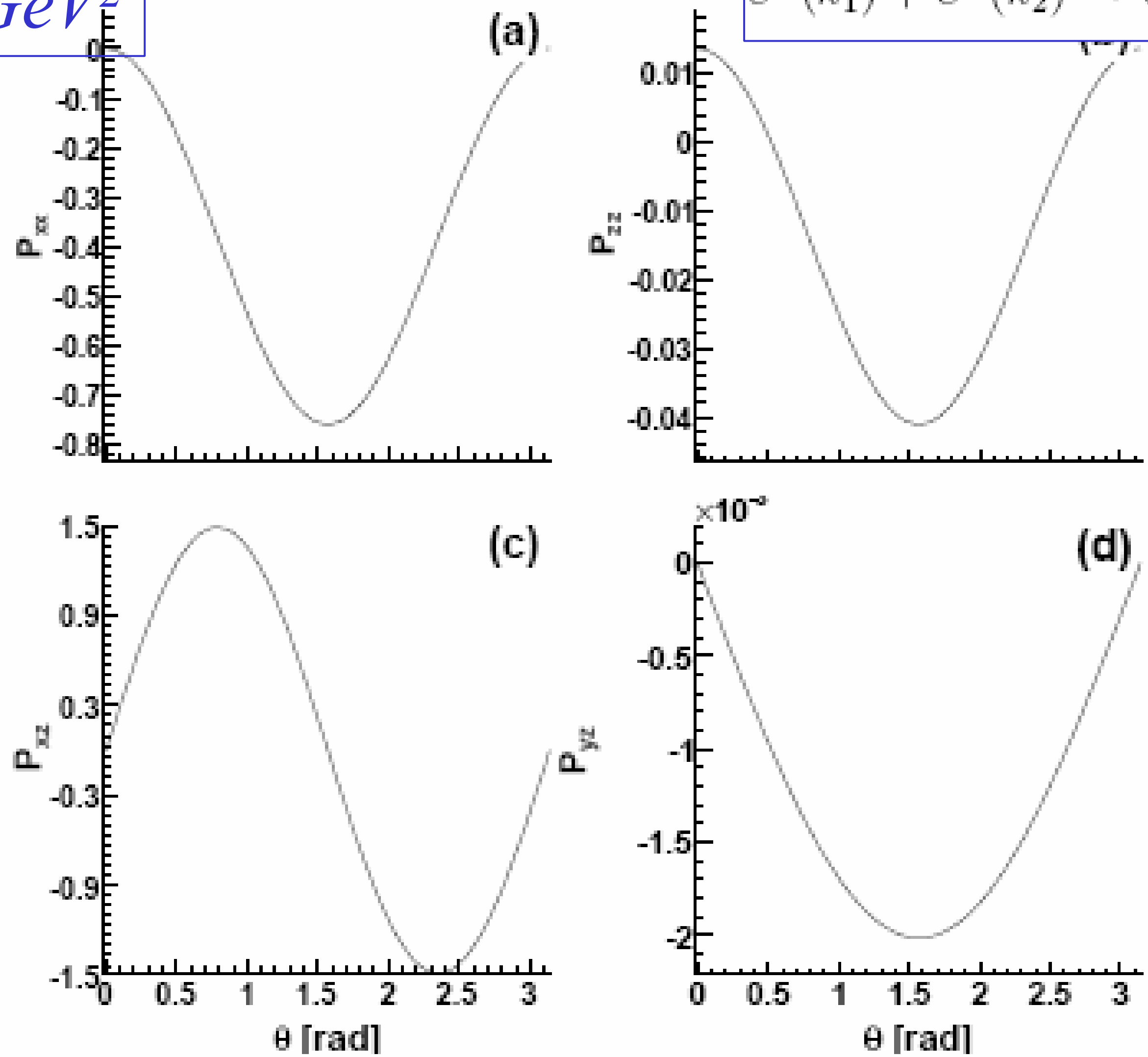
q^2=2 GeV^2



Tensor polarization observables

$q^2 = 2\ GeV^2$

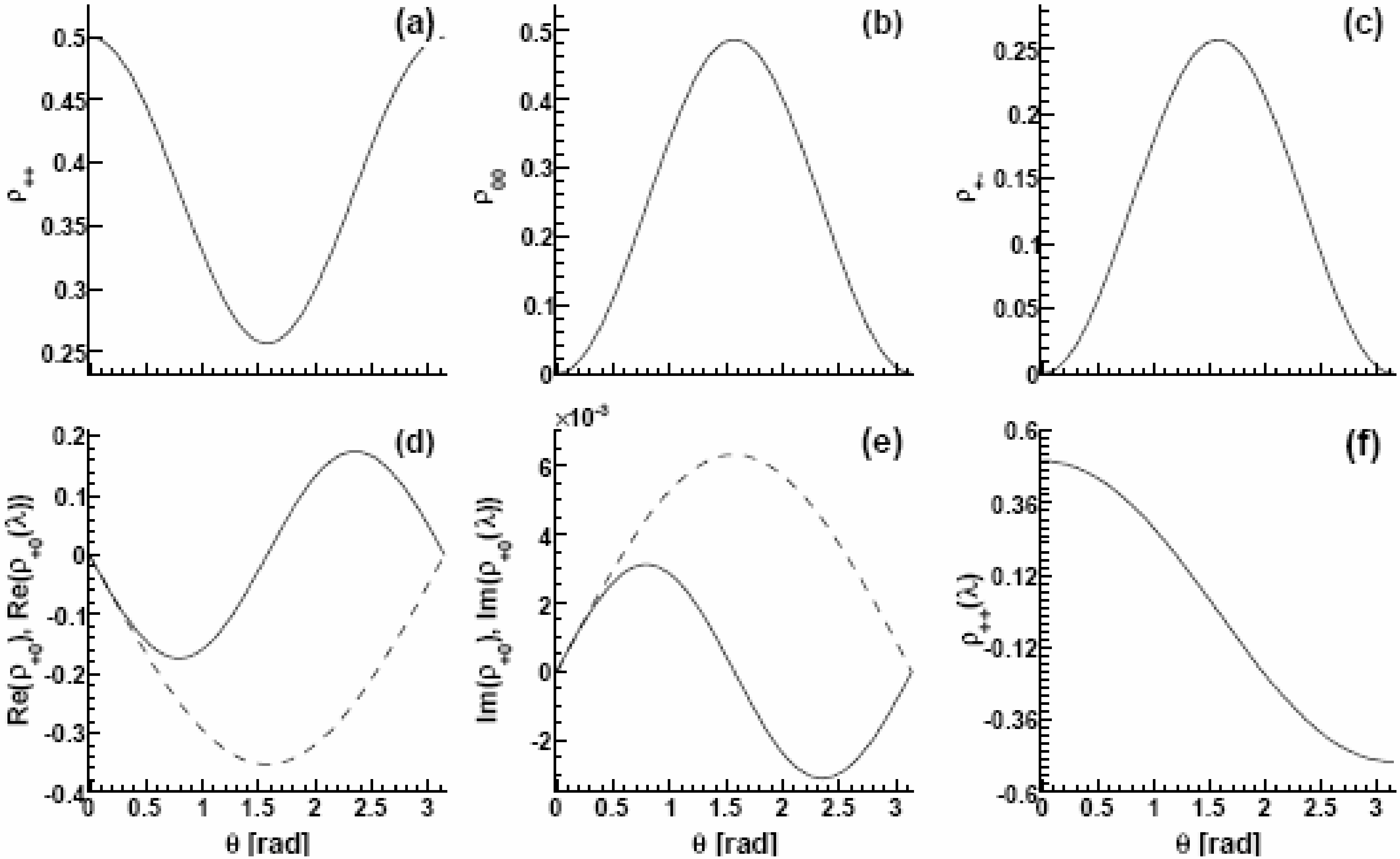
$$e^-(k_1) + e^+(k_2) \rightarrow a_1(p_1) + \pi(p_2)$$



Density matrix

$q^2=2\ GeV^2$

$$S\rho_{mm'}(\lambda) = S_{\mu\nu}(\lambda)U_{\mu}^{(m)}U_{\nu}^{(m')*}$$



Strategy for the complete experiment

1) B/A from unpolarized differential cross section:

$$R_1 = \boxed{\frac{B}{A}} = \frac{p^2}{2M^2}\left[1 + 2r\cos\alpha + \left(1 - \frac{M^2}{q^2}\right) r^2\right]$$
$$\left[1 + \left(1 + \frac{m^2 - M^2}{q^2}\right) r\cos\alpha + \frac{1}{4}\left(1 + \frac{m^2 - M^2}{q^2}\right)^2 r^2\right]^{-1},$$

r, α unknown

$$\boxed{r = |f_2|/|f_1|} \qquad \boxed{\alpha = \alpha_1 - \alpha_2,} \qquad \alpha_1 = \arg f_1 \quad \alpha_2 = \arg f_2.$$

2) Ratio of spin density matrix elements :

$$R_2 = \boxed{\frac{\rho_{00}}{\rho_{+-}}} = 8\tau\left\{\frac{1}{4}(q^2 + M^2 - m^2)^2 + \left[2p^2q^2 + M^2(q^2 - M^2 + m^2)\right] r\cos\alpha + \right.$$
$$\left. +(m^2M^2 + p^2q^2)r^2\right\}\left[q^4 + q^2(q^2 - M^2 + m^2)r\cos\alpha + \frac{1}{4}(q^2 - M^2 + m^2)^2 r^2\right]^{-1}$$

3) Phase α from

$$\boxed{\mathrm{Im}\rho_{+0} = \frac{p^2q^4}{S}\sqrt{\frac{\tau}{2}}\sin 2\theta\, \mathrm{Im} f_1 f_2^*.}$$

No polarized beams required…

Model independent formalism

Annihilation - Scattering

M.P. Rekalo

Spin ½ - Spin 1

One-photon exchange – two photon exchange

$$\bar{p} + p \rightarrow e^+ + e^-$$

$$e^+ + e^- \rightarrow d + \bar{d}$$

$$e^+ + e^- \rightarrow \rho + \rho \rightarrow 4\pi$$

$$e^+ + e^- \rightarrow a_1 + \pi \rightarrow 4\pi$$

$$e^\pm + p(d) \rightarrow e^\pm + p(d)$$

Conclusions

- Model independent Formalism for Scattering and Annihilation
- VDM-like parametrization of form factors extended to Spin 1 particles *(and to axial FF of the proton)*
- Prediction of cross section and polarization observables

$$e^+ + e^- \rightarrow d + \bar{d}$$

$$e^+ + e^- \rightarrow \rho + \rho \rightarrow 4\pi$$

$$e^+ + e^- \rightarrow a_1 + \pi \rightarrow 4\pi$$

in collaboration with G.I. Gakh and C. Adamuscin

Tensor polarization observables

Does not require polarized beams:

$$P_{xx} = -\frac{3}{4}\frac{1}{\sigma_0}\sin^2\theta\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2\right|^2,$$

$$P_{xz} = \frac{3}{4}\frac{\sqrt{\tau}}{\sigma_0}\frac{\sin(2\theta)}{q^2}\left\{2(q^2 + M^2 - m^2)\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2\right|^2 + \right.$$
$$\left. + \left[(q^2 + M^2 - m^2)^2 - 4M^2q^2\right]\left[\frac{1}{2}(q^2 - M^2 + m^2)|f_2|^2 + q^2\mathrm{Re} f_1 f_2^*\right]\right\},$$

$$P_{zz} = \frac{3}{8}\frac{1}{\sigma_0}\frac{1}{M^2q^2}\left[(q^2 + M^2 - m^2)^2 - 4M^2q^2\right]\left\{\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2)f_2\right|^2 + \right.$$
$$+\frac{1}{2}\sin^2\theta\left[-q^4|f_1|^2 + q^2(q^2 + 3M^2 - 3m^2)\mathrm{Re} f_1 f_2^* + \right.$$
$$\left.\left. + \left(2q^2(q^2 - M^2) - \frac{3}{4}(q^2 - M^2 + m^2)^2\right)|f_2|^2\right]\right\}.$$

T-odd observable:

$$P_{yz} = -\frac{3}{2}\frac{\sqrt{\tau}}{\sigma_0}\left[(q^2 + M^2 - m^2)^2 - 4M^2q^2\right]\sin\theta\,\mathrm{Im} f_1 f_2^*.$$

References

eP-elastic scattering

M.P. Rekalo and E.T-G.
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*Nucl.Phys.A **742**, 322 (2004) Nucl.Phys. A**740**, 271 (2004).*

P-Pbar, e+e-

*E. T.-G., F. Lacroix, Ch. Duterte, G.I. Gakh, EPJA **24**, 419 (2005)*
*G.I. Gakh and E.T-G. Nucl.Phys. A **761**,,2005; Nucl.Phys.A**771**:169-183,2006.*

Deuteron FF

*C.Adamuscin, G.I. Gakh and E.T-G., PRC **73**, 045204 (2006)*

D-Dbar

*C. Adamuscin, G.I. Gakh and E.T-G., PRC **74**, 025202 (2006)*

$\rho^+\rho^-$

*C. Adamuscin, G.I. Gakh and E.T-G., PRC **75**, 065202 (2007)*

$a_1\pi$

*C. Adamuscin, G.I. Gakh and E.T-G., PRC **77,** 065214 (2008)*

Axial form factor of the proton

*C. Adamuscin, G.I. Gakh and E.T-G., PRC **78**,035201 (2008)*

Density matrix

$$\rho_{++} = \rho_{--} = \frac{q^2}{2S}(1+\cos^2\theta)\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2,$$

$$\rho_{00} = \frac{q^4}{M^2 S}\sin^2\theta\left\{\frac{1}{4}(q^2+M^2-m^2)^2|f_1|^2 + \left[2p^2q^2 + M^2(q^2-M^2+m^2)\right]\mathrm{Re} f_1 f_2^* + \right.$$
$$\left. +(m^2M^2+p^2q^2)|f_2|^2\right\},$$

$$\rho_{+-} = \rho_{-+} = \frac{q^2}{2S}\sin^2\theta\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2,$$

$$\rho_{+0} = -\frac{q^4}{S}\sqrt{\frac{\tau}{2}}\sin\theta\cos\theta\left\{(q^2+M^2-m^2)|f_1|^2 + (1-\frac{M^2-m^2}{q^2})\left[(q^2+M^2-m^2)\mathrm{Re} f_1 f_2^* + \right.\right.$$
$$\left.\left. + \frac{1}{2}(q^2-M^2-m^2)|f_2|^2\right] + 2p^2 f_2 f_1^*\right\},$$

$$\rho_{-0} = -\rho_{+0},\ \rho_{0+} = \rho_{+0}^*,\ \rho_{0-} = \rho_{-0}^*.$$

$$Tr\rho = 1 \text{ or } \rho_{++} + \rho_{--} + \rho_{00} = 1.$$

$$\mathrm{Re}\rho_{+0} = -\frac{q^4}{S}\sqrt{\frac{\tau}{2}}\sin\theta\cos\theta\left\{\left(q^2+M^2-m^2\right)|f_1|^2 + \frac{1}{2}\left(1-\frac{M^2-m^2}{q^2}\right)\right.$$
$$\left.\left(q^2-M^2-m^2\right)|f_2|^2 + \frac{1}{2}\left[3q^2 - 2M^2 - 2m^2 - \frac{(M^2-m^2)^2}{q^2}\right]\mathrm{Re} f_1 f_2^*\right\},$$

$$\mathrm{Im}\rho_{+0} = \frac{p^2q^4}{S}\sqrt{\frac{\tau}{2}}\sin 2\theta\,\mathrm{Im} f_1 f_2^*.$$

Longitudinally polarized electrons

Irfu cea saclay

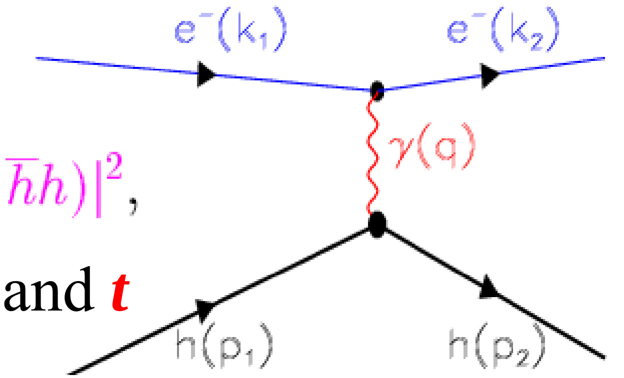
$$S\rho_{mm'}(\lambda) = S_{\mu\nu}(\lambda) U_\mu^{(m)} U_\nu^{(m')*},$$

$$\begin{aligned}
\rho_{++}(\lambda) &= -\rho_{--}(\lambda) = \frac{\lambda}{S} q^2 \cos\theta \left| q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2 \right|^2, \\
\rho_{00}(\lambda) &= \rho_{+-}(\lambda) = \rho_{-+}(\lambda) = 0, \\
\rho_{+0}(\lambda) &= -\frac{\lambda}{S}\sqrt{\frac{\tau}{2}} q^4 \sin\theta \left\{ (q^2 + M^2 - m^2)|f_1|^2 + \right. \\
&\left. + \left(1 - \frac{M^2 - m^2}{q^2}\right)\left[(q^2 + M^2 - m^2)\mathrm{Re} f_1 f_2^* + \frac{1}{2}(q^2 - M^2 - m^2)|f_2|^2 \right] + 2p^2 f_2 f_1^* \right\} \\
\rho_{0+}(\lambda) &= \rho_{+0}^*(\lambda), \;\; \rho_{-0}(\lambda) = \rho_{+0}(\lambda), \;\; \rho_{0-}(\lambda) = \rho_{-0}^*(\lambda).
\end{aligned}$$

$$\begin{aligned}
\mathrm{Re}\rho_{+0}(\lambda) &= -\frac{\lambda}{S}\sqrt{\frac{\tau}{2}} q^4 \sin\theta \left\{ (q^2 + M^2 - m^2)|f_1|^2 + \frac{1}{2}\left(1 - \frac{M^2 - m^2}{q^2}\right)(q^2 - M^2 - m^2)|f_2|^2 + \right. \\
&\left. + \frac{1}{2}\left[(3q^2 - 2M^2 - 2m^2 - \frac{(M^2 - m^2)^2}{q^2} \right] \mathrm{Re} f_1 f_2^* \right\}, \\
\mathrm{Im}\rho_{+0}(\lambda) &= \frac{\lambda}{S}\sqrt{2\tau} p^2 q^4 \sin\theta \, \mathrm{Im} f_1 f_2^*.
\end{aligned}$$

Crossing Symmetry

$e^- + h \to e^- + h$



Scattering and annihilation channels:

- Described by the same amplitude :

$$|\overline{\mathcal{M}}(e^{\pm}h \to e^{\pm}h)|^2 = f(s,t) = |\overline{\mathcal{M}}(e^+e^- \to \overline{h}h)|^2,$$

- function of two kinematical variables, ***s*** and ***t***

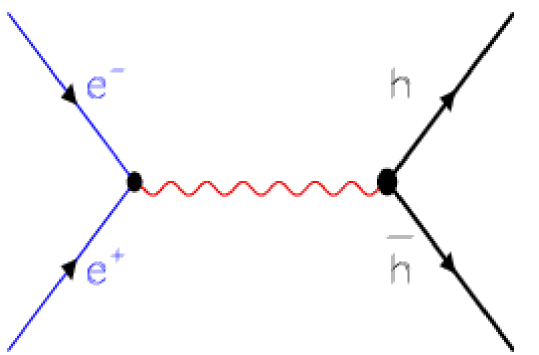
$$s = (k_1 + p_1)^2$$
$$t = (k_1 - k_2)^2$$

$e^- + e^+ \to \overline{h} + h$

- which scan different kinematical regions

$$k_2 \longrightarrow -k_2$$
$$p_2 \longrightarrow -p_2$$

$$cos^2\tilde{\theta} = 1 + \frac{st + (s - M^2)^2}{t(\frac{t}{4} - M^2)} \to 1 + \frac{ctg^2\frac{\theta}{2}}{1+\tau}$$



Density matrix

$$\rho_{++} = \rho_{--} = \frac{q^2}{2S}(1+\cos^2\theta)\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2,$$

$$\rho_{00} = \frac{q^4}{M^2 S}\sin^2\theta \left\{\frac{1}{4}(q^2+M^2-m^2)^2|f_1|^2 + \left[2p^2q^2 + M^2(q^2-M^2+m^2)\right]\mathrm{Re} f_1 f_2^* + \right.$$
$$\left. + (m^2M^2 + p^2q^2)|f_2|^2\right\},$$

$$\rho_{+-} = \rho_{-+} = \frac{q^2}{2S}\sin^2\theta \left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2,$$

$$\rho_{+0} = -\frac{q^4}{S}\sqrt{\frac{\tau}{2}}\sin\theta\cos\theta\left\{(q^2+M^2-m^2)|f_1|^2 + (1 - \frac{M^2-m^2}{q^2})\left[(q^2+M^2-m^2)\mathrm{Re} f_1 f_2^* + \right.\right.$$
$$\left.\left. + \frac{1}{2}(q^2-M^2-m^2)|f_2|^2\right] + 2p^2 f_2 f_1^*\right\},$$

$$\rho_{-0} = -\rho_{+0},\ \rho_{0+} = \rho_{+0}^*,\ \rho_{0-} = \rho_{-0}^*.$$

$$Tr\rho = 1 \text{ or } \rho_{++} + \rho_{--} + \rho_{00} = 1.$$

$$\mathrm{Re}\rho_{+0} = -\frac{q^4}{S}\sqrt{\frac{\tau}{2}}\sin\theta\cos\theta\left\{\left(q^2+M^2-m^2\right)|f_1|^2 + \frac{1}{2}\left(1-\frac{M^2-m^2}{q^2}\right)\right.$$
$$\left.\left(q^2-M^2-m^2\right)|f_2|^2 + \frac{1}{2}\left[3q^2 - 2M^2 - 2m^2 - \frac{(M^2-m^2)^2}{q^2}\right]\mathrm{Re} f_1 f_2^*\right\},$$

$$\mathrm{Im}\rho_{+0} = \frac{p^2q^4}{S}\sqrt{\frac{\tau}{2}}\sin 2\theta\,\mathrm{Im} f_1 f_2^*.$$

Polarized density matrix

Longitudinally polarized electrons

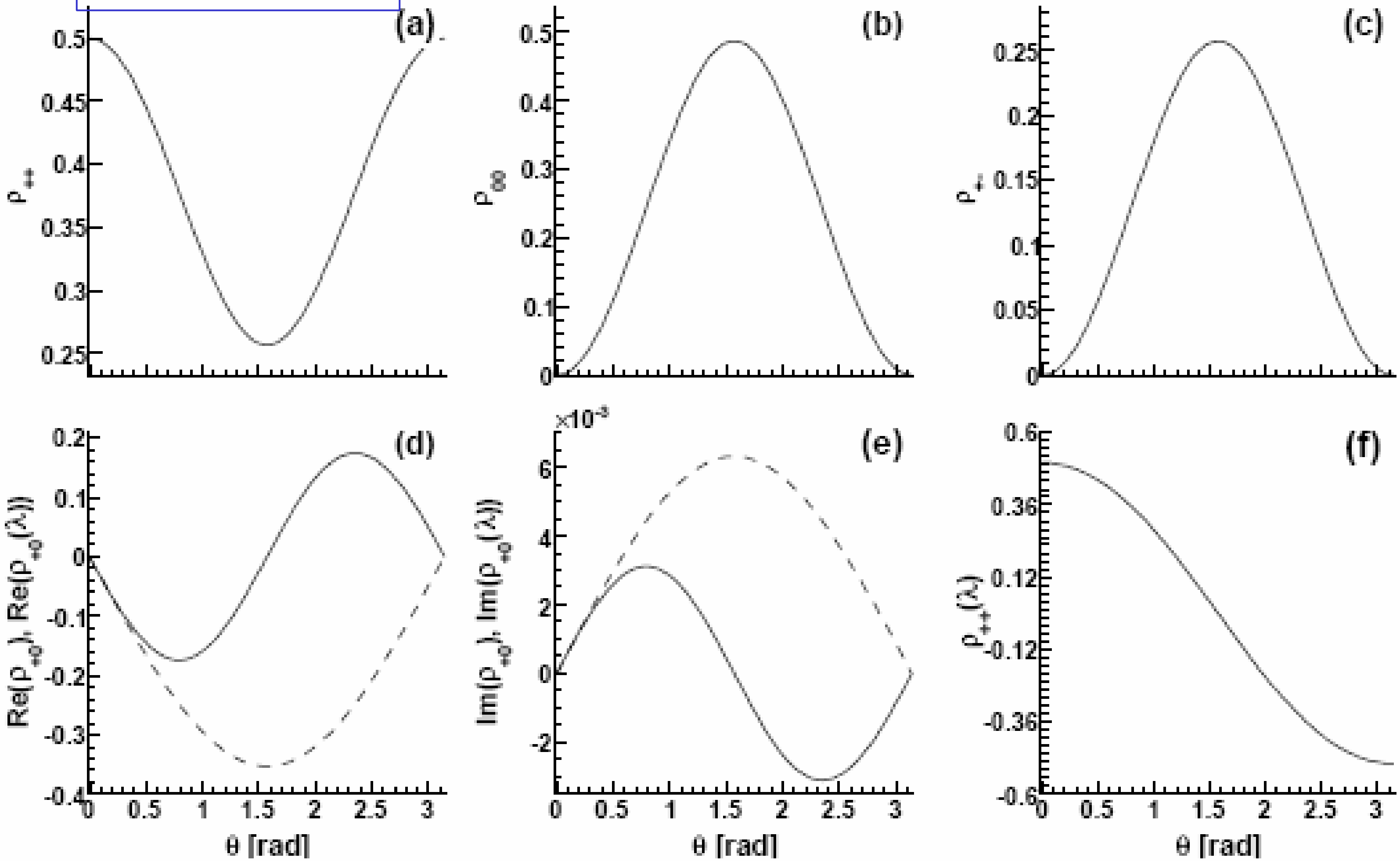
$$S\rho_{mm'}(\lambda) = S_{\mu\nu}(\lambda) U_\mu^{(m)} U_\nu^{(m')*},$$

$$\begin{aligned}
\rho_{++}(\lambda) &= -\rho_{--}(\lambda) = \frac{\lambda}{S} q^2 \cos\theta \left| q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2 \right|^2 , \\
\rho_{00}(\lambda) &= \rho_{+-}(\lambda) = \rho_{-+}(\lambda) = 0, \\
\rho_{+0}(\lambda) &= -\frac{\lambda}{S}\sqrt{\frac{\tau}{2}} q^4 \sin\theta \left\{ (q^2 + M^2 - m^2)|f_1|^2 + \right. \\
&+ \left(1 - \frac{M^2 - m^2}{q^2}\right) \left[(q^2 + M^2 - m^2)\mathrm{Re} f_1 f_2^* + \frac{1}{2}(q^2 - M^2 - m^2)|f_2|^2 \right] \left. + 2p^2 f_2 f_1^* \right\} \\
\rho_{0+}(\lambda) &= \rho_{+0}^*(\lambda),\ \rho_{-0}(\lambda) = \rho_{+0}(\lambda),\ \rho_{0-}(\lambda) = \rho_{-0}^*(\lambda).
\end{aligned}$$

$$\begin{aligned}
\mathrm{Re}\rho_{+0}(\lambda) &= -\frac{\lambda}{S}\sqrt{\frac{\tau}{2}} q^4 \sin\theta \left\{ (q^2 + M^2 - m^2)|f_1|^2 + \frac{1}{2}\left(1 - \frac{M^2 - m^2}{q^2}\right)(q^2 - M^2 - m^2)|f_2|^2 + \right. \\
&\left. + \frac{1}{2}\left[(3q^2 - 2M^2 - 2m^2 - \frac{(M^2 - m^2)^2}{q^2} \right] \mathrm{Re} f_1 f_2^* \right\}, \\
\mathrm{Im}\rho_{+0}(\lambda) &= \frac{\lambda}{S}\sqrt{2\tau} p^2 q^4 \sin\theta \mathrm{Im} f_1 f_2^* .
\end{aligned}$$

Density matrix

$q^2=2\ GeV^2$



Tensor polarization observables

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Does not require polarized beams:

$$P_{xx} = -\frac{3}{4}\frac{1}{\sigma_0}\sin^2\theta\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2,$$

$$P_{xz} = \frac{3}{4}\frac{\sqrt{\tau}}{\sigma_0}\frac{\sin(2\theta)}{q^2}\left\{2(q^2 + M^2 - m^2)\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2 + \right.$$

$$\left. + \left[(q^2 + M^2 - m^2)^2 - 4M^2 q^2\right]\left[\frac{1}{2}(q^2 - M^2 + m^2)|f_2|^2 + q^2 \mathrm{Re} f_1 f_2^*\right]\right\},$$

$$P_{zz} = \frac{3}{8}\frac{1}{\sigma_0}\frac{1}{M^2 q^2}\left[(q^2 + M^2 - m^2)^2 - 4M^2 q^2\right]\left\{\left|q^2 f_1 + \frac{1}{2}(q^2 - M^2 + m^2) f_2\right|^2 + \right.$$

$$+ \frac{1}{2}\sin^2\theta\left[-q^4|f_1|^2 + q^2(q^2 + 3M^2 - 3m^2)\mathrm{Re} f_1 f_2^* + \right.$$

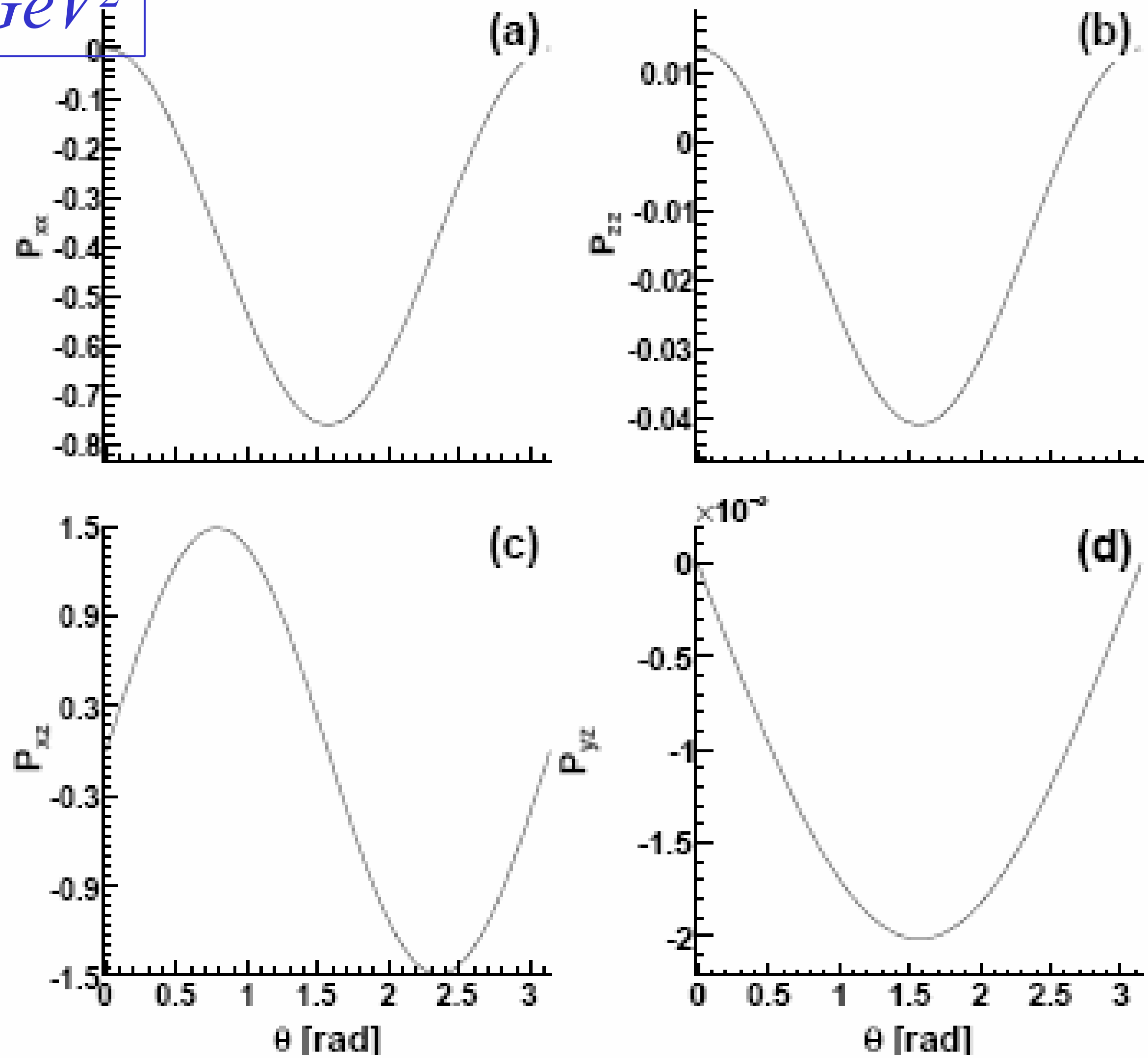
$$\left.\left. + \left(2q^2(q^2 - M^2) - \frac{3}{4}(q^2 - M^2 + m^2)^2\right)|f_2|^2\right]\right\}.$$

T-odd observable:

$$P_{yz} = -\frac{3}{2}\frac{\sqrt{\tau}}{\sigma_0}\left[(q^2 + M^2 - m^2)^2 - 4M^2 q^2\right]\sin\theta \,\mathrm{Im} f_1 f_2^*.$$

Tensor polarization observables

$q^2=2\ GeV^2$

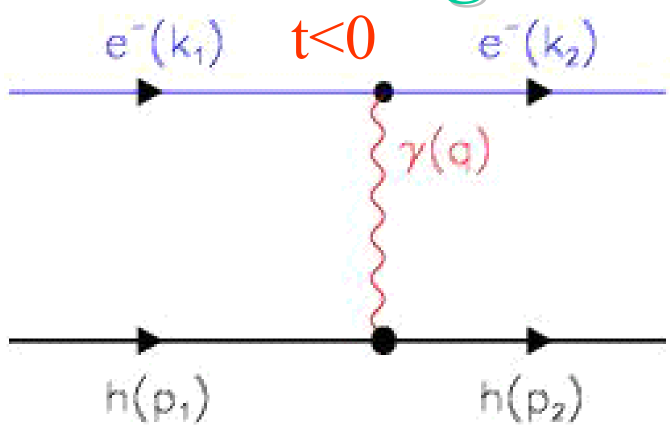


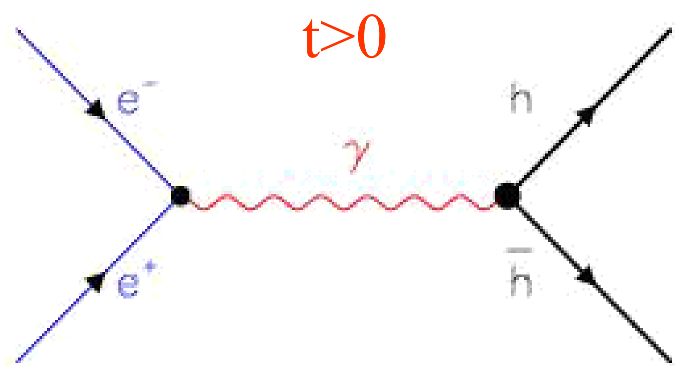
Space-like and Time-like regions

FFs are analytical functions: : ***real in SL region,***
complex in TL region

In framework of one photon exchange, FFs are functions of the momentum transfer squared of the virtual photon, $t = q^2 = -Q^2$.

Scattering | *Annihilation*





$e^- + h \Rightarrow e^- + h$

$e^+ + e^- \Rightarrow \bar{h} + h$

Relative phase accessible ONLY through POLARIZATION