

# T-odd distributions, Formfactors and Duality

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# Outline

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- Duality between quarks and hadrons and between quarks and quarks (= various QCD factorization mechanisms)
- Single Spin Asymmetries in QCD - Sources of (I)FSI
- Non-universality of Sivers function: Colour correlations
- Sivers function and time-like formfactors
- Conclusions



# Duality in QCD

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- Quark-hadron (soliton ...)
- Various factorization mechanisms (“quark-quark” duality; “matching”, ...) - also talk of I. Anikin
- Single Spin Asymmetries: Sivers function (cf talk of I. Cherednikov)
- Dual to twist 3 correlations (small large  $p_T$ )
- Time-like Formfactors – large  $x$  - important for lower energies/forward SSA (cf talk of K. Tanida)



# Single Spin Asymmetries

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Main properties:

- Parity: transverse polarization
- Imaginary phase – can be seen T-invariance or technically - from the imaginary  $i$  in the (quark) density matrix

Various mechanisms – various sources of phases



# Phases in QCD

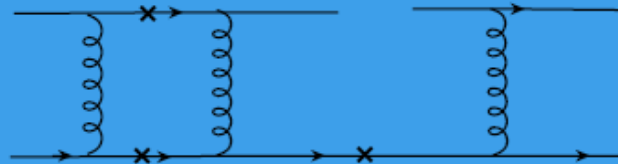
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- QCD factorization – soft and hard parts-
- Phases form soft, hard and overlap
- Assume (generalized) optical theorem – phase due to on-shell intermediate states – positive kinematic variable (= their invariant mass)
- Hard: Perturbative (a la QED: Barut, Fronsdal (1960):  
Kane, Pumplin, Repko (78) Efremov (78)

# Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like  $q - e$  scattering in DIS):

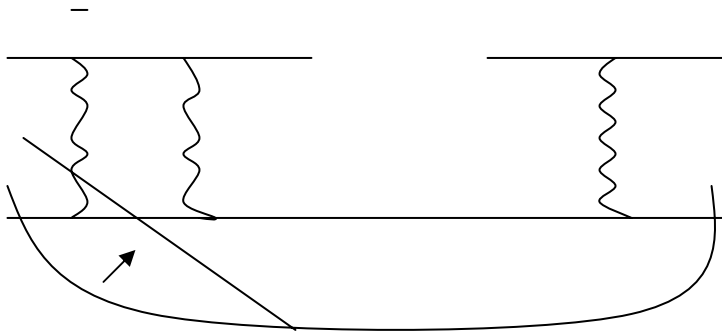


$$A \sim \frac{\alpha_S^{m_{PT}}}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

# Short+ large overlap– twist 3

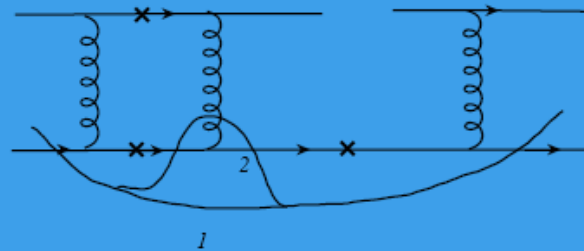
- Quarks – only from hadrons
- Various options for factorization – shift of SH separation (prototype of duality)



- New option for SSA: Instead of 1-loop twist 2  
– Born twist 3: Efremov, OT (85, Fermion poles); Qiu, Sterman (91, GLUONIC poles)

# Twist 3 correlators

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop  $\rightarrow$  Born diagram

At Large distances - quark distribution  $\rightarrow$  quark-gluon correlator.

Physically - process proceeds in the external gluon field of the hadron.

Leads to the shift of  $\alpha_S$  to non-perturbative domain AND

"Renormalization" of quark mass in the external field up to an order of hadron's one

$$\frac{\alpha_S m p_T}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.





# Phases in QCD-Large distances in distributions

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- Distributions: Sivers, Boer and Mulders – no positive kinematic variable producing phase
- QCD: Emerge only due to (initial of final state) interaction between hard and soft parts of the process
- Brodsky -Hwang-Schmidt model: the same SH interactions as twist 3 but non-suppressed by  $Q$ : Sivers function – leading (twist 2).
- Are they related?



# 5 ways from Sivers to twist 3

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- Twist 3 DY - “Effective” or “non-universal” T-odd quark distribution from GP (Boer, Mulders, OT, 97)
- Moment of SF – GP (Boer, Mulders, Pijlman, 03)
- Explicit calculation of SIDVCS for  $Q \gg P_T$  (OT, TRANSVERSITY-05) - compensation of  $1/Q$  suppression by GP)
- Matching of perturbative SF and twist 3 for DY, SIDIS +... (Ji, Qiu, Vogelsang, Yuan, 06; Bachetta, Boer, Diehl, Mulders, 08)
- SF at large  $P_T$  (Ratcliffe, OT, 07)-proof of Torino GPM modified by colour factors
- Follows general line of factorization – all UV to hard part. Also a way to QCD evolution (cf talks of I. Cherednikov, L. Gamberg)?!

# What is “Leading” twist?

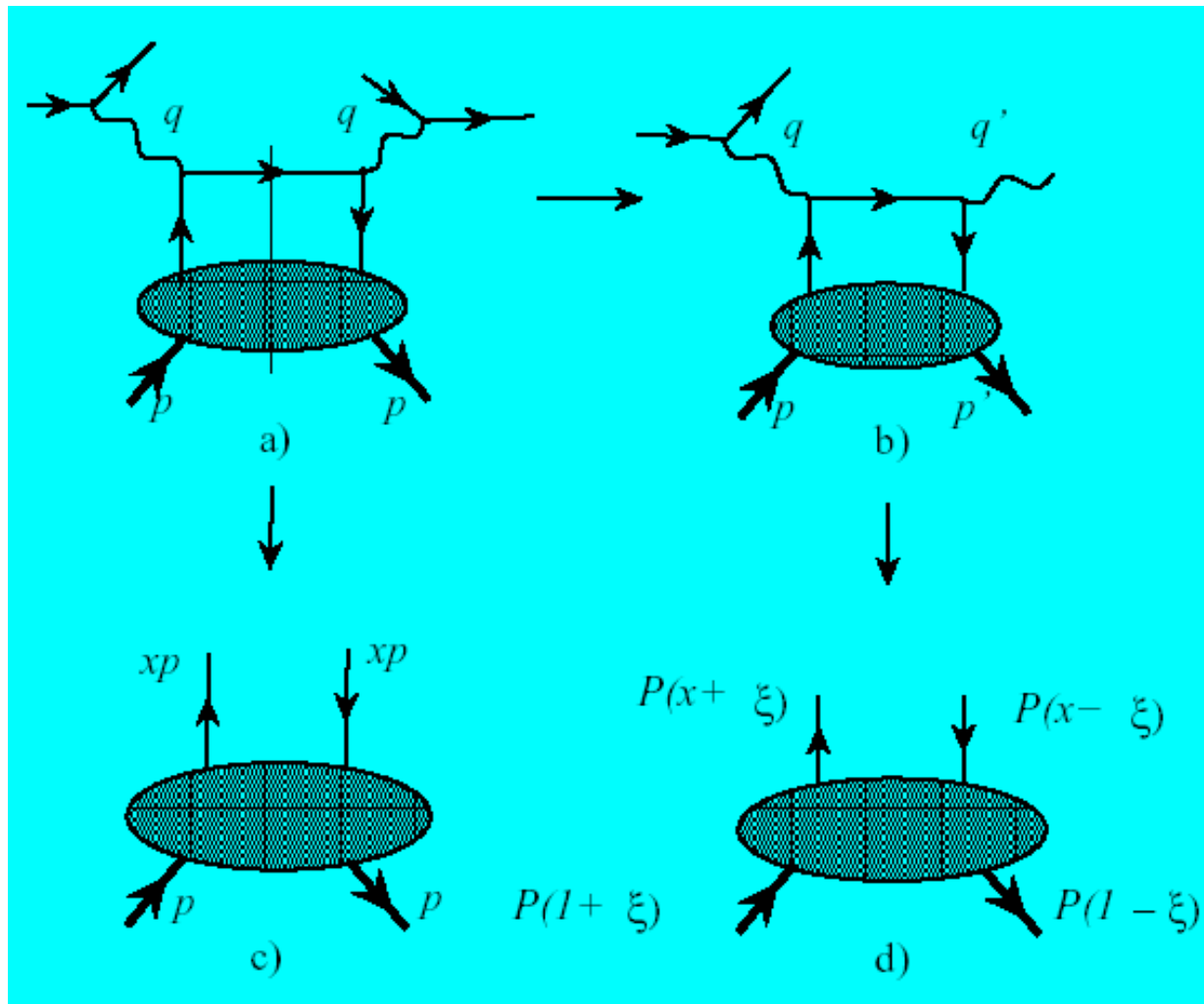
- Practical Definition - Not suppressed as  $M/Q$
- However – More general definition: Twist 3 may be suppressed

as  $M/P_T$

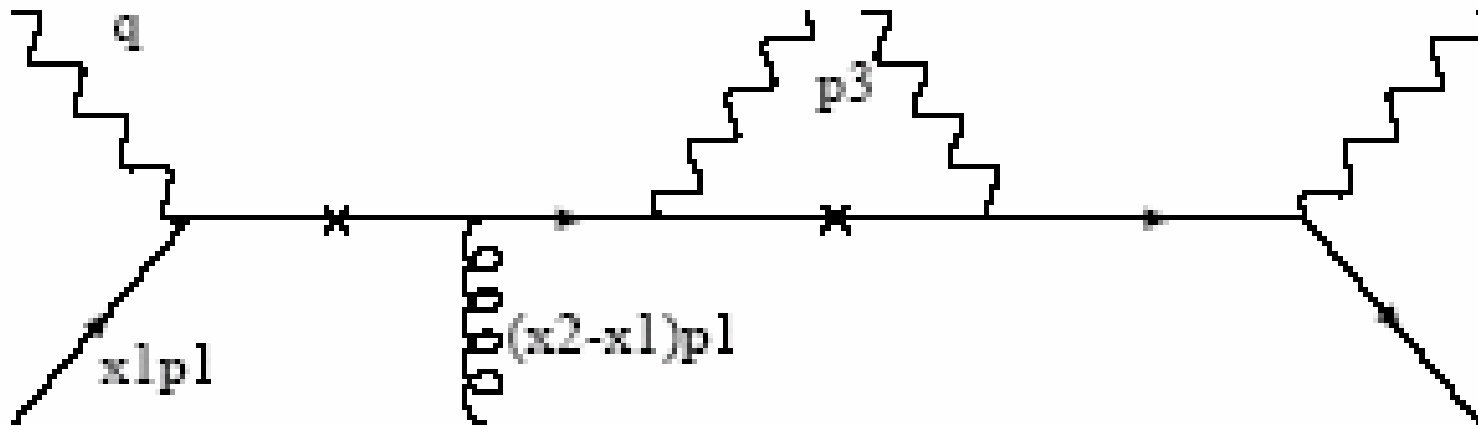
Twist 3 may contribute at leading order in  $1/Q$  !

Does this happen indeed?? – Explicit calculation for the case when  $Q \gg P_T$

# Final Pion $\rightarrow$ Photon: SIDIS $\rightarrow$ SIDVCS (clean, easier than exclusive) - analog of DVCS



# Twist 3 partonic subprocesses for SIDVCS



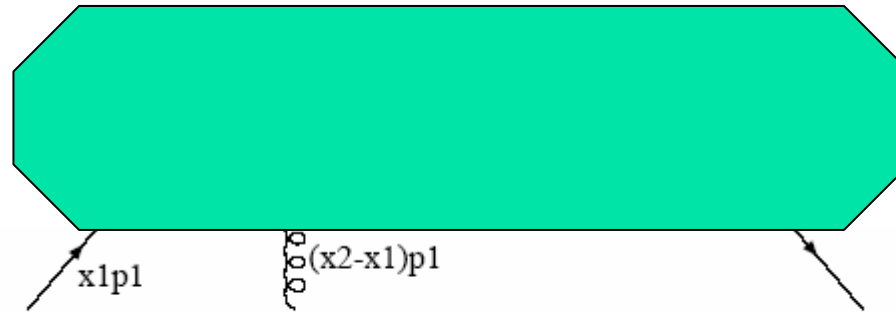


# Real and virtual photons - most clean tests of QCD

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- Both initial and final – real :Efremov, O.T. (85)
- Initial – quark/gluon, final - real : Efremov, OT (86, fermionic poles); Qui, Sterman (91, GLUONIC poles)
- Initial - real, final-virtual (or quark/gluon) – Korotkiian, O.T. (94)
- Initial –virtual, final-real: O.T., Srednyak (05; smooth transition from fermionic via hard to GLUONIC poles).

# Quark-gluon correlators



- Non-perturbative NUCLEON structure – physically mean the quark scattering in external gluon field of the HADRON.
- Depend on TWO parton momentum fractions
- For small transverse momenta – quark momentum fractions are close to each other- gluonic pole; probed if :  
 $Q \gg P_T \gg M$

$$x_2 - x_1 = \delta = \frac{p_T^2 x_B}{Q^2 z}$$

# Cross-sections at low transverse momenta:

$$d\sigma_{total} = f(x_{Bj})8Q^2 \frac{x_{Bj}^2(1+(1-y)^2)(1+(1-z)^2)}{y^2z\delta} \quad (12)$$

$$d\sigma_{ax1x2} = b_A(x_{Bj}, x_2)8M_{PT} \frac{x_{Bj}(1+(1-y)^2)(2-z)}{y^2(1-z)\delta} s_T \sin(\phi_s^h) \quad (13)$$

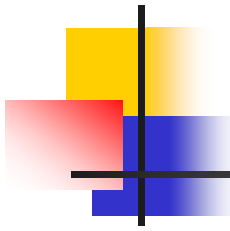
$$d\sigma_{vx1x2} = b_V(x_{Bj}, x_2)8M_{PT} \frac{x_{Bj}(1+(1-y)^2)(1+(1-z)^2)}{y^2z(1-z)\delta} s_T \sin(\phi_s^h) \quad (14)$$

$$d\sigma_{a0x2} = -b_A(0, x_2)8M_{PT} \frac{x_{Bj}^2(2(1-y)(1-2z) + y^2(1-z))}{y^2z^2\delta} s_T \sin(\phi_s^h)$$

(14) - non-suppressed for large Q if Gluonic pole exists=effective Sivvers function; spin-dependent looks like unpolarized (soft gluon)

$$A \propto \frac{2M}{m_T^2} \frac{\mathbf{p}_T \cdot \boldsymbol{\varphi}_V(x_B)}{x_B q(x_B)} s_T \sin \phi_h^s$$





# Other way - NP Sivers and gluonic poles at large PT (P.G. Ratcliffe, OT, [hep-ph/0703293](https://arxiv.org/abs/hep-ph/0703293) )

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- Sivers factorized (general!) expression

$$d\Delta\sigma = \int d^2k_T dx f_S(x, k_T) \text{Tr}[\gamma^\rho H(x, k_T)] \frac{\epsilon^{\rho s P k_T}}{M}$$

- Expand in  $k_T$  = twist 3 part of Sivers

$$d\Delta\sigma = \int dx f_S(x, k_T) \text{Tr} \left[ \gamma^\rho \frac{\partial H(x, k_T = 0)}{\partial k_T^\alpha} k_T^\alpha \right] \epsilon^{\rho s P k_T}$$



# From Sivers to twist 3 - II

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- Angular average :  $\langle k_T^\mu k_T^\nu \rangle = -\frac{g_T^{\mu\nu}}{2} \langle k_T^2 \rangle$

$$g_T^{\mu\nu} = g^{\mu\nu} - P^\mu n^\nu - n^\mu P^\nu$$

- As a result  $d\Delta\sigma = -M \int dx f_S^{(1)}(x) \text{Tr} \left[ \gamma^\rho \frac{\partial H(x, k_T = 0)}{\partial k_T^\alpha} \right]$

$$f_S^{(1)}(x) = \int d^2 k_T f_S(x, k_T) \frac{k_T^2}{2M^2} \quad (\epsilon^{\rho s P \alpha} - P^\alpha \epsilon^{\rho s P n})$$

- M in numerator - sign of twist 3. Higher moments – higher twists. KT dependent function – resummation of higher twists

# From Sivers to gluonic poles -

## III

- Final step – kinematical identity

$$\epsilon^{\rho s P \alpha} = P^{\alpha} \epsilon^{\rho s P n} - P^{\rho} \epsilon^{\alpha s P n}$$

- Two terms are combined to one

$$d\Delta\sigma = M \int dx f_S^{(1)}(x) \text{Tr} \left[ \gamma \cdot P \frac{\partial H(x, k_T = 0)}{\partial k_T^{\alpha}} \right] \epsilon^{\alpha s P n}$$

- Key observation – exactly the form of Master Formula for gluonic poles (Koike et al, 2007)



# Effective Sivers function

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- Follows the expression similar to BMP

$$x f_s^{(1)}(x) = \sum C_i \frac{1}{2M} T_j(x, x),$$

- Up to Colour Factors !
- Defined by colour correlation between partons in hadron participating in (I)FSI
- SIDIS = +1; DY= -1: Collins sign rule
- Generally more complicated
- Factorization in terms of twist 3 but NOT SF



# Colour correlations

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- SIDIS at large  $p_T$  :  $-1/6$  for mesons from quark,  $3/2$  from gluon fragmentation (kaons?)
- DY at large  $p_T$ :  $1/6$  in quark antiquark annihilation,  $-3/2$  in gluon Compton subprocess – Collins sign rule more elaborate – involve crossing of distributions and fragmentations - special role of PION DY (COMPASS).
- **Direct inclusive photons in pp =  $-3/2$**
- Hadronic pion production – more complicated – studied for P-exponentials by Amsterdam group + VW
- IF cancellation – small EFFECTIVE SF
- Vary for different diagrams – modification of hard part
- FSI for pions from quark fragmentation  
 $-1/6 \times (\text{non-Abelian Compton}) + 1/8 \times (\text{Abelian Compton})$

# How to pass from high to low

PT


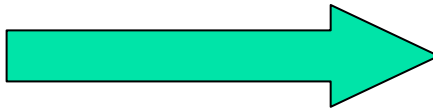

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- Hard poles in correlators (become soft at small PT – c.f. SIDVCS)
- Low pT – cannot distinguish fragmentation from quarks and gluons:  
 $3/2 - 1/6 = 4/3$  (Abelian)
- Strong transverse momentum dependence, very different for mesons from quark and gluon fragmentation



# Colour flow

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- Quark at large PT:  $1/6$  
- Gluon at large PT :  $3/2$  
- Low PT – combination of quark and gluon:  
 $4/3$  (absorbed to definition of Sivers  
function) 
- Similarity to colour transparency  
phenomenon



# Sivers function and formfactors

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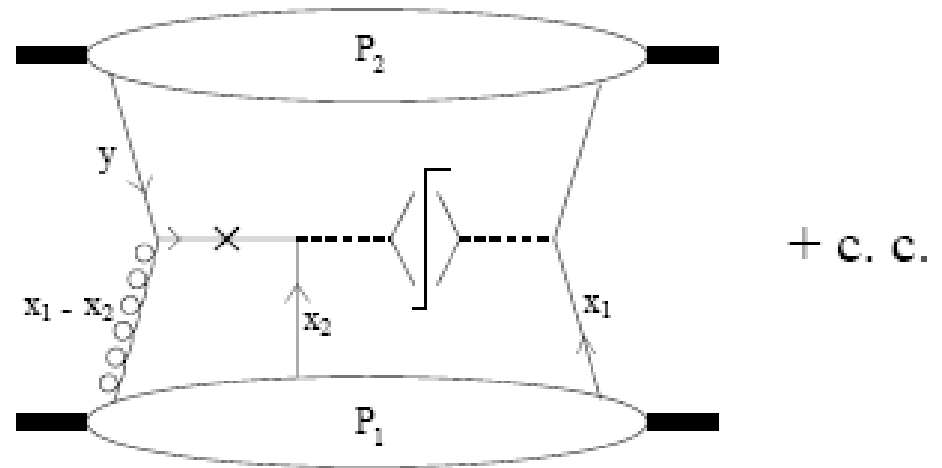
- Relation between Sivers and AMM known on the level of matrix elements (Brodsky, Schmidt, Burkardt)
- Phase?
- Duality for observables?
- Solution: SSA in DY



# SSA in DY

- TM integrated DY with one transverse polarized beam (cf talk of S. Melis)– unique SSA – gluonic pole (Hammon, Schaefer, OT)

$$A = g \frac{\sin 2\theta \cos \phi \left[ T(x, x) - x \frac{dT(x, x)}{dx} \right]}{M [1 + \cos^2 \theta] q(x)}$$





# SSA in exclusive limit

- Proton-antiproton – valence annihilation - cross section is described by Dirac FF squared

$$D = |G_M|^2 (1 + \cos^2 \theta) + \frac{1}{\tau} |G_E|^2 \sin^2 \theta \quad \begin{array}{l} G_M = F_1 + F_2, \\ G_E = F_1 + \tau F_2. \end{array} \quad \tau \equiv q^2 / 4m_B^2 > 1$$

- New kind of duality in time-like region –similar to Drell-Levy-Yan and Bloom-Gilman for DIS
- The same SSA due to interference of Dirac and Pauli FF's with a phase shift

$$\mathcal{P}_y = \frac{\sin 2\theta \operatorname{Im} G_E^* G_M}{D\sqrt{\tau}} = \frac{(\tau - 1) \sin 2\theta \operatorname{Im} F_2^* F_1}{D\sqrt{\tau}}$$

- Exclusive : large  $Q^2 \sim 1/(1-x)$  limit;  $x \rightarrow 1$  :  
 $T(x,x)/q(x) \rightarrow \operatorname{Im} F_2/F_1$  (cf talk of V. Punjabi)
- Sivers/unpolarized  $\sim (1-x)^n$   $n=1-2$





# CONCLUSIONS

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- Dual mechanisms of SSA's
- Effective SF – small in pp - factorization in terms of twist 3 only
- Large  $x$  – relation between SF, GP and time-like FF's
- What do we test? – complementarity (in Bohr/Feynman sense) of various ways of description/degrees of freedom



# Outlook (high energies)

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- TMD vs UGPD
- T-odd UGPD?
- T-odd (P/O) diffractive distributions
- (analogs - also at small energies)
- Quark-hadron duality: description of gluon coupling to exotic objects – diffractive production

# Relation of Sivers function to GPDs

- Qualitatively similar to Anomalous Magnetic Moment (Brodsky et al)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E  
(**hep-ph/0612205**) :  $x f_T(x) \propto xE(x)$
- Burkardt SR for Sivers functions is now related to Ji SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$



# How gravity is coupled to nucleons?

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- Current or constituent quark masses ?—  
neither!
- Energy momentum tensor - like  
electromagnertic current describes the  
coupling to photons



# Equivalence principle

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- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not yet checked
- Anomalous gravitomagnetic moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way





# Gravitational formfactors

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M \right] u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment :  $\mu_G = J$  (g=2)

$$P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity – similar t-dependence to EM FF



# Electromagnetism vs Gravity

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- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$
$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle



# Gravitomagnetism

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- Gravitomagnetic field – action on spin –  $\frac{1}{2}$  from

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

$$\vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i} \quad \text{spin dragging twice smaller than EM}$$

- Lorentz force – similar to EM case: factor  $\frac{1}{2}$  cancelled with 2 from  $h_{00} = 2\phi(x)$

Larmor frequency same as EM  $\vec{H}_L = \text{rot} \vec{g}$

- Orbital and Spin momenta dragging – the same - Equivalence principle  $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$



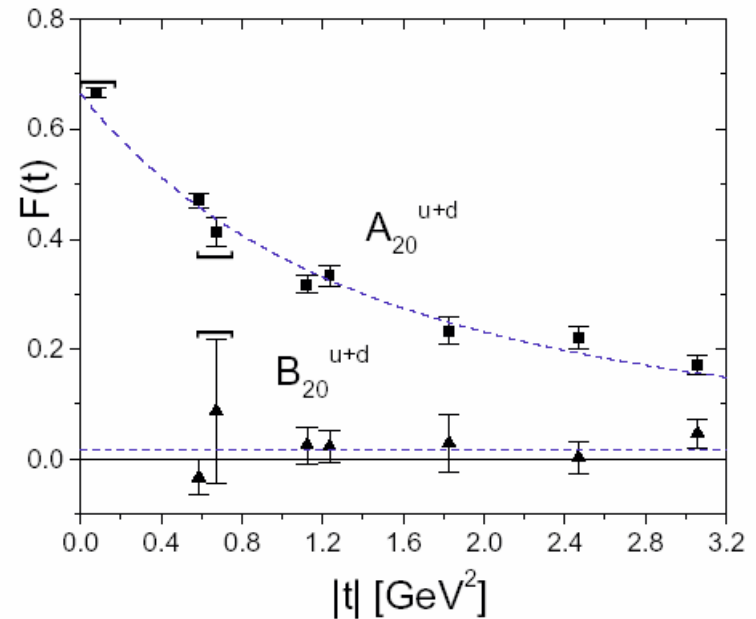
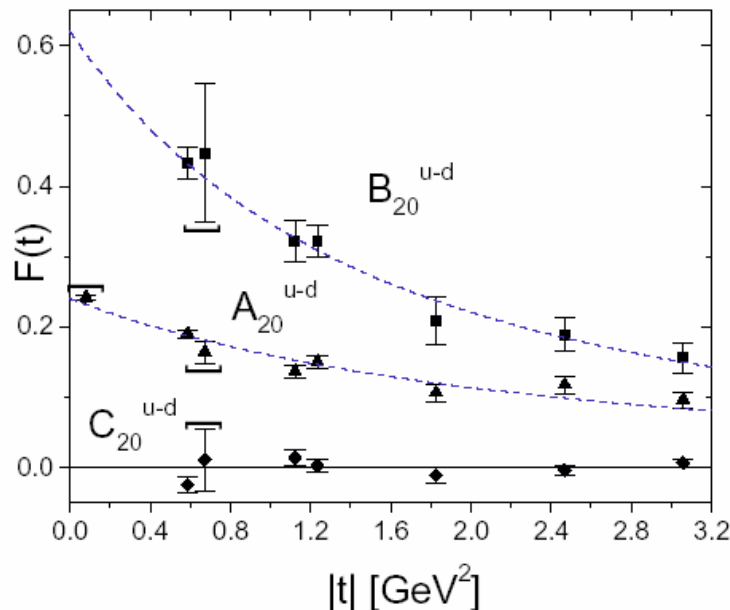
# Sivers function and Extended Equivalence principle

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- Second moment of  $E$  – zero SEPARATELY for quarks and gluons – only in QCD beyond PT (OT, 2001) - supported by lattice simulations etc.. ->
- Gluon Sivers function is small! (COMPASS, STAR, Brodsky&Gardner)
- BUT: gluon orbital momentum is NOT small: total – about  $1/2$ , if small spin – large (longitudinal) orbital momentum
- Gluon Sivers function should result from twist 3 correlator of 3 gluons: remains to be proved!

# Generalization of Equivalence principle

- Various arguments: AGM  $\neq 0$  separately for quarks and gluons – most clear from the lattice (LHPC/SESAM, confirmed recently)





# CONCLUSIONS

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- Sivers and other TMD functions contain infinite tower of twists starting from 3 – special role of moments
- Colour charge of initial/final partons crucial – NO factorization in naive sense
- Transverse momentum dependence of Sivers SSA in SIDIS and DY (PAX) is a new sensitive test of QCD
- Relation of Sivers function to twist 3 in DIS: Reasonable magnitude, but problems with flavor dependence. Bochum results with suppressed singlet twist 3 supported!
- Relation of Sivers to GPD's – link to Nucleon Spin and Equivalence Principle
- Problems: evolution (no WW for Sivers) and SR from twist 3.