KINETICS OF PARTON- ANTIPARTON PLASMA VACUUM CREATION IN THE TIME-DEPENDENT CHROMO- ELECTRIC FIELDS OF ARBITRARY POLARIZATION

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September 30, 2008

XIX International Baldin Seminar

Outline

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- Formula of probability of vacuum pair creation by a constant electric field (Shcwinger, 1951)

$$\omega = \frac{e^2 E^2}{4\pi^3} \exp\{-\frac{E_{cr}}{E}\}, \quad E_{cr} = \frac{m^2 c^3}{e\hbar},$$

J. Schwinger, Phys. Rev. 82, 664 1951

- Critical values of field strength and intensity:

$$E_e = 1.3 \cdot 10^{16} \frac{V}{cm}, \quad I_e \sim 10^{29} \frac{W}{cm^2}, \quad \textit{for electrons}$$
$$E_\pi = 10^{21} \frac{V}{cm}, \quad I_\pi \sim 10^{39} \frac{W}{cm^2}, \quad \textit{for pions}$$

- Laser intensity:
  the achieved level $I \sim 10^{23} \frac{W}{cm^2}$
  prognosis (Paris, ZettaWatt laser) : $I \sim 10^{28} \frac{W}{cm^2}$
- Experimental verification is lacking, because of too high value of the critical field strength.

Outline

- Very different situation occurs when the electric field is time-dependent.
- It was shown in previous works on electron- positron pairs creation. the main density is $n = 10^7 \lambda^{-3}$ or $n = 10^{20} cm^{-3}$. Registration of created plasma – by two- photon annihilation

- In the present work we make the first step on the way of theoretical research of PAP vacuum creation in the nonstationary chromo- electric field of arbitrary polarization. The corresponding kinetic equation (KE) will be derived below on the strict non- perturbation dynamics basis. We will restrict ourselves here by consideration of the nonstationary Schwinger effect in vacuum only leaving in a site the analysis of this effect in some plasma-similar medium. We use the oscillator representation (OR) for the construction of the kinetic theory (initially, this representation was be suggested in the scalar QED (Pervushin et. Al.).
  V.N. Pervushin et. al., Int. J. Mod. Phys. A.2005, V20; hep-ph/0307200.

Oscillator representation

- Let us consider the QED system in the presence of an external quasi-classical spatially homogeneous time-dependent field of arbitrary polarization with the 4-potential $A^{\mu}(t) = (0, \vec{A}(t))$ and the corresponding field strength $\vec{E}(t) = -\dot{\vec{A}}(t)$.

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- The Lagrange function is

$$\mathcal{L} = \frac{i}{2}\{\overline{\psi}\gamma^\mu D_\mu \Psi - (D^*_\mu \overline{\psi})\gamma^\mu \Psi\} - m\overline{\psi}\psi. \tag{1}$$

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- The corresponding equations of motion are

$$\begin{aligned} (i\gamma^\mu D_\mu - m)\psi &= 0, \\ \overline{\psi}(i\gamma^\mu \overleftarrow{D}^*_\mu + m) &= 0. \end{aligned} \tag{2}$$

Oscillator representation

- In the considered case, the system is space homogeneous and nonstationary. Therefore the transition in the Fock space can be realized on the basis functions $\phi = \exp\left(\pm i\mathbf{kx}\right)$ and creation and annihilation operators become the time dependent one, generally speaking. Hence, we have the following decompositions of the field functions in the discrete momentum space :

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$$\psi(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \sum_{\alpha=1,2} \left\{ e^{i\vec{k}\vec{x}} a_\alpha(\vec{k}, t) u_\alpha(\mathbf{k}, t) + e^{-i\mathbf{kx}} b^+_\alpha(\mathbf{k}, t) v_\alpha(\mathbf{k}, t) \right\},$$

$$\bar{\psi}(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \sum_{\alpha=1,2} \{ e^{-i\mathbf{kx}} a^+_\alpha(\mathbf{k}, t) \bar{u}_\alpha(\mathbf{k}, t) + e^{i\mathbf{kx}} b_\alpha(\mathbf{k}, t) \bar{v}_\alpha(\mathbf{k}, t) \}. \quad (3)$$

Oscillator representation

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- Thus, the following "free-like" equations for the spinors are postulated in OR:

$$
\begin{aligned}
[\gamma p - m] u(\mathbf{k}, t) &= 0, \\
[\gamma p^c + m] v(\mathbf{k}, t) &= 0,
\end{aligned}
\tag{4}
$$

where $p^0 = \omega(\mathbf{p}) = \sqrt{m^2 + \mathbf{p}^2}$.

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$$H(t) = \sum_{\mathbf{k},\alpha} \omega(\vec{k}, t) \left[a^{+}_{\alpha}(\mathbf{k}, t) a_{\alpha}(\vec{k}, t) - b_{\alpha}(-\vec{k}, t) b^{+}_{\alpha}(-\vec{k}, t) \right] . \tag{5}$$

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- Such form of the Hamiltonian is necessary for interpretation of the time dependent operators a^{+}, a (and b^{+}, b) as the operators of creation and annihilation of quasi-particles (anti-quasi-particles). Thus, this way results to QP representation at once.

Kinetic equation

- In order to get KE for time dependent electric fields of arbitrary polarization, let us introduce the one particle correlation functions of electrons and positrons

$$
\begin{aligned}
f_{\alpha\beta}(\mathbf{k},t) &= < a^{+}_{\beta}(\mathbf{k},t)a_{\alpha}(\mathbf{k},t) >, \\
f^{c}_{\alpha\beta}(\mathbf{k},t) &= < b_{\beta}(-\mathbf{k},t)b^{+}_{\alpha}(-\mathbf{k},t) >,
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- the auxiliary correlation functions was introduced

$$
\begin{aligned}
f^{(+)}_{\alpha\beta}(\mathbf{k}, t) &= < a^{+}_{\beta}(\mathbf{k}, t) b^{+}_{\alpha}(-\mathbf{k}, t) >, \\
f^{(-)}_{\alpha\beta}(\mathbf{k}, t) &= < b_{\beta}(-\mathbf{k}, t) a_{\alpha}(\mathbf{k}, t) > .
\end{aligned} \tag{7}
$$

Kinetic equation

- Differentiation over time leads to equations

$$\begin{aligned} \dot{f} &= [f, U_{(1)}] - \left(U_{(2)} f^{(+)} + f^{(-)} U_{(2)}\right), \\ \dot{f}^c &= [f^c, U_{(1)}] + \left(f^{(+)} U_{(2)} + U_{(2)} f^{(-)}\right), \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{f}^{(+)} &= [f^{(+)}, U_{(1)}] + \left(U_{(2)} f - f^c U_{(2)}\right) + 2i\omega f^{(+)}, \\ \dot{f}^{(-)} &= [f^{(-)}, U_{(1)}] + \left(f U_{(2)} - U_{(2)} f^c\right) - 2i\omega f^{(-)}, \end{aligned} \quad (9)$$

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- The Eqs.(8) and(9) represent the closed system of 16 ordinary differential equations. Accounting of charge symmetry allows to reduce this number up to 12.

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$$\dot{f}(t) = [f(t), U_{(1)}] - U_{(2)}(t)S(t)\int_{t_0}^{t} dt' S^{+}(t')[U_{(2)}(t')f(t') - f^{c}(t')U_{(2)}(t')]S(t')S^{+}(t')e^{2i\theta(t,t')}$$

$$- S(t)\int_{t_0}^{t} dt' S^{+}(t')[f(t')U_{(2)}(t') - U_{(2)}(t')f^{c}(t')] \cdot S(t')S^{+}(t')U_{(2)}(t)e^{-2i\theta(t,t')}, \quad (10)$$

Kinetic equation

- In comparison with the KE for the known case of the linear polarized field

$$\mathbf{A}(t) = \{0, 0, A^3(t) = A(t)\}, \tag{11}$$

KE (10) has more complicated form because nontrivial spin effects.

Perturbation theory

- Let us write the source term (the right hand side) of KE (10) in the leading (second) order of the perturbative theory with respect to weak external field. It means, that the adiabaticity parameter [Popov] is large

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$$\gamma = \frac{m\nu}{eE_m} \gg 1, \tag{12}$$

Perturbation theory

- In the considering leading approximation, the diagonal terms of the correlation functions U if small in comparison with unit, $f_{\alpha\alpha}$ and the non-diagonal terms $f_{\alpha\beta} \sim E^2$ for $\alpha \neq \beta$, that allows to omit the corresponding contribution in the source term,

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  then

$$\dot{f}(t) = \int_{t_0}^{t} Sp\{U^0_{(2)}(t) U^0_{(2)}(t')\} \cos 2\theta(t, t'). \qquad (13)$$

Perturbation theory

- As it follows from correlation functions U_i

$$Sp\{U^0_{(2)}(t)U^0_{(2)}(t')\} = \frac{e^2}{2\omega_0^4}\left\{\mathbf{E}(t)\mathbf{E}(t')\omega_0^2(\mathbf{E}(t)\mathbf{p})(\mathbf{E}(t')\mathbf{p})\right\} = \Phi(\mathbf{p}|t,t'). \qquad (14)$$

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- If at the initial time before switch-on of an electric field the particles are absent, we can write the total density

$$n(t) = \frac{1}{4\pi^3}\int d^3p \int\limits_{t_0}^{t} dt_1 \int\limits_{t_0}^{t_1} dt_2 \Phi(\mathbf{p}|t_1,t_2)\cos\left[2\omega_0(t_1-t_2\right]. \qquad (15)$$

Perturbation theory

- In the case of the linear polarization (11), from Eqs. (14) and (15) it follows the well known result [Prozorkevich et. al., Bulanov et.al.]:

$$n(t) = \frac{1}{4\pi^3} \int d^3p \left| \int\limits_{t_0}^{t} dt' \lambda(t') \exp\left(2i\omega_0(t - t')\right) \right|^2, \qquad (16)$$

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where $\lambda(\mathbf{p}, t) = eE(t0\varepsilon_\perp/2\omega_0^2$ and $\varepsilon_\perp^2 = m_2 + p_\perp^2$, $\mathbf{p}_\perp$ is the transversal momentum relatively of the vector $\mathbf{E}(t)$.

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- The relations (14) and (15) are convenient for the numerical analysis.

Conclusion

Thus, it was shown that the oscillator representation may be used in the spinor QED for the KE derivation in the rather non-trivial case of the time-dependent electric field of arbitrary polarization. The obtained KE's can be used for investigation of EPP creation in strong laser fields of optical and X-ray range. The used method opens prospects for further generalization (e.g., the account of a constant magnetic field).