

# Study of Nuclear Suppression at Large Forward Rapidities in d+Au Collisions at RHIC

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# Outline

## Reactions at large $x_F$

High- $p_T$  hadrons at large  $\eta$

Coherence effects

$J/\psi$  production

Leading hadrons with small  $p_T$

## Multiple scattering approach

Leading hadrons with small  $p_T$

Comparison with RHIC data

Predictions for RHIC

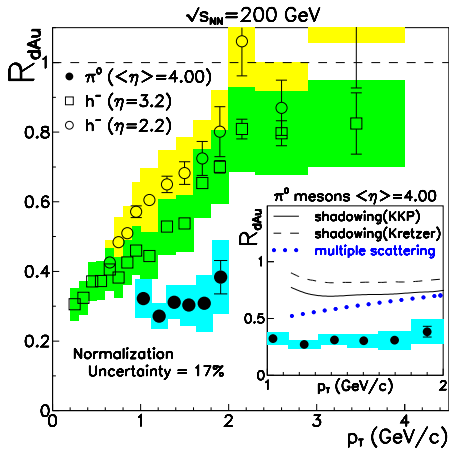
## Summary

## Back up slides

# *Reactions at large $x_F$*

# Hadron production at large $\eta$ at RHIC

$$R_{dAu}^Y = \frac{\sigma_{inel}^{pp}}{\langle N_{bin} \rangle \sigma_{had}^{dAu}} \frac{Ed^3\sigma/dp^3(d+Au \rightarrow Y+X)}{Ed^3\sigma/dp^3(p+p \rightarrow Y+X)}$$



Nuclear modification factor for hadrons in  
 $d + Au$  collisions at  $\sqrt{s_{NN}} = 200 \text{ GeV}$

BRAHMS; Phys.Rev.Lett. 91, 072305 (2003)

BRAHMS; Phys.Rev.Lett. 93, 242303 (2004)

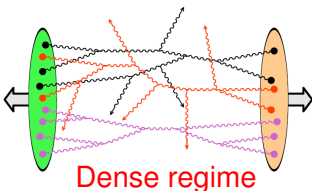
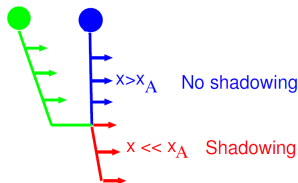
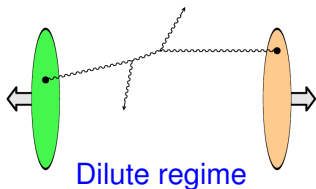
STAR; Phys.Rev.Lett. 97, 152302 (2006)

- ▶ at  $\eta = 3.2 - 4.0$   
 $x_F \approx \frac{p_T}{\sqrt{s}} e^\eta \approx 0.5 - 0.6$
- ▶ the data reach large- $x_F$  region
- ▶ both shadowing and multiple scattering models fail to describe the data
- ▶ nuclei are known to suppress reactions at large  $x_F$

# Leading order kinematics

- ▶ light-front momentum fraction for projectile:  $x_1 = \frac{m_T}{\sqrt{s}} e^y$
- ▶ light-front momentum fraction for target:  $x_2 = \frac{m_T}{\sqrt{s}} e^{-y}$
- ▶ Feynman variable:  $x_F = x_1 - x_2 = \frac{2 m_T}{\sqrt{s}} \sinh(y)$
- ▶ forward rapidity  $\leftrightarrow$  the beam fragmentation region at large  $x_F$ :
  - ▶ **PROJECTILE**:  $x_1 \approx 0.5 - 1$   
mostly valence quarks contribute
  - ▶ **TARGET**:  $x_2 < 0.01$   
gluons dominate

# Coherence effects



- ▶ When longitudinal localization of partons exceeds thickness of Lorentz-contracted nucleus

$$\frac{1}{k_z} = \frac{1}{xp_N} > R_A \frac{m_N}{p_N}$$

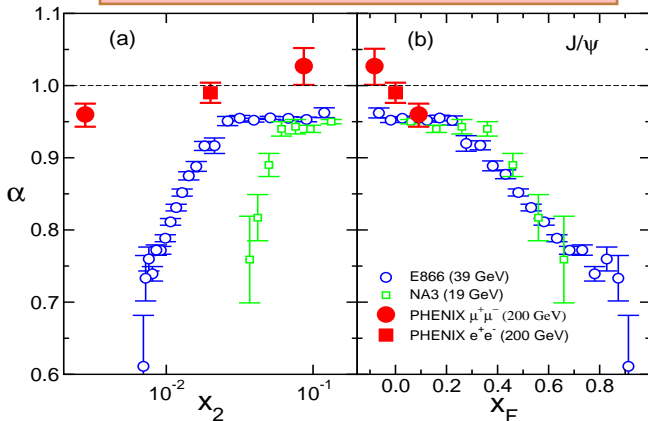
the spatial overlap of partons from different nucleons having the same impact parameter reduces density of small- $x$  partons. The small means:

$$x \lesssim x_A = \frac{1}{R_A m_N} \ll 0.1$$

- ▶ Partonic fusion reflects the fact that several nucleons share the same sea.
- ▶ **At forward rapidities  $x_2 = x_1 - x_F \rightarrow 0$  one can access the strongest coherence effects**

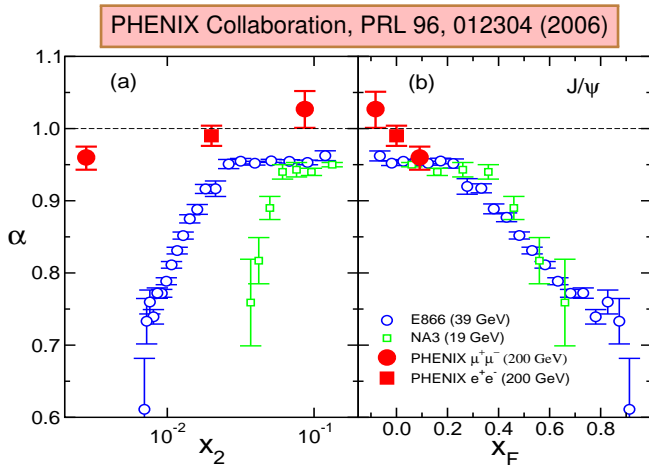
$J/\psi$  production: unexpected results from d+Au

PHENIX Collaboration, PRL 96, 012304 (2006)



$x_2$ -,  $x_F$ -dependence of the exponent  $\alpha$ :  $\sigma_{dA} = \sigma_{pp} \times (2A)^\alpha$  parametrizing the  $A$ -dependence of the nucleus-to-proton ratio

*Clear demonstration of  $x_F$ - , rather than of  $x_2$ - scaling*

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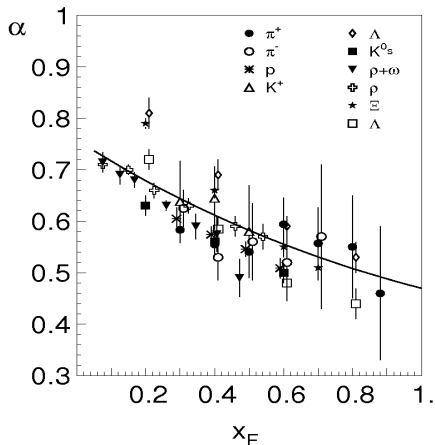
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**Clear demonstration of  $x_F$ - , rather than of  $x_2$ - scaling**



# Production of leading hadrons with small $p_T$

B.Z.Kopeliovich et al. Phys.Rev.C72,054606 (2005)



Exponent describing the  $A$ -dependence ( $\propto A^\alpha$ ) of the nucleus-to-proton ratio for production of different hadrons as a function of  $x_F$

- ▶ data for production of different hadrons in p+A collisions
- ▶ exhibit quite strong and universal nuclear suppression at large  $x_F$
- ▶ an approximate  $x_F$ - scaling in the laboratory energy range from 70 to 400 GeV
- ▶ Why do we observe a common feature of all known reactions on nuclear targets - a significant suppression at large  $x_F$  ( $x_1$ ) ?

# Coherent vs. Incoherent

- ▶ Nuclear modifications of the  $p_T$ -distribution occur in both the coherent and incoherent regimes. Only the coherent regime can be a manifestation of the Color Glass Condensate.
- ▶ **In incoherent scenario** nuclear suppression at large  $x_F$  can be interpreted alternatively as being due to:
  - ▶ reduced survival probability for large rapidity gap (LRG) processes in nuclei
  - ▶ enhanced resolution of higher Fock states by nuclei
  - ▶ effective energy loss that rises linearly with energy.
- ▶ **Driving mechanism at large  $x_F$  is the energy conservation:** there can not be more than one particle with  $x_F > 1/2$ , more than two with  $x_F > 1/3$ , etc.

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# *Multiple scattering approach*

# Leading hadrons with small $p_T$

- ▶ Large- $x_F$  suppression is related to the survival probability of LRG in multiple interaction with the nucleus at impact parameter  $b$ :

$$W_{LRG}^{hA}(b) = \exp[-\sigma_{in}^{hN} T_A(b)] \sum_{n=1}^A \frac{1}{n!} [\sigma_{in}^{hN} T_A(b)]^n S(x_F)^{n-1}$$

- ▶ The Sudakov factor  $S(x_F)$ :

$$S(x_F) = \exp[-\langle n_G(\Delta y) \rangle] = (1 - x_F)^{dn_G/dy} \approx (1 - x_F)$$

is the probability that none of the Poisson-distributed gluons enters the rapidity interval  $\Delta y = -\ln(1 - x_F)$  left behind the large- $x_F$  particle.

# Leading hadrons with small $p_T$

- ▶ Nuclear effect can then be calculated summing over  $n$  and integrating over impact parameter  $b$  in expression for a survival probability of the LRG:

$$R_{A/N}(x_F) = \frac{\int d^2b \exp[-\sigma_{\text{eff}} T_A(b)] \{ \exp[(1 - x_F)\sigma_{\text{eff}} T_A(b)] - 1 \}}{(1 - x_F) \sigma_{\text{eff}} A}$$

- ▶ N.B. Within the Glauber model  $\sigma_{\text{eff}} = \sigma_{\text{in}}^{hN}$ . However Gribov's corrections make medium more transparent and substantially reduce  $\sigma_{\text{eff}}$ . For  $A = 40$ ,  $\sigma_{\text{eff}} = 20 \text{ mb}$ .
- ▶ ***Above simple expression describes rather well soft  $p + A$  collisions and explains the observed  $x_F$ -scaling.***

# Multiple scattering of valence quarks

- ▶ For large values of hadron transverse momenta the cross section for hadron production in  $dA(pp)$  can be written as a convolution of the distribution function for the projectile valence quark with the quark scattering cross section and the fragmentation function:

$$\frac{d^2\sigma}{d^2p_T d\eta} = \sum_q \int_{z_{min}}^1 dz f_{q/d(p)}(x, q_T^2) \left. \frac{d^2\sigma[qA(p)]}{d^2q_T d\eta} \right|_{\vec{q}_T = \vec{p}_T/z} D_{h/q}(z)$$

$$x \equiv \frac{q_T}{\sqrt{s}} e^\eta$$

- ▶ Multiple rescattering of the projectile valence quark inside the target nucleus modifies the quark distribution function:

$$f_{q/N}^{(A)}(x, q_T^2, b, z) = \sum_{n=0}^A v_n(b, z) f_{q/N}^n(x, q_T^2)$$

$$v_n(b, z) = \frac{[\sigma_{eff} T_A(b, z)]^n}{[1 + \sigma_{eff} T_A(b, z)]^{n+1}}$$

$$f_{q/N}^n(x, q_T^2) = C_n f_{q/N} S(x)^n$$

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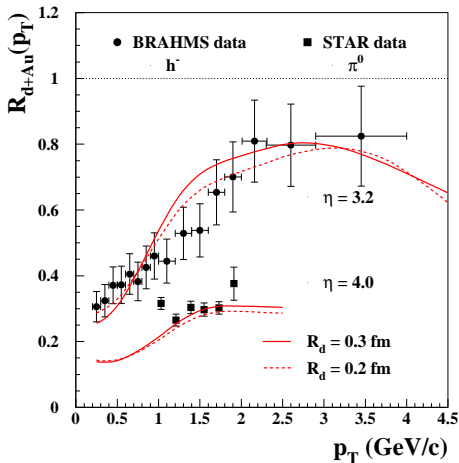
- ▶ **Projectile quark distribution correlates with the target  $\Rightarrow$  interaction with a nuclear target does not obey factorization**

# Quark scattering cross section on the target

Using **light cone dipole approach** quark scattering cross section on the target  $\frac{d^2\sigma[qA(p)]}{d^2q_T d\eta}$  was calculated for contributions characterized by different initial transverse momenta of the partons  $q_T$ :

- ▶ Quark-diquark break up:  $p \rightarrow \hat{q}q + q$   
We treat the diquark as point-like and integrate over its momentum. This process dominates at  $q_T < 1 \text{ GeV}/c$
- ▶ Diquark break up:  $\hat{q}q \rightarrow q + q$   
At larger  $q_T$  the interaction resolves the diquark. The valence quark from the diquark break up has much larger primordial transverse momentum.
- ▶ Hard gluon radiation:  $q \rightarrow Gq$   
At large  $q_T$  the dipole approach should recover the parton model  $\Rightarrow$  radiation of a gluon equilibrating  $q_T$  i.e. process  $qN \rightarrow qGN$  has to be included in the dipole description.

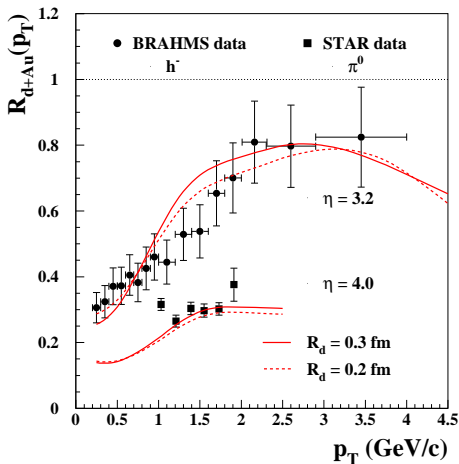
# RHIC: Comparison with BRAHMS and STAR data



Nuclear modification factor for hadrons  
in  $d + Au$  collisions at  $\sqrt{s_{NN}} = 200 \text{ GeV}$

- Fragmentation functions  $D_{\pi^-/u}(z)$  for  $u \rightarrow \pi^-$  and  $D_{\pi^-/d}(z)$  for  $d \rightarrow \pi^-$  from D. de Florian et al., Phys.Rev.D76,074033(2007)
- Enhancement (w.r.t.  $\pi^0$  case) of the ratio  $R_{d+Au}(p_T)$  for  $h^-$  by 3/2 at large  $p_T$  is due to isospin effects - negative hadrons with large  $p_T$  close to the kinematic limit are produced mainly from  $u$  rather than from  $d$  quarks.

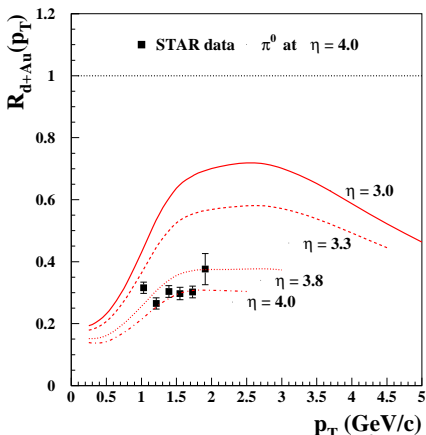
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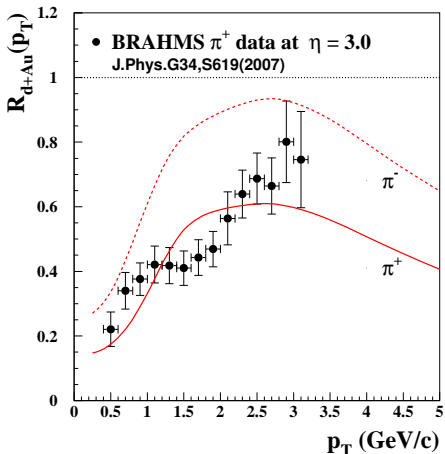
# RHIC: predictions for $\pi^0$ -production at different $\eta$



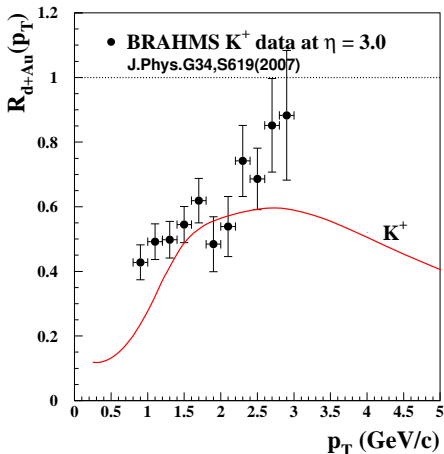
Nuclear modification factor for  $\pi^0$  in  
 $d + Au$  collisions at  $\sqrt{s_{NN}} = 200\text{GeV}$

- ▶ Study of  $\pi^0$ -production eliminates isospin effects present in  $d + Au$
  - ▶  $\frac{R_{d+Au}(p_t)|_{\eta=3}}{R_{d+Au}(p_t)|_{\eta=4}} \approx 2$
  - ▶ Variation of  $R_{d+Au}$  with  $\eta$  is due to a stronger onset of the Sudakov factors  $S(x_F)^n$  at larger  $x_F$  in the PDFs.
  - ▶ Prediction can be tested using latest d+Au and p+p data.
- Forward Meson Spectrometer** - recent upgrade of the STAR experiment has  $2.5 \lesssim \eta \lesssim 4.0$ .

# RHIC: Comparison to BRAHMS data on $\pi^+$ and $K^+$

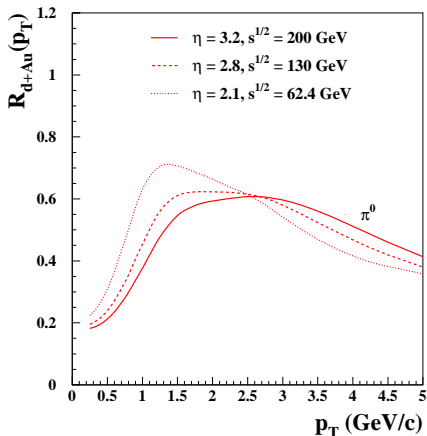


Nuclear modification factor for  $\pi^\pm$   
in  $d + Au$  collisions at  $\sqrt{s_{NN}} = 200\text{GeV}$



Nuclear modification factor for  $K^+$   
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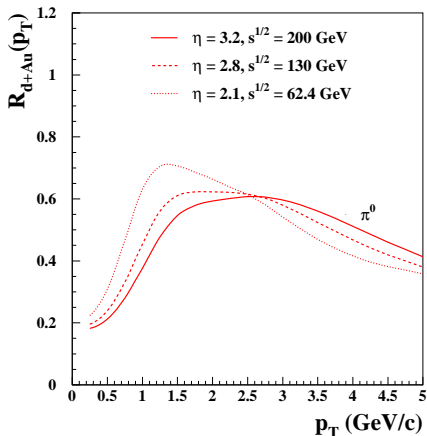
# $x_1$ - scaling at RHIC?



Theoretical predictions of an approximate  $\exp(\eta)/\sqrt{s}$  - scaling in  $d + Au$  collisions

- ▶ At fixed  $m_T$  universal and strong nuclear suppression caused by the Sudakov factor  $S(x_1)$  leads to approximately same nuclear effects at different  $\sqrt{s}$  and  $\eta$  corresponding to the same value of  $x_1 \approx m_T \exp(\eta) / \sqrt{s}$ .
- ▶ This prediction can be tested using future  $d+Au$  and  $p+p$  data at lower RHIC energies. **Forward Meson Spectrometer** - recent upgrade of the STAR experiment - covering  $2.5 \lesssim \eta \lesssim 4.0$  has much wider acceptance than so far published data.

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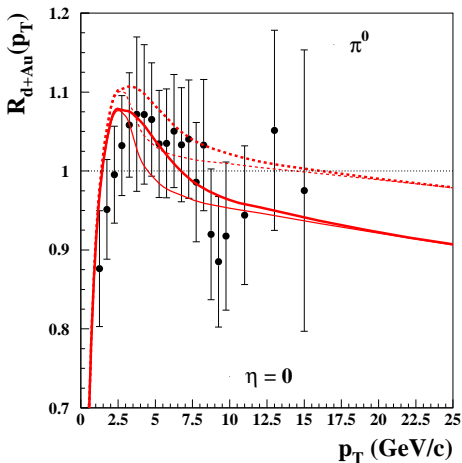
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# RHIC: predictions for $\pi^0$ -production at midrapidity

- PHENIX Coll., PRL98,172302 (2007)



Nuclear modification factor for  $\pi^0$ -mesons from  $d + Au$  collisions at  $\sqrt{s_{NN}} = 200\text{GeV}$

- ▶ At fixed  $\sqrt{s}$  and large  $p_T$  corresponding to the same  $x_1$ -values as those at forward rapidities the  $x_1$ -scaling allows one to study nuclear effects at midrapidities.
- ▶ Solid and dashed lines represent the calculations with and without multiple parton rescatterings, respectively.
- ▶ Thick and thin lines represent the calculations with and without finite coherence length correction, respectively.

# Summary

- ▶ Main source of nuclear suppression at large  $p_T$  and large forward rapidities is multiple parton rescattering in the nuclear matter.
- ▶ QCD factorization at the kinematic limits,  $x_F \rightarrow 1$ ,  $x_1 \rightarrow 1$ , ... fails.
- ▶ Suppression of high- $p_T$  hadrons at large rapidity observed by the BRAHMS and STAR Collaborations is well explained.
- ▶ We predict  $x_1$  ( $x_F$ )- scaling: the same nuclear effects at different energies and rapidities corresponding to the same value of  $x_1$ . It is in accord with the observed  $x_F$ - scaling of nuclear suppression for  $J/\psi$ .
- ▶ At RHIC the data at midrapidities cover rather large  $x_2$ -interval where no effect of coherence is possible ( $0.05 < x_2 < 0.1$ ). It allows to exclude the models based on CGC from interpretation of observed nuclear suppression.

*Thank you*

# *Back up slides*

# The Sudakov factor

- ▶ Assuming an uncorrelated Poisson distribution for gluons, (i.e. the energy conservation is not an issue) probability to have a rapidity gap  $\Delta y$ , becomes  $S(x_F) = \exp[-\langle n_G(\Delta y) \rangle]$  where  $n_G(\Delta y)$  is the mean number of gluons that would be radiated within  $\Delta y$ .
- ▶ N.B. Even in the case where no gluon is radiated within the rapidity gap, the hadronization can easily fill the gap with particles. The probability that this does not happen is another suppression factor which, however, is independent of target and cancels in the nucleus-to-proton ratio.
- ▶ The mean number  $\langle n_G(\Delta y) \rangle$  of gluons is related to the height of the plateau in the gluon spectrum,  $\langle n_G(\Delta y) \rangle = \Delta y dn_G/dy$
- ▶ Then, the Sudakov factor acquires the simple form

$$S(x_F) = \exp[-\langle n_G(\Delta y) \rangle] = (1 - x_F)^{dn_G/dy}$$

# The Sudakov factor

- ▶ The height of the gluon plateau was estimated in J. F. Gunion and G. Bertsch, *Phys. Rev. D* 25, 746 (1982) as

$$\frac{dn_G}{dy} = \frac{3\alpha_s}{\pi} \ln \frac{m_p^2}{\Lambda_{QCD}^2}$$

- ▶ The value of  $\alpha_s$  was fitted to data on pion multiplicity in  $e^+e^-$  annihilation and was found that  $\alpha_s = 0.45$ . This is close to the value  $\langle \alpha_s \rangle = 0.38$  calculated within a model of small gluonic spots when averaged over the gluon radiation spectrum. For further calculations, we take  $\alpha_s = 0.4$ , which gives with high accuracy  $dn_G/dy = 1$ , i.e. the Sudakov factor:  $S(x_F) = (1 - x_F)$
- ▶ N.B. this coincides with the suppression factor applied to every additional Pomeron exchange in the quark-gluon string and dual parton models based on the Regge approach.

# Quark-diquark break up $p \rightarrow \hat{q}q + q$

- ▶ The first possibility is to break up the proton in such a way which leaves the diquark intact,  $p \rightarrow \hat{q}q + q$ . This process dominates at  $q_T < 1 \text{ GeV}/c$
- ▶ We treat the diquark as point-like and integrate over its momentum
- ▶  $q_T$ -distribution of the projectile valence quark, after propagation through the nucleus at impact parameter  $\vec{b}$ , is given as:

$$\frac{d\sigma(NA \rightarrow qX)}{d^2q_T d^2b} = \int \frac{d^2r_1 d^2r_2}{(2\pi)^2} e^{i\vec{q}_T(\vec{r}_1 - \vec{r}_2)} \Psi_N^\dagger(r_1) \Psi_N(r_2) \times [1 + g(\vec{r}_1 - \vec{r}_2) - g(\vec{r}_1) - g(\vec{r}_2)]$$

$$g(\vec{r}) \equiv e^{-\frac{1}{2}\sigma_{\bar{q}q}^N(\vec{r})T_A(b)}$$

- ▶  $\sigma_{\bar{q}q}^N(\vec{r})$  is color-dipole cross section on nucleon at transverse  $\bar{q}q$  separation  $\vec{r}$
- ▶  $\Psi_N(r)$  is quark-diquark wave function of the nucleon matching the known pQCD behavior at large  $q_T$ :  $\Psi_N(r) \propto K_0(r/R_p)$

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# Diquark break up $\hat{q}q \rightarrow q + q$

- ▶ At larger  $q_T$  the interaction resolves the diquark and its break up should be included
- ▶ The valence quark from the diquark break up has much larger primordial transverse momentum
- ▶  $q_T$ -distribution of the projectile valence quark from the diquark break up reads:

$$\frac{d\sigma(\hat{q}qA \rightarrow qX)}{d^2q_T d^2b} = \int \frac{d^2r_1 d^2r_2}{2(2\pi)^2} e^{i\vec{q}_T(\vec{r}_1 - \vec{r}_2)} \Psi_D^\dagger(r_1) \Psi_D(r_2) \times$$

$$\left[ 2 - g(\vec{r}_1) - g(\vec{r}_2) - g\left(\frac{\vec{r}_1}{2}\right) - g\left(\frac{\vec{r}_2}{2}\right) - g\left(\vec{r}_1 - \frac{\vec{r}_2}{2}\right) - g\left(\vec{r}_2 - \frac{\vec{r}_1}{2}\right) - 2g(\vec{r}_1 - \vec{r}_2) - 2g\left(\frac{\vec{r}_1 - \vec{r}_2}{2}\right) \right]$$

- ▶ Diquark wave function  $\Psi_D(r)$  is also assumed to behave as  $\Psi_D(r) \propto K_0(r/R_D)$  but with a mean separation  $R_D = 0.2 - 0.3 \text{ fm}$

# Hard gluon radiation $q \rightarrow Gq$

- ▶ in the dipole approach the cross section is given by the same formula:

$$\frac{d\sigma(qA \rightarrow qX)}{d^2q_T d^2b} = \int \frac{d^2r_1 d^2r_2}{(2\pi)^2} e^{i\vec{q}_T(\vec{r}_1 - \vec{r}_2)} \Psi_{qG}^\dagger(r_1) \Psi_{qG}(r_2) \times [1 + G(\vec{r}_1 - \vec{r}_2) - G(\vec{r}_1) - G(\vec{r}_2)]$$

where  $G(\vec{r}) \equiv e^{-\frac{1}{2}\sigma_{GG}^N(\vec{r})T_A(b)}$

- ▶ the nucleon wave function is replaced by the quark-gluon light-cone wave function,  $\Psi_N(r_T) \rightarrow \Psi_{qG}(r_T)$ , where

$$\Psi_{qG}(\vec{r}_T) = -\frac{2i}{\pi} \sqrt{\frac{\alpha_s}{3}} \frac{\vec{r}_T \cdot \vec{e}^*}{r_T^2} e^{-\frac{r_T^2}{2r_0^2}} \text{ with } r_0 = 0.3 \text{ fm}$$

⇒ small gluonic spots