Novel approach to the analysis of nucleus-nucleus collisions at relativistic energies.

E. I. Shahaliev[1], R. G. Nazmitdinov[1,2], M. K. Suleymanov[1,3]

[1] Joint Institute for Nuclear Research, Dubna, Russia
[2] Departament de Física, Universitat de les Illes Balears, Palma de Mallorca, Spain
[3] COMSATS Institute of Information Technology, Islamabad, Pakistan

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RELATIVISTIC NUCLEAR PHYSICS
& QUANTUM CHROMODYNAMICS

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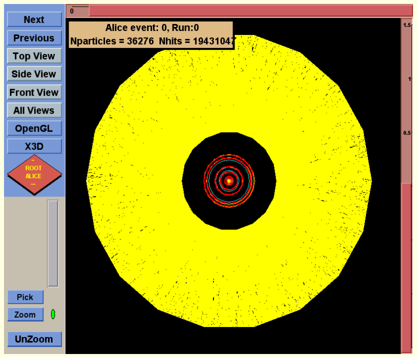
1. Motivation

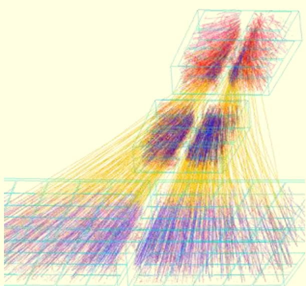
When we pass from hadron-light nucleus, light nucleus-light nucleus collisions at low, middle and high energies to relativistic and ultrarelativistic heavy ion collisions we get the new and unequal possibility: to create the high density and high temperature hadronic matter and to get the information on the properties of the matter under extreme conditions. In such new situation the volume of information increases sharply as well as the background information. The Figure illustrates how the volume of the information increase with energy and the mass of beams. Some time the background information can grow faster than useful signal information due to the reason that the number of secondary multiparticle interactions become more and more it is very essential in case of central collisions.



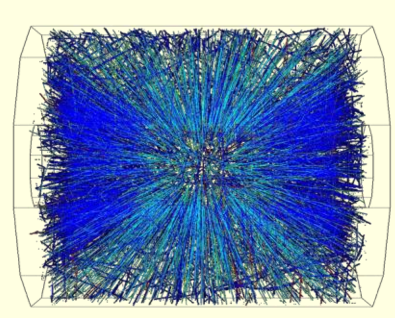
PbPb $\sqrt{s_{NN}}$= 5.5 TeV

Alice LHC CERN

π,p + p,A



Pb+Pb $\sqrt{sNN}$ = 17.3 GeV
NA 49 SPS CERN



Au+Au $\sqrt{s_{NN}}$ = 200 GeV
STAR RHIC BNL

It is expected that in central collisions, at energies that are and will be soon available at SPS (CERN), RHIC (BNL) and LHC (CERN) the new phases of strongly interacting matter as well as Quark Gluon Plasma *[S. Jeon and V. Koch Review for Quark-Gluon Plasma 3 eds. R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2004) pp 430-490.]* could be formed. In these collisions a number of produced particles can be around of 10^5 (last picture in Fig.1) and a natural question arises how could a useful signal separate ? The most popular traditional methods of analysing data coming from high-energy nucleus-nucleus collisions are: i) the

correlation analysis [E. A. De Wolf , I. M. Dremin, and W. Kittel, Usp. Fiz. Nauk **163**, 3 (1993) [Phys. Usp. **36**, 225 (1993)]. ii) the analysis of

missing masses [E. Byckling and K. Kajantie, *Particle Kinematics* (Wiley, New York, 1973).] and effective mass spectra

[V. I. Gol'danckiy, Yu. P. Nikitin, and I. L. Rozental', *Kinematic methods in high energy physics* (Nauka, Moscow, 1987) (in Russian).], iii) the interference method of identical

particles [M. I. Podgoretsky, Fiz. Elem. Chastits At.Yadra, **20**, 628 (1989).] etc. These ones are very sensitive to background information.

Therefore it is necessary to look for and apply new methods, new approach to data analyses at relativistic and ultrarelativistic heavy ion collisions which would be not so sensitive to background information.

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2. Random Matrix Theory for data process at low energy.

The Ref. [M. L. Mehta, Random Matrices, Second ed.(Academic, New York, 1991).] introduced method on basic of Random Matrix Theory to explain the statistical fluctuations of neutron resonances in compound nuclei [C. E. Porter, Statistical Theories of Spectra: Fluctuations (Academic, New York, 1965).] (see also Ref.[T. A. Brody, J. Flores, J. B. French, P. A. Mello, A.Pandy, and S. S. M. Wong, Rev. Mod. Phys.**53**, 385 (1981).]) which dose not depend on the background of measurements. Let us consider the main points of this method.

Rashid Nazmitdinov -- BLTP

To analysis the energetic levels of compound nuclei the *p(s)* function is defined. This is the function of distances between two energetic levels. It can be defined using the general distributions for probability of all kinds of ensembles [T. Ghur, A. Muller-Groeling, and H. A. Weidenmuller, Phys. Rep. 299, 189 1998.]

$$P(E_1, \cdots, E_N) \sim \prod_{n>m} (E_n - E_m)^{\nu} exp\left(-A \sum_n E_n^2\right)$$

Here ν is index of universality which can get the values 1,2 or 4 for different statistics. If ν=0 so we will get the Poisson type distributions.

Theory

In case of two dimension matrix using the formula for probability one can get

$$p(s) = \int_{-\infty}^{\infty} dE_1 \int_{-\infty}^{\infty} dE_2 P(E_1, E_2)\delta(s - |E_1 - E_2|) =$$

$$= C \cdot \int_{-\infty}^{\infty} dE_1 \int_{-\infty}^{\infty} dE_2 |E_1 - E_2|^{\nu} \exp\left(-A \sum_n E_n^2\right) \times \delta(s - |E_1 - E_2|.$$

here $s = |E_{n+1} - E_n|/D$ the distance between two neighbor levels , D average distance between the levels. The values of the parameters A and C can be obtained from conditions:

$$\int_0^{\infty} p(s) ds = 1,$$

$$\int_0^{\infty} s p(s) ds = 1.$$

The first one is a condition of normalized total for probability and second one does a condition of normalized average between levels distances.

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At *ν=1* the integral for *p(s)* gives Wigner type behavior

$$P_w(s) = \frac{\pi}{2} \cdot s \cdot e^{-\frac{\pi}{4}s^2}, \qquad s \geq 0$$

It means the repulsive force appeared between the levels. Therefore if we have got the energetic levels iv normal conditions so their *p(s)* distribution will have Passion type behavior and if the levels were excited so we will get Wigner type distribution.

Ehtiram Shahaliev has prepared the package of program to define the p(s) functions for all kind of reactions (shah@sunhe.jinr.ru).

Theory

Let us consider the discrete spectrum $\{E_i\}, i = 1, ..., N$ of a d-dimensional quantum system (d is a number of degrees of freedom). A separation of fluctuations of a quantum spectrum can be based on the analysis of the density of states below some threshold E

$$S(E) = \sum_{i=1}^{N} \delta(E - E_i)\,. \tag{1}$$

We can define a staircase function

$$N(E) = \int_{-\infty}^{E} S(E')dE' = \sum_{i=1}^{N} \theta(E - E'), \tag{2}$$

giving the number of points on the energy axis which are below or equal to E. Here

$$\theta(x) = \begin{cases} 0 & for \;\; x < 0 \\ 1 & for \;\; x > 1 \end{cases} \tag{3}$$

We separate $N(E)$ in a smooth part $\zeta(E)$ and the reminder that will define the fluctuating part $N_{fl}(E)$

$$N(E) = \zeta(E) + N_{fl}(E) \tag{4}$$

The smooth part $\zeta(E)$ can be determined either from semiclassical arguments or using a polynomial or spline interpolation for the staircase function.

To study fluctuations we have to get rid of the smooth part. The usual procedure is to "unfold" the original spectrum $\{E_i\}$ through the mapping $E \to x$

$$x_i = \zeta(E_i), \qquad i = 1, ..., N \tag{5}$$

T h e o r y

Now we can define spacings $s_i = x_{i+1} - x_i$ between two adjacent points and collect them in a histogram. The effect of mapping is that the sequence $\{x_i\}$ has on the average a constant mean spacing (or a constant density), irrespective of the particular form of the function $\zeta(E)$. To characterize fluctuations one deals with different correlation functions . In this paper we will use only a correlation function related to spacing distribution between adjacent levels. Below, we follow a simple heuristic argument due to Wigner that illustrates the presence or absence of level repulsion in an energy spectrum.

For a random sequence, the probability that the level will be in the small interval $[x_0 + s, x_0 + s + ds]$ is independent of whether or not there is a level at x_0. Given a level at x_0, let the probability that the next level be in $[x_0 + s, x_0 + s + ds]$ be $p(s)ds$. Then for $p(s)$, the nearest-neighbor spacing distribution, we have

$$p(s)ds = p(1 \in ds|0 \in s)p(0 \in s) \tag{6}$$

Here, $p(n \in s)$ is a probability that the interval of length s contains n levels and $p(n \in ds|m \in s)$ is the conditional probability that the interval of length ds contains n levels, when that of length s contains m levels. One has $p(0 \in s) = \int_s^\infty p(s')ds'$, the probability that the spacing is larger than s. The term $p(1 \in ds|0 \in s) = \mu(s)ds$ [$\mu(s)$ is the density of spacings s], depends explicitly on the choices, 1 and 0, of the discrete variables n, m. As a result, one obtains $p(s) = \mu(s) \int_s^\infty p(s')ds'$ which can be solved to give

$$p(s) = \mu(s) \exp(-\int_0^s \mu(s')ds') \tag{7}$$

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The function $p(s)$ and its first moment are normalized to unity,

$$\int_0^s p(s)ds = 1, \qquad \int_0^s sp(s)ds = 1. \tag{8}$$

For a linear repulsion $\mu(s) = \pi s/2$ one obtains the Wigner surmise,

$$p(s) = \frac{\pi}{2} s \exp(-\frac{\pi}{4} s^2), \qquad s \geq 0 \tag{9}$$

For a constant value $\mu(s) = 1$ one obtains the Poisson distribution

$$p(s) = \exp^{-s}, \qquad s \geq 0 \tag{10}$$

As discussed above, when quantum numbers of levels are well defined, one should expect for the spacings the Poisson type distribution, while a Wigner type distribution occurs due to either internal or external perturbations that destroy these quantum numbers.

3. The applying of RMT for high energy physics.

It is idea of E. Shahaliev to apply method of RMT to data process for nucleus – nucleus collisions at relativistic and ultrarelativistic energies. The secondary particles produced at low energies and defined the p(s) functions. In this step we try only to get some signal of appearing the Wigner type distribution in the experiment.

Experiment

To test the utility and the validity of the proposal we use the experimental data that have been obtained from the 2-m propane bubble chamber of LHE, JINR . The chamber, placed in a magnetic field of 1.5 T, was exposed to beams of light relativistic nuclei at the Dubna Synchrophasotron. Practically all secondaries, emitted at a 4π total solid angle, were detected in the chamber. All negative particles, except those identified as electrons, were considered as π^--mesons. The contaminations by misidentified electrons and negative strange particles do not exceed 5% and 1%, respectively. The average minimum momentum for pion registration is about 70 MeV/c. The protons were selected by a statistical method applied to all positive particles with a momentum of $|p| > 500$ MeV/c (we identified slow protons with $|p| \leq 700$ MeV/c by ionization in the chamber). In this experiment, we had got 20407 ^{12}CC interactions at a momentum of 4.2A GeV/c

We considered the events with more than 10 particles as well as the method is statistical one.

Our analyses has been done for different range of values of the momentum for pecondary particles.

Experiment

3.1 First signal.

On Fig. 1 the dependence $dN/d|p|$ as a function of the measured momentum (0.15-7.5 GeV/c) of the secondary particles is displayed . The numerical data $N(p)$ were approximated by the polynomial function of the sixth order and we obtain the distribution of various spacings s_i in 2636 events satisfying the condition of χ^2 per degree of freedom less than 1.0. In the Figs the data were presented for: the 0.15-1.14 GeV/c (region I, Fig. 2a) particles; (region II, Fig. 2b) covers the values 1.14-4.0 GeV/c. ;the region 4.0-7.5 GeV/c

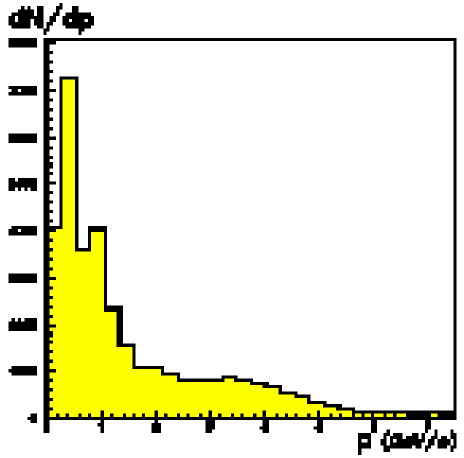


Fig.1

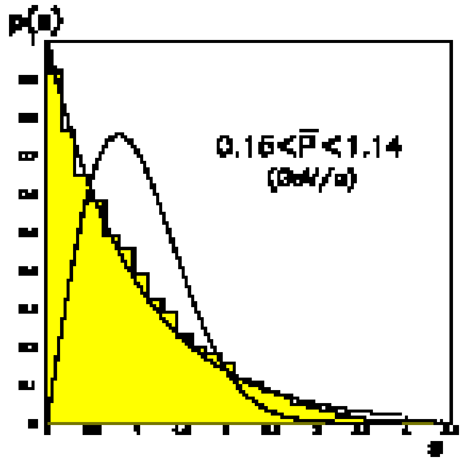


Fig.2 a

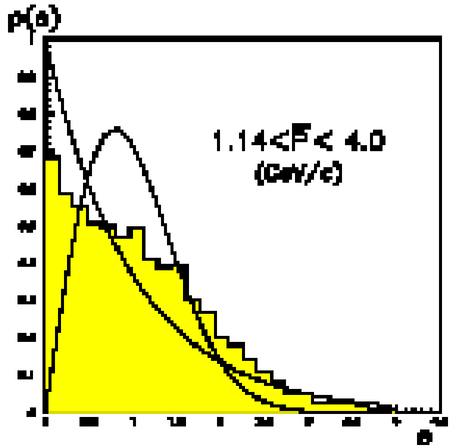
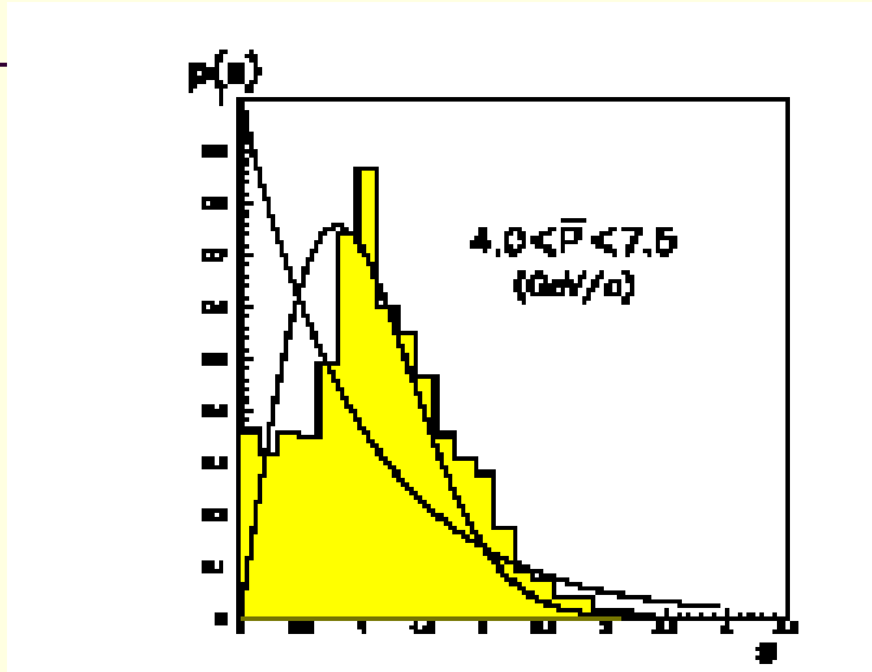


Fig.2 b

(Fig. 2c). The spacing probability nicely reproduces this tendency depending on the region of the momentum distribution. The function $p(s)$ has the Poisson distribution for the

Experiment

First signal.



To test the scale dependences of the results we changed of the scale of momentum in I, II and III regions. The results did not change qualitatively. We used as a main parameter to create the p(s) distribution total energy of secondary particles instead the momentum too. The pictures did not change qualitatively.

region I and the region II corresponds to the intermediate situation, when the spacing distribution lies between the Poisson and the Wigner distributions. In third region we have got a Wigner type distribution for the spacing probability .

Yes, the Method works!!!!!!

Experiment

4. Comparison

Above we have noted that amongst the most popular traditional methods of analysing data produced at high-energy nucleus-nucleus collisions are: i) the correlation analysis , ii) the analysis of missing masses and effective mass spectra , iii) the interference method of identical particles etc. The results based on such methods are very sensitive to background information. On the other hand all new methods cannot give the results which would be against these methods. So we tried to compare the results coming from these methods and from one on Random Matrix Theory.

4.1 The two particle correlation analysis.

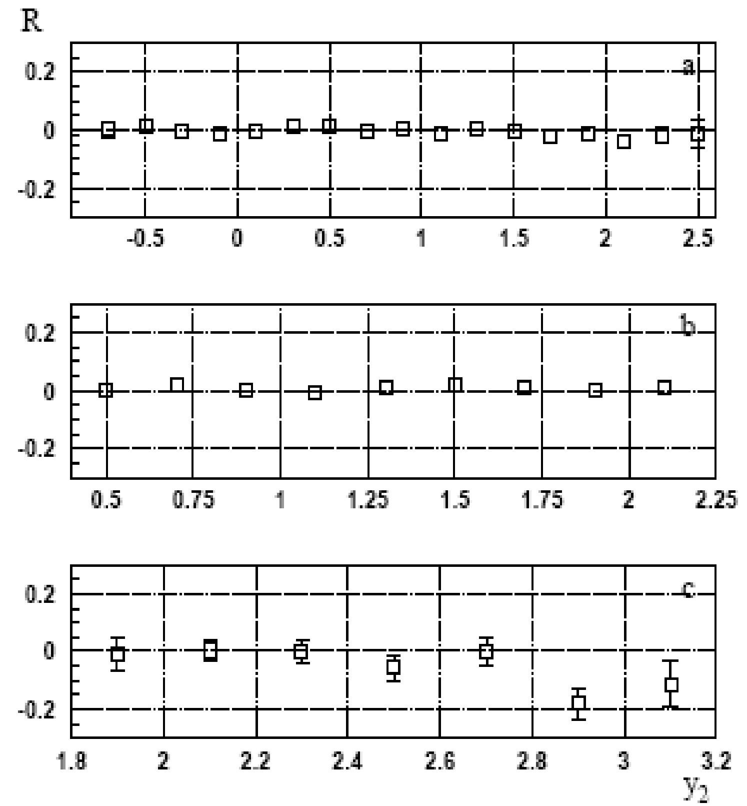
The validity of the RMT analysis is confirmed by an independent analysis of the data with the aid of the standard pair-correlation function

$$R(y_1, y_2) = \sigma \frac{d^2\sigma/dy_1 dy_2}{(d\sigma/dy_1)(d\sigma/dy_2)} - 1$$

Here, the quantity σ is the cross section of the inclusive reaction and $y = \frac{1}{2} ln \frac{E+P_{||}}{E-P_{||}}$ is the rapidity, which depends on the particle energy E and its longitudinal momentum $P_{||}$. The rapidity is one of the main characteristics widely used in relativistic nuclear physics . In particular, the change of the reference frame leads to a trivial shift Δy in the rapidity.

Experiment

The pair-correlation function manifests the difference between the probability density of two-particle events and the product of the probability densities of independent particle events. It vanishes if the particle rapidities are independent. Figure demonstrates the results for particles obtained in ^{12}CC-interactions. For the sake of illustration, we integrate the function $R(y_1, y_2)$ over one of the variables, say y_1, and consider the dependence on y_2. For different momentum distributions, there are three intervals of integration for the variable y_1: a)for $0.1 < |p| < 1.14$ GeV/c the function $R(y_1, y_2)$ is integrated in the interval $-0.9 < y_1 < 2.5$; b)for $1.14 < |p| < 4.0$GeV/c it is integrated in the interval $0.5 < y_1 < 2.4$; and c)for $4.0 < |p| < 7.5$ GeV/c it is integrated in the interval $2.5 < y_1 < 3.5$. The results for the function $R = \int_{y_1} R(y_1, y_2)dy_1$ clearly indicate the presence of correlations between particles in the region $4.0 < |p| < 7.5$ GeV/c : there is a strong deviation from zero in the interval $2.7 < y_2 < 3.2$.



4.2 The method of effective mass

Now we consider the method of effective mass (MEM), which is a standard tool to extract the information on the correlation between secondary particles.

Experiment

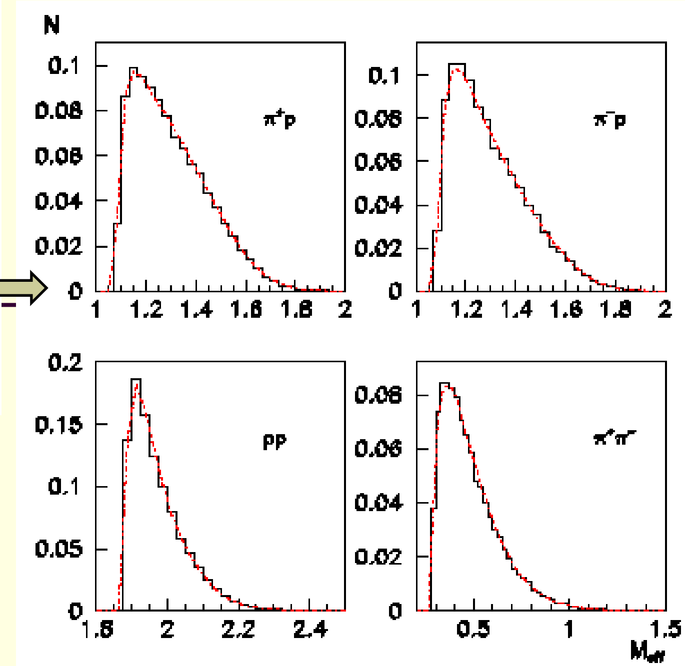
Before we proceed a few remarks are in order: in various theoreical approaches it is usually assumed that at high-energy nucleus-nucleus collisions: i) a majority of produced secondary pions are emitted basically through the mechanism of production and decay of the light resonances; ii) a significant portion of the protons are produced as a result of Δ-isobar decays |. Further, it is also assumed that such processes consist of two steps. First, $(n-k)$ particles and the resonance are produced. Second, the resonance decays on k-particles and one may expect that due to kinematics there are some correlations between these k-particles. In order to extract these correlations we are forced to consider all possible combinations of the k-particle-participants and compare with the known resonance masses. As a result, this procedure gives rise to a large nonphysical background contribution to the analysis.

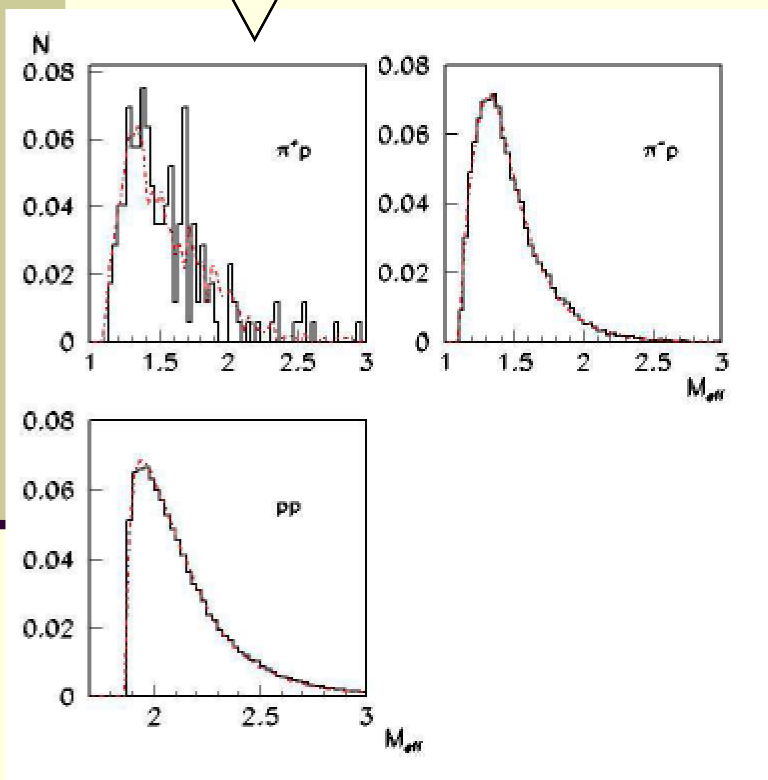
In particular, in the MEM one considers the effective mass for k secondary particles as

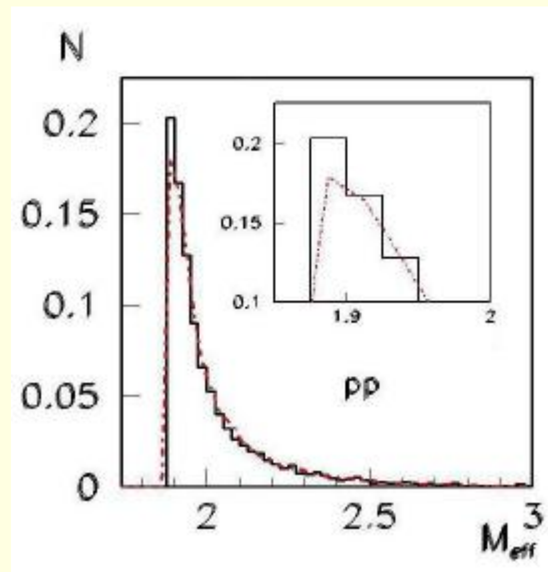
$$M_{12..k} = \left\{ (\sum_{i=1}^{k} E_i)^2 - (\sum_{i=1}^{k} \vec{p}_i)^2 \right\}^{1/2} . \qquad (2)$$

If the condition $M_{12..k} = M^{res}_{12..k}$ holds, one may conclude that a resonance with the mass $M^{res}_{12..k}$ is identified in the data. In addition, one assumes that each identified resonance contributes to the total cross section with own weight (probability). To carry on this idea, each resonance is approximated by the Breight-Wigner distribution with the identified mass and the resonance width. Varying the weights of identified resonances, one is aiming to reach the best agreement with the observed inclusive cross sections of resonances production.

To elucidate a relation between RMT and MEM, we consider the distributions of charged particle pairs emitted in ^{12}CC -interactions at 4.2 A GeV/c . Figs. demonstrate the distributions of $(\pi^+ p)-$, $(\pi^- p)-$, $(pp)-$, and $(\pi^+\pi^-)-$pairs emitted in three ranges of the momentum distribution of the secondary particles: $0.1 - 1.14$; $1.14 - 4.0$ and $4.0 - 7.5$ GeV/c . All distributions are normalized to the total number of pairs.







Experiment

To extract the information on the resonance production from data presented in these figures it is necessary to evaluate a background contribution. It is indeed a difficult problem which cannot be solved completely. Therefore, to construct the background we use the method of mixing events . In this method the background is defined by the pairs constructed from the particles produced in different events only. Needless to say, a method that is independent of such a background is required to interpret the data correctly. In fact, the RMT results, discussed in the previous section, serves the useful purpose of giving an independent view of the latter analysis.

One observes that in the interval of momentum $0.1 - 1.14$ GeV/c no clear-cut distinction exists between experimental and background distributions. In this interval we have obtained very good statistical conditions: 41615 (π^+p)–, 43626 (π^-p)–, 52992 (pp)– and 39112 $(\pi^+\pi^-)$–pairs. One concludes that there is an absence of any manifestation of resonance production.

Experiment

duces the Poisson distribution for the behavior of the $p(s)$-function

In the second interval we have obtained 173 $(\pi^+ p)$–, 16470 $(\pi^- p)$–, 167094 (pp)– and only 16 $(\pi^+\pi^-)$–pairs (are not shown). There is some deviation of the experimental distribution from the background only for $(\pi^+ p)$–pairs. We believe that it is connected with the production of the Δ^{++}-isobars with masses $m_{\Delta^{++}} = 1.232$ and 1.650 (GeV/c^2). Note that the observation of Δ^{++}-isobars with masses $m_{\Delta^{++}} = 1.232$ GeV/c^2 has been reported , while the ones with masses 1.650 GeV/c^2 are observed for the first time. There are not any visible deviations between the experimental data and the background for the distribution of the $(\pi^- p)$– and (pp)–pairs. In this interval the RMT approach produces a visible deviation from the Poisson distribution for the behavior of $p(s)$-function .
It appears that the RMT is able to provide a hint on the resonance production.

Experiment

In the third interval of the momentum distribution of charged secondary particles we have 10 $(\pi^- p)$– (are not shown) and 9522 (pp)–pairs). Here, the $(\pi^+ p)$–pairs are absent. The deviation of the signal relative to the background is above 20%. It is well known that in this interval the striping protons are the dominant ones (with a small contribution of deuterons, tritons and others)]. These protons carry a maximum momentum near the value 4.2 GeV/c. It results in a very small deviations of the particle trajectories in the magnetic field of the setup. In fact, it is the worst situation for the accurate determination of the errors in the momentum distribution. The RMT approach produces in this interval a distribution of the density $p(s)$ close to the Wigner

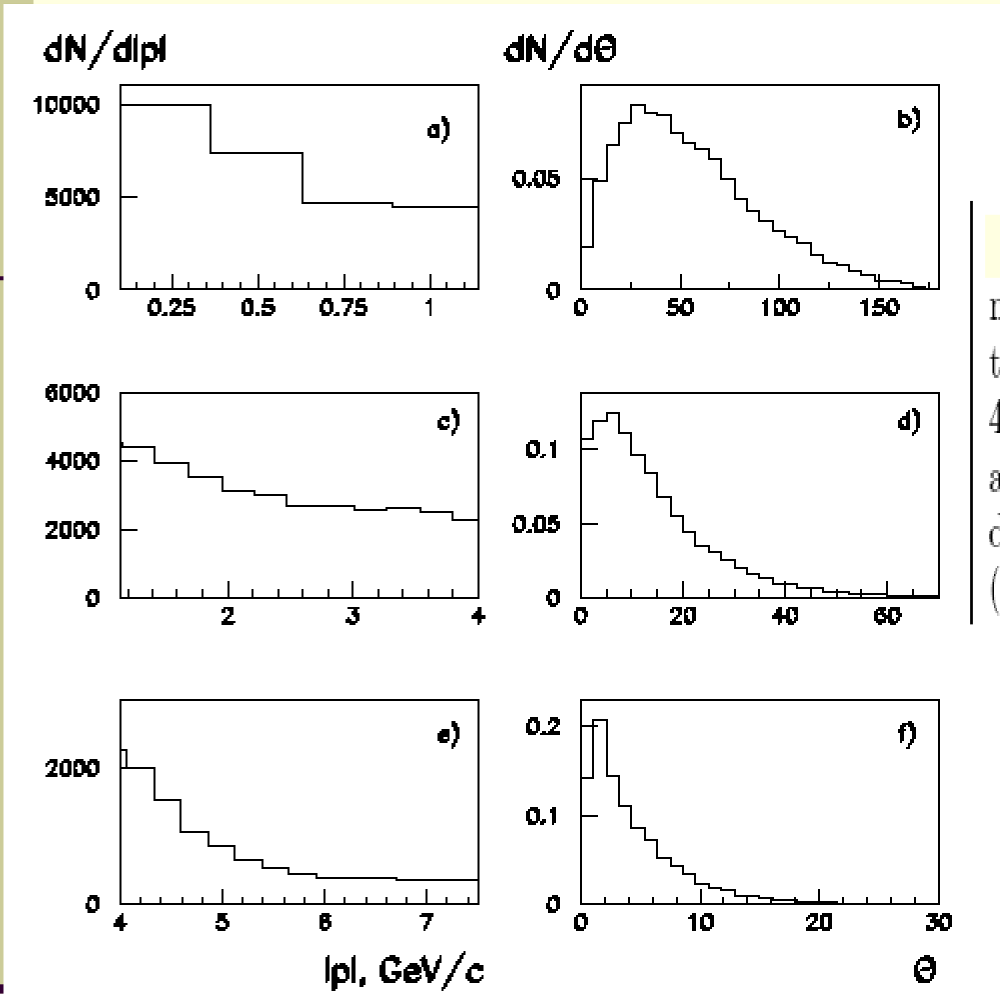
surmise form . As stressed above, such a distribution is associated with the breaking of regularity in the spectral properties of a quantum system due to either external or internal sources. We have already mentioned that the onset of the Wigner distribution for the density $p(s)$ (breaking of the regularity) could indicate the presence of errors in the measurement.

Experiment

One observes a clear cut distinction between the background and experimental distribution of (pp)–pairs in $4.0 - 7.5$ GeV/c . However, there is not a solid basis to associate such a strong deviation with a production of di-baryon resonances since the inclusive cross section of such "resonances" would exceed essentially those that are predicted by various theoretical models.

In all considered distributions there is an evident dominance of the pp-pairs. To trace the evolution of the pp correlations we select only the momentum and angular distributions of the protons in three intervals
In the first interval the angular distribution (
of the pairs covers almost all angles of the semisphere with some concentration around $\sim 50^0$. In the second interval the momentum distribution of the pairs (
is spread smoothly over all considered values of the momentum $(1.14 - 4.0$ GeV/c). There is some concentration of the emitted pairs in the angular distribution, which covers a solid angle $\sim 20^0$. In the third interval one observes that striping protons have similar momenta and almost zero angle in the distribution. Evidently, under such conditions, one may expect a large probability for the interaction in the final state, which leads to the narrow peak appearances in the effective mass spectrum of the proton pairs. Such interaction effects in a final state are well known for the particle production and decay process at high energies We recall that for this interval the RMT approach provides the Wigner distribution for the behavior of the density $p(s)$.

Experiment



Left: The distributions $dN/d|p|$ as a function of the measured momentum in three ranges of the momentum distribution of the protons: 0.1 – 1.14 (a); 1.14 – 4.0 (c) and 4.0 – 7.5 (e) (GeV/c). Right: The distributions $dN/d\theta$ as a function of the angle in the lab frame for the momentum distriubution: 0.1 – 1.14 (b); 1.14 – 4.0 (d) and 4.0 – 7.5 (f) (GeV/c). The angular distribution is normalized to unity.

The comparison of the RMT results with the MEM analysis manifests in fact that there are evident correlations between behavior of the density $p(s)$ in different energy (momentum) intervals and the appearance of new sources that breaks the regularity in the momentum distribution of the charged particles.

Conclusion

We propose a novel statistical approach to the analysis of experimental data obtained in nucleus-nucleus collisions at high energies which borrows from methods developed within the context of Random Matrix Theory. It is applied to the detection of correlations in momentum distributions of emitted particles. We find good agreement between the results obtained in this way and a standard analysis based on the method of effective mass spectra and two-pair correlation function often used in high energy physics. The method introduced here is free from unwanted background contributions.

Ehtiram Shahaliev – shah@sunhe.jinr.ru