Possibility of measurement of the tensor electric and magnetic polarizabilities of the deuteron and other nuclei in experiments with polarized beams

Alexander J. Silenko

Research Institute for Nuclear Problems of Belarusian State University

XIX International Baldin Seminar on High Energy Physics Problems

Dubna 2008

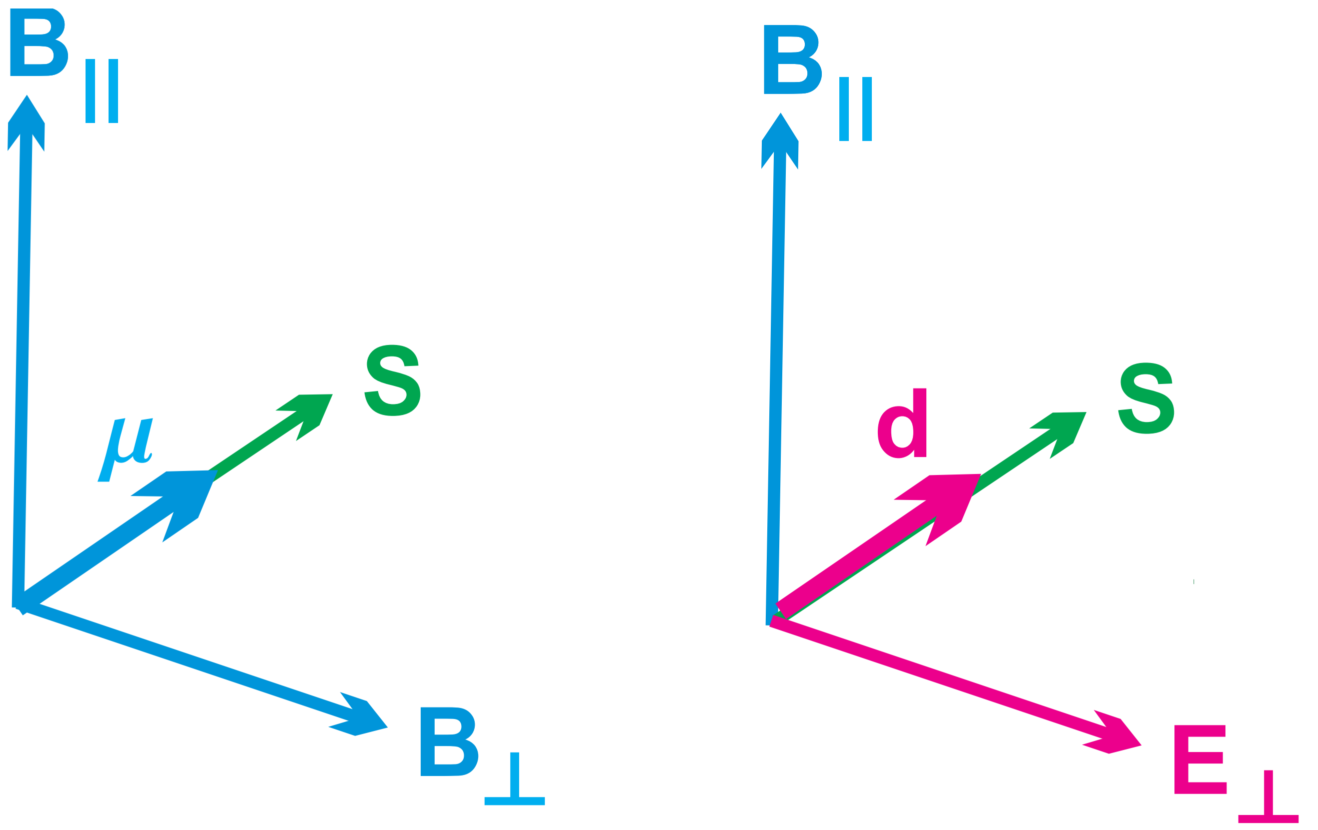
OUTLINE

- Overview
- Measurement of tensor electric polarizability of the deuteron by the resonance method
- Tensor magnetic polarizabilities of the deuteron and other nuclei in storage ring experiments
- Measurement of tensor electric and magnetic polarizabilities of the deuteron by the frozen spin method
- Summary

Methods of measurement of tensor polarizabilities of the deuteron and other nuclei

Methods of measurement of tensor polarizabilities of the deuteron and other nuclei have been proposed by V. Baryshevsky and co-workers:

- V. Baryshevsky and A. Shirvel, hep-ph/0503214.
- V. G. Baryshevsky, STORI 2005 Conference Proceedings, Schriften des Forschungszentrums Jülich, Matter and Materials, Vol. 30 (2005), pp. 227–230; J. Phys. G: Nucl. Part. Phys. **35,** 035102 (2008); hep-ph/0504064; hep-ph/0510158; hep-ph/0603191.
- V. G. Baryshevsky and A. A. Gurinovich, hep-ph/0506135.



B ||
μ
S
B ⊥
B ||
d
S
E ⊥

V.G. Baryshevsky and co-authors:

The tensor electric and magnetic polarizabilities of the deuteron manifest in the EDM experiment with vector-polarized deuterons and can be measured

- **The tensor electric polarizability stimulates the buildup of the vertical polarization of vector-polarized deuteron beam**
- **The tensor magnetic polarizability produces the spin rotation with two frequencies instead of one, beating and causes transitions between vector and tensor polarizations**

The tensor electric and magnetic polarizabilities affect the spin

$$V = -\frac{\alpha_T}{\gamma}(\mathbf{S}\cdot\mathbf{E}')^2 - \frac{\beta_T}{\gamma}(\mathbf{S}\cdot\mathbf{B}')^2.$$

$\mathbf{E}'$ and $\mathbf{B}'$ are the rest frame fields

$$V = -\frac{\alpha_T}{\gamma}\left(\beta\gamma B_z S_\rho + E_\phi S_\phi\right)^2$$

$$-\beta_T \gamma B_z^2 S_z^2.$$

There are two resonance frequencies, **$\omega \approx \omega_0$** and **$\omega \approx 2\omega_0$**:

$$\beta^2\gamma = \beta_0^2\gamma_0 + \left(2 + \beta_0^2\gamma_0^2\right)\beta_0\gamma_0 \cdot \Delta\beta_0 \cos\left(\omega t + \varphi\right)$$

$$+ \frac{1}{4}\left(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4\right)$$

$$\times \gamma_0 \left(\Delta\beta_0\right)^2 \left\{1 + \cos\left[2\left(\omega t + \varphi\right)\right]\right\}.$$

A. J. Silenko, Phys. Rev. C **75**, 014003 (2007).

Measurement of tensor electric polarizability of the deuteron by the resonance method

- We use the matrix Hamiltonian for determining the evolution of the spin wave function:

$$i\frac{d\Psi}{dt} = H\Psi, \qquad \Psi = \begin{pmatrix} C_1(t) \\ C_0(t) \\ C_{-1}(t) \end{pmatrix}.$$

The polarization vector and tensor:

$$P_i = \frac{\langle S_i \rangle}{S}, \qquad P_{ij} = \frac{3\langle S_i S_j + S_j S_i \rangle - 2S(S+1)\delta_{ij}}{2S(2S-1)},$$

The spin matrices:

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

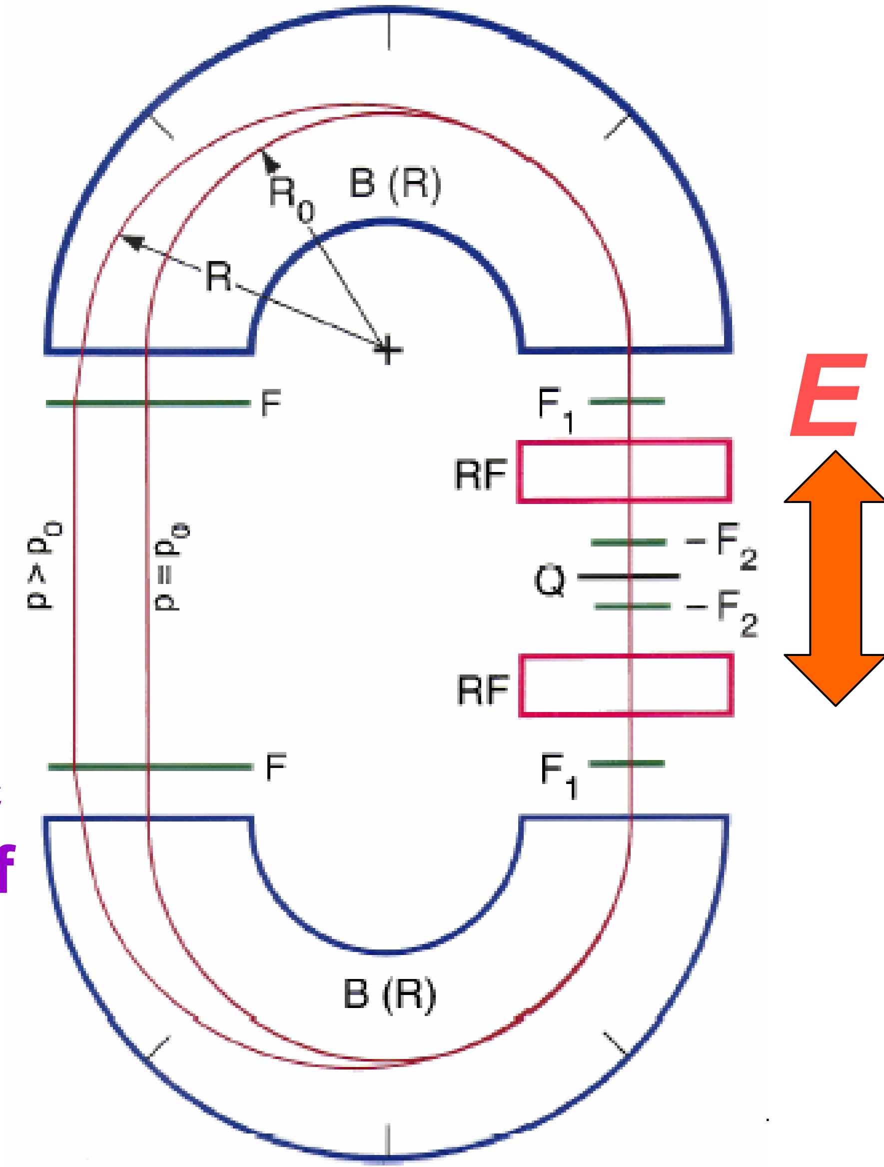
$$S_x^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \qquad S_y^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix},$$

$$S_x S_y + S_y S_x = i\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad S_z^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

Measurement of tensor electric polarizability of the deuteron by the resonance method

Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006).

Resonance Method of Electric-Dipole-Moment Measurements in Storage Rings



Oscillating longitudinal electric field stimulates an oscillation of particle velocity in resonance with spin precession

Initial matrix Hamiltonian:

$$\mathcal{H} = \begin{pmatrix} E_0 + \omega_0 + \mathcal{A} + \mathcal{B} & 0 & \mathcal{A} \\ 0 & E_0 + 2\mathcal{A} & 0 \\ \mathcal{A} & 0 & E_0 - \omega_0 + \mathcal{A} + \mathcal{B} \end{pmatrix},$$

$$\mathcal{A} = a_0 + a_1 \cos(\omega t + \varphi) + a_2 \cos[2(\omega t + \varphi)],$$

$$\mathcal{B} = b_0 + b_1 \cos(\omega t + \varphi) + b_2 \cos[2(\omega t + \varphi)],$$

$$a_0 = -\tfrac{1}{2}\alpha_T B_z^2 \gamma_0 \left[\beta_0^2 + \tfrac{1}{4}\left(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4\right)(\Delta\beta_0)^2\right],$$

$$a_1 = -\tfrac{1}{2}\alpha_T B_z^2 \left(2 + \beta_0^2\gamma_0^2\right)\beta_0\gamma_0 \cdot \Delta\beta_0,$$

$$a_2 = -\tfrac{1}{8}\alpha_T B_z^2 \left(2 + 5\beta_0^2\gamma_0^2 + 3\beta_0^4\gamma_0^4\right)\gamma_0(\Delta\beta_0)^2,$$

$$b_0 = -\beta_T B_z^2 \gamma_0 \left[1 + \tfrac{1}{4}\left(1 + 3\beta_0^2\gamma_0^2\right)\gamma_0^2(\Delta\beta_0)^2\right],$$

$$b_1 = -\beta_T B_z^2 \beta_0 \gamma_0^3 \cdot \Delta\beta_0,$$

$$b_2 = -\tfrac{1}{4}\beta_T B_z^2 \left(1 + 3\beta_0^2\gamma_0^2\right)\gamma_0^3(\Delta\beta_0)^2$$

Connection between the polarization vector and tensor and the spin amplitudes:

$$P_\rho = \frac{1}{\sqrt{2}}(C_1C_0^* + C_1^*C_0 + C_0C_{-1}^* + C_0^*C_{-1}),$$

$$P_\phi = \frac{i}{\sqrt{2}}(C_1C_0^* - C_1^*C_0 + C_0C_{-1}^* - C_0^*C_{-1}),$$

$$P_z = (C_1C_1^* - C_{-1}C_{-1}^*),$$

$$P_{\rho\rho} = \frac{3}{2}(C_1C_{-1}^* + C_1^*C_{-1} + C_0C_0^*) - \frac{1}{2},$$

$$P_{\phi\phi} = -\frac{3}{2}(C_1C_{-1}^* + C_1^*C_{-1} - C_0C_0^*) - \frac{1}{2},$$

$$P_{\rho\phi} = i\frac{3}{2}(C_1C_{-1}^* - C_1^*C_{-1}).$$

Vector- polarized beam
(initial horizontal polarization):

$$\boldsymbol{P}(0) = \sin\theta\cos\psi\,\boldsymbol{e}_\rho + \sin\theta\sin\psi\,\boldsymbol{e}_\phi + \cos\theta\,\boldsymbol{e}_z$$

$$P_{\rho\rho} = \frac{1}{2}\left[3\sin^2(\theta)\cos^2(\psi) - 1\right],$$

$$P_{\phi\phi} = \frac{1}{2}\left[3\sin^2(\theta)\sin^2(\psi) - 1\right], \quad P_{\rho\phi} = \frac{3}{4}\sin^2(\theta)\sin(2\psi)$$

Θ and ψ are spherical angles

Tensor- polarized beam
(initial horizontal polarization and $S_l = 0$):

$$\boldsymbol{P}(0) = 0, \quad P_{\rho\rho}(0) = 1 - 3\sin^2\theta\cos^2\psi,$$

$$P_{\phi\phi}(0) = 1 - 3\sin^2\theta\sin^2\psi, \quad P_{zz}(0) = 1 - 3\cos^2\theta,$$

$$P_{\rho\phi}(0) = -\frac{3}{2}\sin^2\theta\sin(2\psi),$$

$$P_{\rho z}(0) = -\frac{3}{2}\sin(2\theta)\cos\psi, \quad P_{\phi z}(0) = -\frac{3}{2}\sin(2\theta)\sin\psi.$$

Weaker resonance at the frequency $\omega \approx \omega_0$

$$P_z(t) = \left[1 - \frac{\mathcal{E}_0^2}{\omega'^2}\left(1-\cos\left(2\omega' t\right)\right)\right] P_z(0)$$

$$+\frac{2\mathcal{E}_0}{3\omega'}\left\{\frac{1}{2}[P_{\rho\rho}(0) - P_{\phi\phi}(0)]\left[\frac{\omega_0-\omega}{\omega'}\cos\left(2\varphi\right)\left(1-\cos\left(2\omega' t\right)\right) - \sin\left(2\varphi\right)\sin\left(2\omega' t\right)\right]\right.$$

$$\left.+P_{\rho\phi}(0)\left[\frac{\omega_0-\omega}{\omega'}\sin\left(2\varphi\right)\left(1-\cos\left(2\omega' t\right)\right) + \cos\left(2\varphi\right)\sin\left(2\omega' t\right)\right]\right\}, \quad \mathcal{E}_0 = \frac{a_2}{2},$$

Stronger resonance at the frequency $\omega \approx 2\omega_0$

$$P_z(t) = \left[1 - \frac{4\mathcal{E}_0'^2}{\omega''^2}\left(1-\cos\left(\omega'' t\right)\right)\right] P_z(0)$$

$$+\frac{2\mathcal{E}_0'}{3\omega''}\left\{[P_{\rho\rho}(0) - P_{\phi\phi}(0)]\left[\frac{2\omega_0-\omega}{\omega''}\cos\left(\varphi\right)\left(1-\cos\left(\omega'' t\right)\right) - \sin\left(\varphi\right)\sin\left(\omega'' t\right)\right]\right.$$

$$\left.+2P_{\rho\phi}(0)\left[\frac{2\omega_0-\omega}{\omega''}\sin\left(\varphi\right)\left(1-\cos\left(\omega'' t\right)\right) + \cos\left(\varphi\right)\sin\left(\omega'' t\right)\right]\right\}, \quad \mathcal{E}_0' = \frac{a_1}{2},$$

Final vertical beam polarization for initial horizontal vector polarization and $\omega \approx \omega_0$:

$$P_z(t) = \left[1 - \frac{\mathcal{E}_0^2}{\omega'^2}[1 - \cos(2\omega' t)]\right]\cos(\theta)$$
$$+\frac{\mathcal{E}_0}{2\omega'}\sin^2(\theta)\left\{\frac{\omega_0 - \omega}{\omega'}\cos[2(\psi - \varphi)][1 - \cos(2\omega' t)]\right.$$
$$\left. + \sin[2(\psi - \varphi)]\sin(2\omega' t)\right\}$$

Final vertical beam polarization for initial horizontal tensor polarization and $\omega \approx 2\omega_0$:

$$P_z(t) = -\frac{2\mathcal{E}_0'}{\omega''}\sin^2(\theta)\left\{\frac{2\omega_0 - \omega}{\omega''}\cos(2\psi - \varphi)[1 - \cos(\omega'' t)]\right.$$
$$\left. + \sin(2\psi - \varphi)\sin(\omega'' t)\right\}.$$

Theoretical data

Tensor electric polarizability of deuteron:

$\alpha_T = -6.2 \times 10^{-41}\ \mathrm{cm}^3$ — J.-W. Chen, H. W. Grießhammer, M. J. Savage, and R. P. Springer, Nucl. Phys. **A644**, 221 (1998).

$\alpha_T = -6.8 \times 10^{-41}\ \mathrm{cm}^3$ — X. Ji and Y. Li, Phys. Lett. **B591**, 76 (2004).

$\alpha_T = 3.2 \times 10^{-41}\ \mathrm{cm}^3$ — J. L. Friar and G. L. Payne, Phys. Rev. C **72**, 014004 (2005).

Tensor magnetic polarizability of deuteron:

$\beta_T = 1.95 \times 10^{-40}\ \mathrm{cm}^3$ — J.-W. Chen, H. W. Grießhammer, M. J. Savage, and R. P. Springer, Nucl. Phys. **A644**, 221 (1998).

X. Ji and Y. Li, Phys. Lett. **B591**, 76 (2004).

Estimated expirimental sensitivity of the resonance method

Tensor electric polarizability of deuteron:

$\delta\alpha_T \sim 10^{-45} \div 10^{-44}$ cm^3

(initial tensor-polarized beam)

A. J. Silenko, Phys. Rev. C **75**, 014003 (2007).

Tensor magnetic polarizabilities of the deuteron and other nuclei in storage ring experiments

Tensor magnetic polarizability of the deuteron can be measured without both a resonance field and a restriction of spin rotation

Only vertical magnetic field is used

A. J. Silenko, Phys. Rev. C **77**, 021001(R) (2008).

$$\mathcal{A} = -\frac{1}{2}\alpha_T \beta^2 \gamma B_z^2,$$

$$\mathcal{B} = -\beta_T \gamma B_z^2.$$

When the direction of the initial tensor polarization is defined by the spherical angles ***θ*** and ***ψ***,

$$P_\rho(t) = \sin(2\theta)\sin(\omega_0 t + \psi)\sin(bt),$$

$$P_\phi(t) = -\sin(2\theta)\cos(\omega_0 t + \psi)\sin(bt),$$

$$P_z(t) = 0,$$

$$b = \mathcal{B} - \mathcal{A} = -\left(\beta_T - \frac{1}{2}\alpha_T \beta^2\right)\gamma B_z^2.$$

When *t* ~ 1000 s, $P_{\|}$ ~ 1 %.

This polarization is measurable.

Measurement of tensor electric and magnetic polarizabilities of the deuteron by the frozen spin method

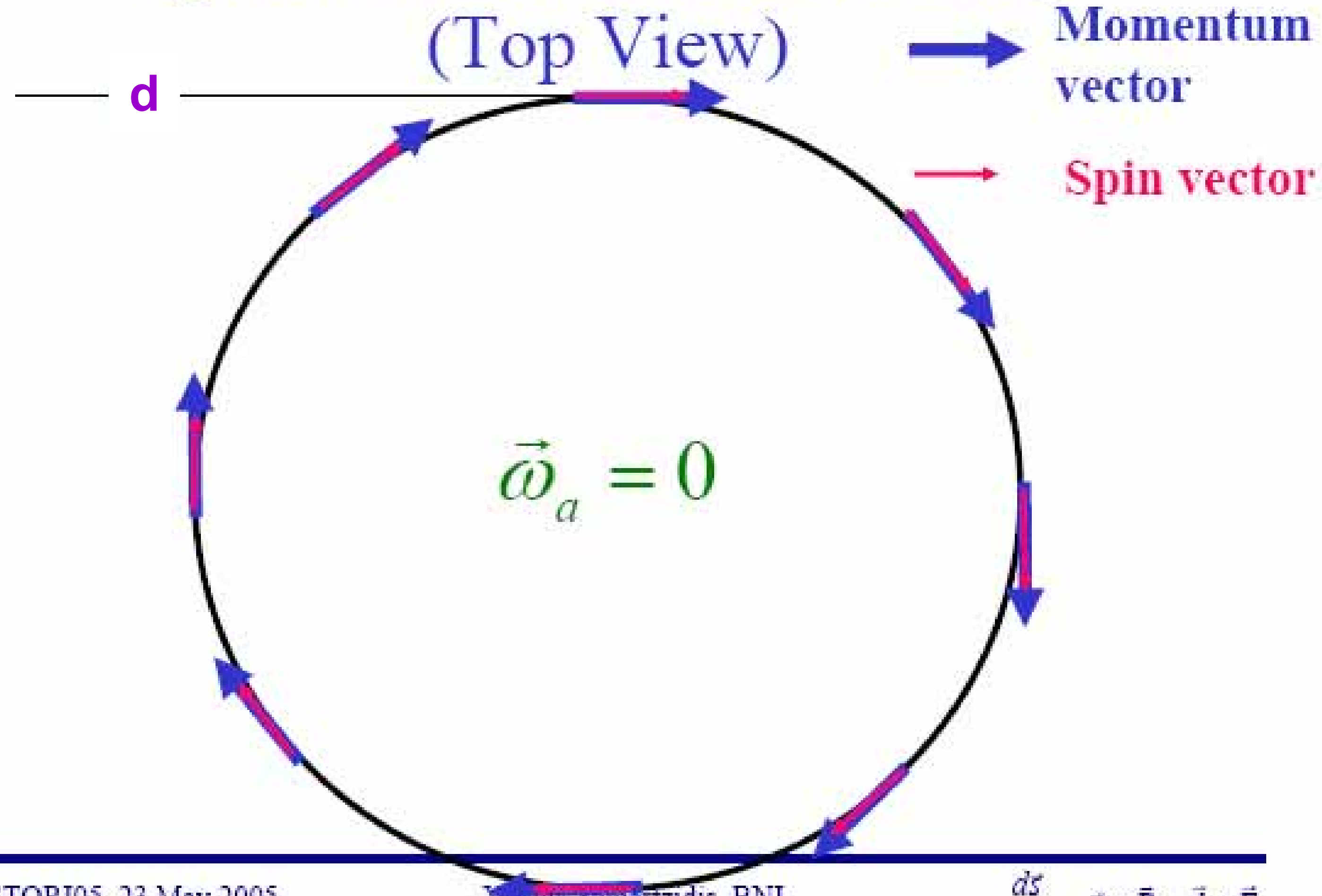
$$\text{щ}_a = -\frac{e}{m}\left[a\mathbf{B} - \frac{a\gamma}{\gamma+1}\,\textbf{в}\left(\textbf{в}\cdot\mathbf{B}\right) + \left(\frac{1}{\gamma^2-1} - a\right)\textbf{в}\times\mathbf{E} \right].$$

$$a = \frac{g-2}{2}, \qquad \textbf{в}\cdot\mathbf{B} = 0, \qquad E_\rho = \frac{a\beta_\phi\gamma^2}{1-a\beta^2\gamma^2}B_z.$$

The spin orientation relatively the momentum direction remains unchanged!

F. J. M. Farley, K. Jungmann, J. P. Miller, W. M. Morse, Y. F. Orlov, B. L. Roberts, Y. K. Semertzidis, A. Silenko, and E. J. Stephenson, Phys. Rev. Lett. **93**, 052001 (2004).

Spin Precession in EDM Ring
(Top View)



$$\frac{d\vec{s}}{dt} = \vec{\mu} \times \vec{B} + \vec{d} \times \vec{E}$$

Correction to the Hamilton operator

$$V = -\alpha_T \gamma \left(\beta_\phi B_z + E_\rho\right)^2 S_\rho^2$$

$$-\beta_T \gamma \left(B_z + \beta_\phi E_\rho\right)^2 S_z^2.$$

$$V = -\frac{\gamma}{\left(1 - a\beta^2\gamma^2\right)^2} B_z^2 \left[\alpha_T (1+a)^2 \beta^2 S_\rho^2 + \beta_T S_z^2\right].$$

$$\mathcal{A} = -\alpha_T \frac{(1+a)^2 \beta^2 \gamma}{2\left(1 - a\beta^2\gamma^2\right)^2} B_z^2,$$

$$\mathcal{B} = -\beta_T \frac{\gamma}{\left(1 - a\beta^2\gamma^2\right)^2} B_z^2.$$

When the direction of the initial tensor polarization is defined by the spherical angles ***θ*** and ***ψ***,

$$P_\rho(t) = \sin\left(2\theta\right)\left[bt\cdot\sin\left(\omega_0 t+\psi\right)+\frac{\mathcal{A}}{\omega_0}\sin\left(\omega_0 t\right)\sin\psi\right],$$

$$P_\phi(t) = \sin\left(2\theta\right)\left[-bt\cdot\cos\left(\omega_0 t+\psi\right)+\frac{\mathcal{A}}{\omega_0}\sin\left(\omega_0 t\right)\cos\psi\right],$$

$$P_z(t) = -\frac{2\mathcal{A}}{\omega_0}\sin^2\theta\sin\left(\omega_0 t\right)\sin\left(\omega_0 t+2\psi\right),$$

$$b=\mathcal{B}-\mathcal{A}=-\frac{\gamma}{\left(1-a\beta^2\gamma^2\right)^2}B_z^2\left[\beta_T-\frac{1}{2}\alpha_T\left(1+a\right)^2\beta^2\right].$$

The frozen spin method provides a weaker magnetic field than other methods

The frozen spin method provide a less experimental sensitivity of measurement of the tensor electric polarizability of the deuteron than the resonance method

Nevertheless, well-controlled dynamics of the horizontal spin components can improve the experimental conditions of measurement of tensor magnetic polarizability of the deuteron by the frozen spin method as compared with other methods

V.G. Baryshevsky, A.A. Gurinovich,
Nucl. Instrum. Meth. B **252**, 136 (2006).

- **Particle motion in bent and straight crystals is accompanied by a rotation and an oscillation of the particle spin that allows to measure the tensor electric and magnetic polarizabilities of nuclei and elementary particles**
- **When $1<g<2$ and $\gamma=g/(g-2)$, the electric field of a bent crystal does not influence the spin and the effect of the tensor polarizabilities can be measured with a vector-polarized beam**
- **If one uses an initial tensor-polarized beam, the effect of the tensor polarizabilities can be measured for arbitrary *g* and *γ* in both bent and straight crystals**
- **However, the effects are very small: an additional vector polarization of the beam is of order of 10^{-4} or less**

Summary

- Spin dynamics caused by the tensor electric and magnetic polarizabilities of the deuteron and other nuclei in storage rings is calculated
- Potential for the measurement of the tensor polarizabilities of the deuteron and other nuclei in experiments with polarized beams is investigated
- For the tensor electric polarizability of the deuteron, the resonance method provides the experimental sensitivity of the order of

  $10^{-45} \div 10^{-44}$ cm^3
- Both a purely magnetic ring and the frozen spin method can be used for the measurement of the tensor magnetic polarizability of the deuteron

Thank you for attention