

Multichannel Description of Light and Intermediate Scalar Mesons

George Rupp

CFIF, Instituto Superior Técnico, Lisbon

Present collaborators: Eef van Beveren and Susana Coito
Previously: Eef van Beveren, David Bugg and Frieder Kleefeld

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I. Introduction to Scalar Mesons

⇒ Scalar mesons have been very problematic in standard quark models:

- ground-state nonet seems too light for P -wave $q\bar{q}$ states, i.e., $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$, and $a_0(980)$;
- approximate mass degeneracy of $f_0(980)$ and $a_0(980)$ is puzzling;
- $f_0(600)$ and $K_0^*(800)$ are very broad, while $f_0(980)$ and $a_0(980)$ are much narrower;
- $K_0^*(800)$ still controversial experimentally (in PDG2008, but omitted (?) from summary table);
- disagreement on interpretation of $f_0(1370)$ (existence?) and especially $f_0(1500)$ (glueball?), besides $f_0(1710)$ and tentative $f_0(1770)$.

⇒ Some major theoretical approaches:

- $qq\bar{q}\bar{q}$ states (*Jaffe*, 1977; many followers more recently), explaining light scalars with large colour-hyperfine interaction, and also $f_0(980)/a_0(980)$ degeneracy;
- quark-level Linear σ Model (qLSM) (*Scadron*, 1982; *Delbourgo & Scadron*, 1995, 1998; *Scadron et al.*, 2000 →), self-consistent nonperturbative theory of light scalars and pseudoscalars as fundamental fields as well as $q\bar{q}$ states;

- mesonic t -channel exchanges produce $a_0(980)$ and $f_0(980)$ as $K\bar{K}$ molecules, and σ (too light) as a purely dynamical resonance (*Isgur & Weinstein*, 1982, 1990; *Janssen, Pearce, Holinde, Speth*, 1990, 1995);
- effective-Lagrangian model (*Schechter et al.*, 1995 \rightarrow), light scalars are mixtures of $q^2\bar{q}^2$ (dominant) and $q\bar{q}$ states;
- K -matrix method to extract “bare” states from the data (*Anisovich, Sarantsev*, 1995 \rightarrow), K -matrix poles are claimed to be linked to QCD model predictions;
- unitarised Chiral Perturbation Theory (ChPT) (*Oller, Oset, Peláez*, 1998 \rightarrow), light scalars are dynamical poles in meson-meson scattering that disappear (?) in the $N_c \rightarrow \infty$ limit;
- Roy-equation approach to the $\pi\pi$ S -wave, with input from ChPT, produces dynamical σ pole (*Caprini, Colangelo, Leutwyler*, 2006).

\Rightarrow Alternative is unitarisation of bare scalar states, via their strong coupling to S -wave two-meson channels:

- Helsinki unitarised quark model of *Törnqvist* (1982, 1995) and *Törnqvist, Roos* (1996); doubling of some scalar states, producing e.g. a light σ , but **no** light κ ;
- Unitarised quark-meson models of *van Beveren, Dullemond, Rijken, Rupp et al.* (1983, 1986) and *van Beveren, Rupp* (1998 \rightarrow); unitarisation gives rise to **two complete** light scalar nonets, starting from only one bare P -wave $q\bar{q}$ nonet. Also new charmed scalars $D_{s0}^*(2317)$ and $D_{sJ}^*(2860)$ reproduced (2003) resp. predicted (2006).

⇒ Note:

- Original quark-meson model, published in *Z. Phys. C* **30** (1986) 615 predicted the poles of the light scalar nonet at:

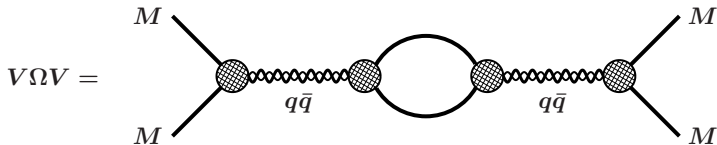
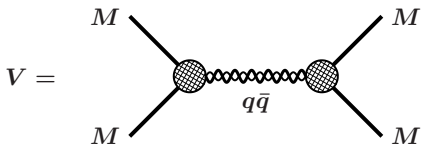
$$\sigma: 470 - i208 \text{ MeV}; \quad \kappa: 727 - i263 \text{ MeV};$$

$$f_0(980): 994 - i20 \text{ MeV}; \quad a_0(980): 968 - i28 \text{ MeV}.$$

- These were **not** the result of a fit;
- The few parameters had been fixed previously to charmonium and bottomonium, as well as some light vector and pseudoscalar mesons;
- The employed confining potential for the bare spectrum was a harmonic oscillator (HO) with mass-independent frequency;
- This model predicted a $\rho(1250)$ resonance as the first radially excited ρ meson;
- The $\rho(1250)$ has now been confirmed once again in the analysis of *Surovtsev and Bydžovský*, *Nucl. Phys. A* **807** (2008) 145;
- In the recent formulation of the model (see next slide), only the confinement spectrum enters the equations, thus allowing much more flexibility;
- Nevertheless, in **all** phenomenological applications, we have been sticking to the HO, simply because it works;
- Anyone who disagrees, may try their favourite confinement spectrum in our expressions, to see whether better results can be obtained!

II. Resonance Spectrum Expansion Model and Applications

⇒ Building blocks of Resonance Spectrum Expansion (RSE) are:



- V is the effective two-meson potential;
- Ω is the two-meson loop function;
- the blobs are the 3P_0 vertex functions, modelled by a spherical δ shell in coordinate space, i.e., a spherical Bessel function in momentum space;
- the wiggly lines stand for s -channel exchanges of infinite towers of $q\bar{q}$ states, i.e., a kind of Regge propagators (see [arXiv:0809.1149 \[hep-ph\]](https://arxiv.org/abs/0809.1149)).

Explicitly:

$$V_\ell(p) = \frac{\lambda^2}{r_0} \sum_{N=0}^{\infty} \frac{|g_{NL}|^2}{E(p) - E_{NL}} \quad , \quad \Omega_\ell(p) = -2i\mu p r_0^2 j_\ell(pr_0) h_\ell^{(1)}(pr_0) \quad , \quad (1)$$

where ℓ refers to the ℓ -th two-meson partial wave, and N, L are the radial resp. angular $q\bar{q}$ quantum numbers.

The (amputated) amplitude for non-exotic two-meson scattering is then just

$$t = V + V\Omega V + V\Omega V\Omega V + \dots = V [1 - \Omega V]^{-1} \quad , \quad (2)$$

while the unitarised partial-wave T -matrix is given by

$$T_\ell = X_\ell^\dagger t_\ell X_\ell = X_\ell^\dagger V_\ell [1 - \Omega_\ell V_\ell]^{-1} X_\ell \quad , \quad X_\ell^\dagger X_\ell = X_\ell X_\ell^\dagger = \Im m(\Omega_\ell) \quad . \quad (3)$$

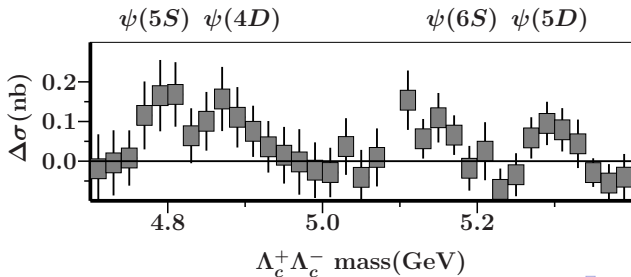
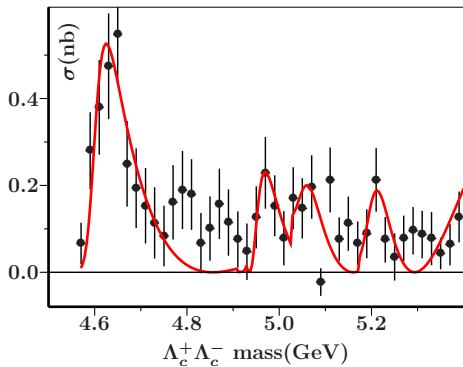
⇒ **Note:**

- This model does **not** correspond to the exchange of an s -channel resonance only, but of a whole spectrum of states;
- From duality arguments, this will effectively also account for some t -channel phenomena;
- Concretely, approximating the infinite sum in Eq. (1) by one or more s -channel states plus a remainder, one obtains the typical s -channel seed(s) plus contact term of (unitarised) chiral models. See [arXiv:0809.1149 \[hep-ph\]](https://arxiv.org/abs/0809.1149) for details.

⇒ Applications:

- Single-channel description of $K_0^*(800)$, $K_0^*(1430)$ (EPJC 22 (2001) 493), of $D_{s0}(2317)$ (PRL 91 (2003) 012003), and of $D_{s1}(2460)$, $D_1(2400)$ (EPJC 32 (2001) 493);
- Multichannel description of $f_0(600)$, $K_0^*(800)$, $f_0(980)$, and $a_0(980)$ (PLB 641 (2006) 265, see below), and of $D_{sJ}(2860)$ (PRL 97 (2006) 202001);
- RSE formulation of production amplitudes, manifestly satisfying generalised unitarity (AOP 323 (2008) 1215);
- Single-channel application of production formalism to production processes of $K_0^*(800)$ and $f_0(600)$ (JPG 34 (2007) 1789);
- Deriving a **complex** relation between production and scattering amplitudes, while respecting (generalised) unitarity, allows to separate dynamics from kinematics when describing production processes (EPL 81 (2008) 61002);
- Applying the formalism to $\pi\pi$ production processes at very different mass scales allows to deduce the effective string-breaking distance, in agreement with recent lattice calculations (arXiv:0712.1771 [hep-ph]);
- Production amplitudes exhibit zeros in leading order, which are not present in scattering amplitudes. Application to the recent BELLE enhancement at 4.634 GeV in $\Lambda_c^+\Lambda_c^-$ leads to an indication of 5 new charmonium vector states, viz. $\psi(5S, 6S, 3D, 4D, 5D)$ (arXiv:0809.1151 [hep-ph]; also see Fig. 1 on next slide).

Figure 1:



III. Light and Intermediate Scalar Mesons

⇒ The T -matrix can be solved in closed form, even with 2 coupled $q\bar{q}$ channels.

Writing $T_{ij}(E) = N_{ij}(E)/D(E)$, we have

$$N_{ij}(E) = -2r_0\lambda^2 \sqrt{\mu_i\mu_j k_i k_j} j_{\ell_i}(k_i r_0) j_{\ell_j}(k_j r_0) \left(\sum_{\alpha=1}^2 \left\{ \sum_{s=0}^{\infty} \frac{g_{\alpha i}(s) g_{\alpha j}(s)}{E - E_{\alpha s}} \right\} + \right. \\ \left. + 2ir_0\lambda^2 \sum_{n=1}^{N_f} \left\{ \sum_{s,s'=0}^{\infty} \frac{\begin{vmatrix} g_{1i}(s) & g_{1n}(s) \\ g_{2i}(s') & g_{2n}(s') \end{vmatrix} \begin{vmatrix} g_{1j}(s) & g_{1n}(s) \\ g_{2j}(s') & g_{2n}(s') \end{vmatrix}}{(E - E_{1s})(E - E_{2s'})} \right\} \mu_n k_n j_{\ell_n}(k_n r_0) h_{\ell_n}^{(1)}(k_n r_0) \right)$$

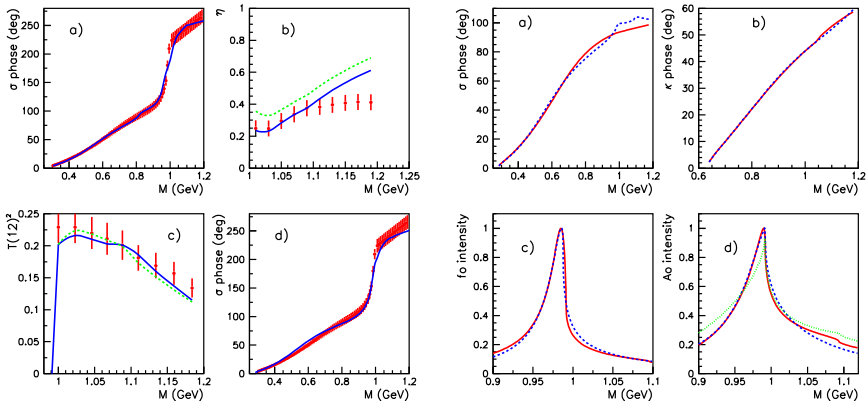
and

$$D(E) = 1 + 2ir_0\lambda^2 \sum_{\alpha=1}^2 \sum_{n=1}^{N_f} \left\{ \sum_{s=0}^{\infty} \frac{g_{\alpha n}^2(s)}{E - E_{\alpha s}} \right\} \mu_n k_n j_{\ell_n}(k_n r_0) h_{\ell_n}^{(1)}(k_n r_0) - 2r_0^2\lambda^4 \times \\ \times \sum_{\substack{n,n'=1 \\ n \neq n'}}^{N_f} \left\{ \sum_{s,s'=0}^{\infty} \frac{\left| \begin{vmatrix} g_{1n}(s) & g_{1n'}(s) \\ g_{2n}(s') & g_{2n'}(s') \end{vmatrix} \right|^2}{(E - E_{1s})(E - E_{2s'})} \right\} \mu_n k_n j_{\ell_n}(k_n r_0) h_{\ell_n}^{(1)}(k_n r_0) \mu_{n'} k_{n'} j_{\ell_{n'}}(k_{n'} r_0) h_{\ell_{n'}}^{(1)}(k_{n'} r_0),$$

where the index $\alpha = 1, 2$ refers to the two confinement channels, respectively.

⇒ Published work:

- E. van Beveren, D. V. Bugg, F. Kleefeld, and G.R., *Phys. Lett. B* **641** (2006) 265. Only pseudoscalar-pseudoscalar (PP) channels were included, i.e., $\pi\pi$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$ for coupled σ - $f_0(980)$ system; $\eta\pi$, KK , $\eta'\pi$, for $a_0(980)$; $K\pi$, $K\eta$, $K\eta'$ for κ . Results:



- Pole positions:

σ : 530 - i 226 MeV; κ : 745 - i 316 MeV;

$f_0(980)$: 1007 - i 38 MeV; $a_0(980)$: 1021 - i 47 MeV.

⇒ Preliminary new results:

- Now we also include all vector-vector (VV) channels (for $L = 0$ and approximately for $L = 2$), i.e., $\rho\rho$, $\omega\omega$, K^*K^* , $\phi\phi$ for coupled σ - $f_0(980)$ system; $\rho\omega$, K^*K^* for $a_0(980)$; ρK^* , ωK^* , ϕK^* for κ .
- Two-parameter fit to data compiled by Bugg and Leutwyler is carried out in the isoscalar case, up to 1.9 GeV. The fit is very good up to 1 GeV, reasonable up to 1.2 GeV, and quite off thereabove, though not crazy (see Fig. 2 on next slide). Clearly, structure is lacking here in our approach. See the conclusions for possible remedies.

Poles for PP+VV fit:

σ : 457 - i 233 MeV; $f_0(980)$: 995 - i 55 MeV (second sheet!);
 $f_0(1370)$ (?): 1211 - i 106 MeV; $f_0(1500)$ (?): 1519 - i 194 MeV.

⇒ Note: Turning off the $\eta\eta$ channel changes the $f_0(980)$ pole only slightly:
999 - i 39 MeV!

- In the isodoublet case, no good fit is possible with the VV channels. In Fig. 3 below we show the two-parameter fit to LASS data up to 1.5 GeV, with only PP channels, and then the computed result with the inclusion of the VV channels, without any refit. For remedies, see conclusions.

Poles for PP fit:

κ : 758 - i 295 MeV;
 $K_0^*(1430)$: 1410 - i 124 MeV;

Figure 2:

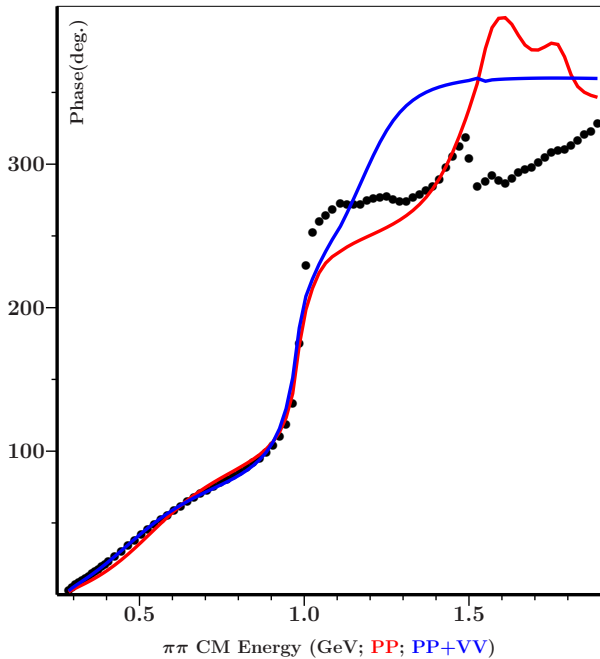
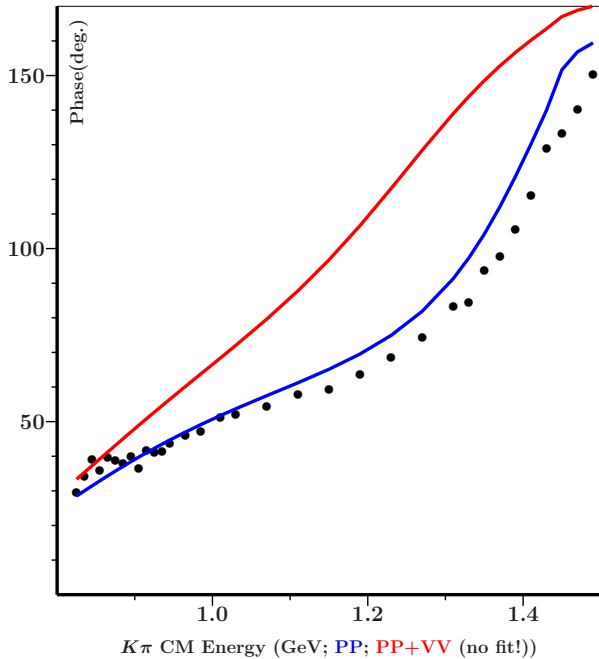


Figure 3:



- In the isotriplet case, inclusion of the VV channels does improve the fit to the $a_0(980)$ line shape derived by Bugg in a data analysis. Letting now also the pseudoscalar mixing angle vary freely, the latter quantity settles at the reasonable value of 43.7° in the flavour basis.

Poles for PP+VV fit:

$a_0(980)$: $1018 - i44$ MeV (second sheet); $a_0(1450)$: $1410 - i261$ MeV.

IV. Conclusions and Outlook

- Inclusion of VV channels has a sizable influence on the S -wave phase shifts above 1 GeV, and in the isodoublet case at even lower energies;
- In the isoscalar and isotriplet case, some improvement can be observed, also allowing to extract reasonable to good values for resonances in the 1.2–1.6 GeV region.
- The situation in the isodoublet case is puzzling and requires further analysis.
- Planned improvements of the present approach amount to the inclusion of scalar-scalar channels as well, such as $\sigma\sigma$ in the isoscalar case.
- Perhaps more importantly, we intend to allow for truly unstable particles in the final state, via complex masses, which can nevertheless be done satisfying unitarity, viz. through Eq. (3). This will make it possible to simulate e.g. the important 4π decays, via $\sigma\sigma$ and $\rho\rho$.