Kaon wave function at NNLO in QCD

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XIX International Baldin Seminar on high energy physics problems
Relativistic Nuclear Physics and Quantum Chromodynamics
Dubna, Russia, Sep 29 – Oct 4, 2008

Outline

- Light-cone distribution amplitudes
- Correlation function at NNLO
- Numeric analysis with QCD sum rules
- Conclusion

1. Light-cone distribution amplitudes (LCDA)

LCDA enter factorization formulae used for description of exclusive processes in QCD. Typical representation for a physical amplitude $\mathcal{A}(Q)$ is

$$\mathcal{A}(Q) = \int d\xi \ C(Q, \alpha_s(Q), \mu_F; \xi) \otimes \phi(\Lambda_{\rm QCD}, \mu_F; \xi)$$

with $C(Q, \alpha_s(Q), \mu_F; \xi)$ - hard part computed in PT in $\alpha_s(Q)$ at scale Q, and $\phi(\Lambda_{\rm QCD}, \mu_F; \xi)$ - soft part characterizing an hadron.

Examples:
Pion EM form factor at large Q^2
Light cone sum rules for form factors of heavy hadrons
QCD factorization in B-meson decays

The formal framework at present – soft-collinear effective theory

Twist-2 LCDA of the kaon $\varphi_K(u,\mu)$ is defined through a matrix element of nonlocal quark-antiquark operator with light-cone like separation

$$\langle K^-(q)|\bar{s}(z)\gamma_\mu\gamma_5\,[z,-z]\,u(-z)|0\rangle_{z^2=0}$$

$$= -iq_\mu f_K \int_0^1 du\; e^{iuq\cdot z - i\bar{u}q\cdot z}\varphi_K(u,\mu)$$

s- and $\bar{u}$ carry the momentum fractions u and $\bar{u}=1-u$; the Wilson line

$$[x_1,x_2] = P\exp(i\int_0^1 dv(x_1-x_2)_\rho A^\rho(vx_1+\bar{v}x_2))$$

makes the matrix element gauge invariant;

μ – the normalization scale.

Gegenbauer polynomials $C_n^{3/2}(x)$ expansion

$$\varphi_K(u,\mu) = 6u\bar{u}\left(1 + \sum_{n=1}^{\infty} a_n^K(\mu) C_n^{3/2}(u-\bar{u})\right)$$

$a_n^K(\mu)$ - Gegenbauer moments.

a_1^K is related to the difference between the longitudinal momenta of the strange and nonstrange quarks in the kaon.

We determine a numerical value of this asymmetry parameter $a_1^K(\mu)$ at a low scale $\mu \sim 1 \text{ GeV}$ with NNLO accuracy
(published in K. G. Chetyrkin, A. Khodjamirian, and AAP,
Phys. Lett. B **661**, 661 (2008))

The method of calculation is based on QCD sum rules.

a_1^K reduces to the vacuum-to-kaon matrix element of a local operator

$$\langle K^-(q)|\bar{s}\gamma_\nu\gamma_5 i\overleftrightarrow{D}_\lambda u|0\rangle = -iq_\nu q_\lambda f_K \frac{3}{5} a_1^K$$

$$\overleftrightarrow{D}_\lambda = \overrightarrow{D}_\lambda - \overleftarrow{D}_\lambda$$

Previous results (average)

$$a_1^K(1 \text{ GeV}) = 0.06 \pm 0.03$$

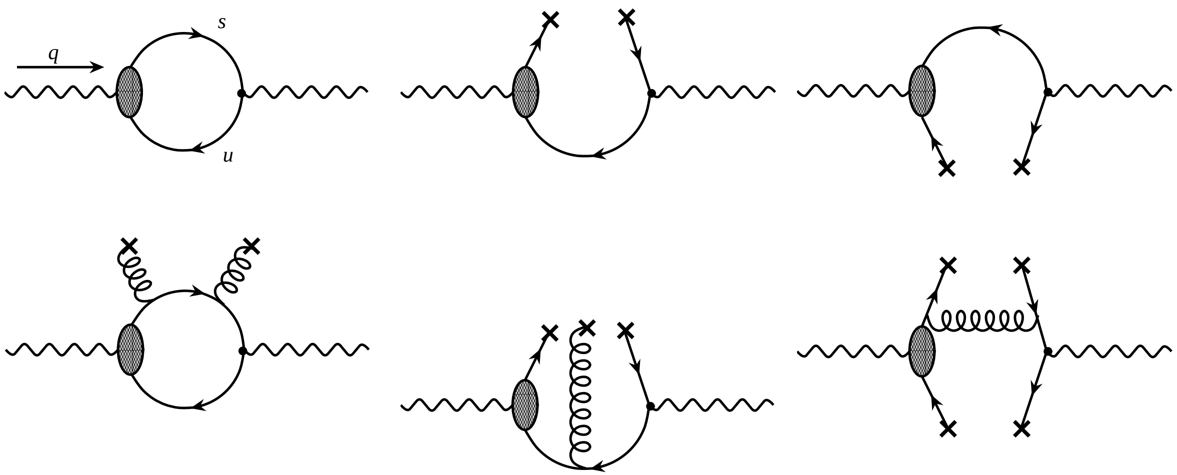
The error of the prediction is large: 50%

2. Correlation function

Correlation function for a_1^K reads

$$\Pi_{\mu\nu\lambda}(q) = i\int d^4x\ e^{iq\cdot x}\langle T\left\{\bar{u}(x)\gamma_\mu\gamma_5 s(x), \bar{s}(0)\gamma_\nu\gamma_5 i\overleftrightarrow{D}_\lambda u(0)\right\}\rangle$$

Diagrams for OPE at LO: PT loop, quark-condensate, gluon-, quark-gluon and four-quark condensate diagrams.



OPE gives an expansion for $\Pi(Q^2)$ in powers of $1/Q$ at large Q

$$\Pi(Q^2,\mu) = \frac{\mathcal{A}_2(Q^2,\mu)}{Q^2} + \frac{\mathcal{A}_4(Q^2,\mu)}{Q^4} + \frac{\mathcal{A}_6(Q^2,\mu)}{Q^6} + ...$$

$\mathcal{A}_j$ has a double expansion in α_s and m_s^2 (u,d-quark masses are neglected)

$$\mathcal{A}_d = a_d^{(0,0)} + \left(\frac{\alpha_s}{\pi}\right) a_d^{(1,0)} + \left(\frac{\alpha_s}{\pi}\right)^2 a_d^{(2,0)} + \left(\frac{m_s^2}{Q^2}\right) a_d^{(0,1)}$$

$$+ \left(\frac{m_s^2}{Q^2}\right)^2 a_d^{(0,2)} + \left(\frac{\alpha_s}{\pi}\right)\left(\frac{m_s^2}{Q^2}\right) a_d^{(1,1)} + \ldots$$

Numerical role of small parameters at $Q^2 \simeq 1\ \text{GeV}^2$:
for $\alpha_s(1\ \text{GeV}) = 0.47$ and $m_s(1\ \text{GeV}) < 150\ \text{MeV}$,
one has $m_s^2/Q^2 \leq 0.02 \ll \alpha_s/\pi \simeq 0.15$.

So far only the $O(\alpha_s)$ correction (the NLO accuracy in α_s) to the quark-condensate contribution $\mathcal{A}_4$ was calculated.
The result has been obtained by two groups that contradict each other.

In our work, for the largest $d = 2, 4$ terms of the OPE the NNLO accuracy in α_s ($O(\alpha_s^2)$ order) is achieved.
The discrepancy between previous calculations has been resolved.

The techniques of multi-loop calculations are employed. Programs:

QGRAF – diagram generation,

FORM – symbolic manipulation for large (in fact, huge) expressions,

MINCER – a routine for three-loop diagrams computation.

Results of the calculation for the correlator at NNLO:

$$d = 2: \quad \mathcal{A}_2(Q^2,\mu) = \frac{m_s^2}{4\pi^2}\Bigg(1 + \frac{\alpha_s}{\pi}\left[\frac{26}{9} + \frac{10}{9} l_Q\right]$$

$$+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{366659}{11664} - \frac{29}{9}\zeta(3) + \frac{14449}{972} l_Q + \frac{605}{324} l_Q^2\right] + 3\frac{m_s^2}{Q^2}\left(\frac{5}{2} + l_Q\right)\Bigg);$$

$$d = 4: \quad \mathcal{A}_4(Q^2,\mu) = -m_s\langle\bar{s}s\rangle\Bigg(1 - \frac{\alpha_s}{\pi}\left[\frac{112}{27} + \frac{8}{9} l_Q\right]$$

$$-\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{28135}{1458} - 4\zeta(3) + \frac{218}{27} l_Q + \frac{49}{81} l_Q^2\right] + 2\frac{m_s^2}{Q^2}\Bigg)$$

$$-m_s\langle\bar{u}u\rangle\left(\frac{4\alpha_s}{9\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{59}{54} + \frac{49}{81} l_Q\right]\right)$$

For completeness I give here the expression for dimension six contribution to the correlation function.

It is not large and known at the LO in $O(\alpha_s)$ only.

$$d = 6:$$

$$\mathcal{A}_6(Q^2,\mu) = \frac{2}{3}m_s\langle\bar{s}Gs\rangle + \frac{1}{3}m_s^2\langle G^2\rangle\,(1+l_Q) - \frac{32}{27}\pi\alpha_s\Big(\langle\bar{s}s\rangle^2 - \langle\bar{u}u\rangle^2\Big)$$

Here $l_Q = \ln(\mu^2/Q^2)$.

Now we turn to a physical representation of the correlation function necessary for the QCD sum rules analysis

The hadronic dispersion relation reads

$$\Pi(q^2) = \frac{\frac{3}{5} a_1^K f_K^2}{m_K^2 - q^2} + \int\limits_{s_h}^{\infty} ds \frac{\rho^h(s)}{s - q^2} \,.$$

The $\rho^h(s)$ includes contributions of $K\pi\pi$, $K^*\pi$, $K\rho$, $K_1(1270)$, $K_1(1400)$,... To approximate $\rho^h(s)$, we employ the quark-hadron duality

$$\rho^h(s) = \rho^{OPE}(s)\Theta(s - s_0^K) \,,$$

where s_0^K is the effective threshold.

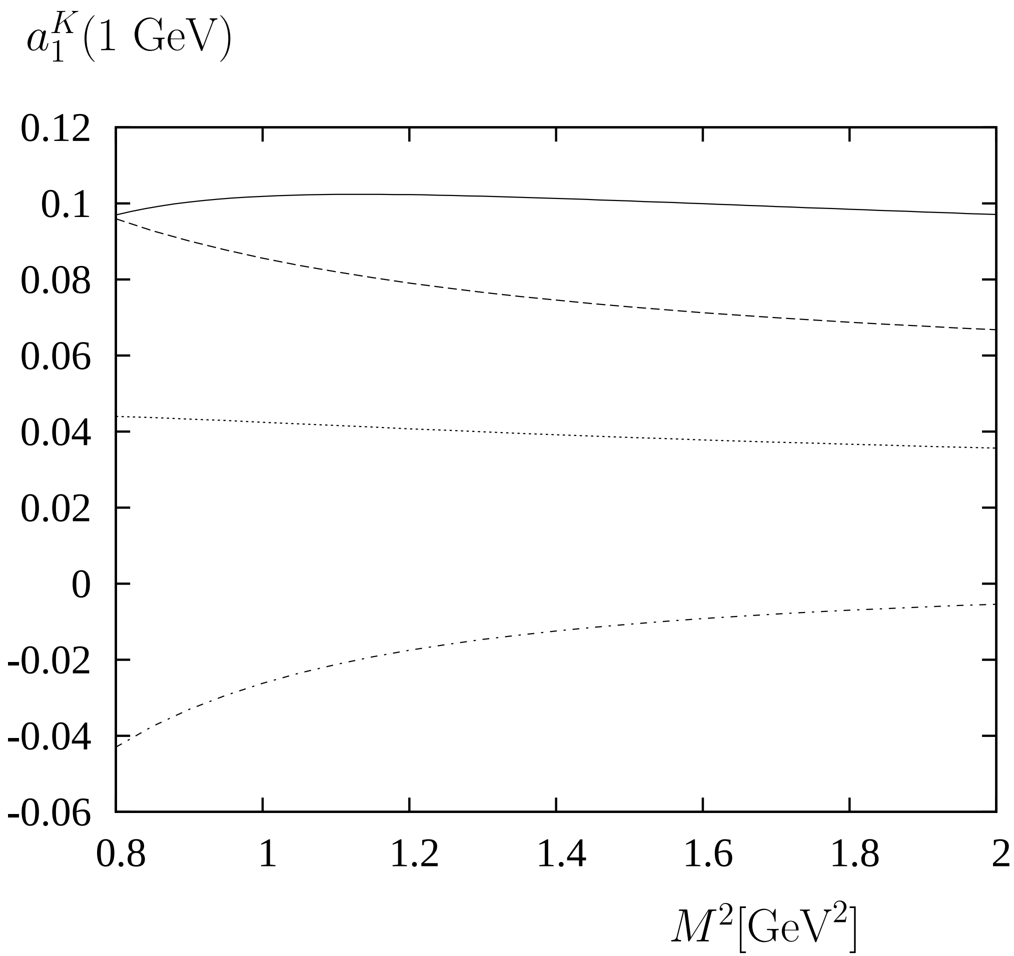
Finally, the sum rule for a_1^K takes the form

$$a_1^K = \frac{5}{3f_K^2} e^{m_K^2/M^2} \left(\Pi(M^2) - \int\limits_{s_0^K}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2} \right)$$

3. Numerical analysis

Input parameters:

- kaon mass $m_K^{\pm} = 493.58$ MeV
- kaon decay constant $f_K = 159.8 \pm 1.4 \pm 0.44$ MeV
- strange quark mass $m_s(2\text{ GeV}) = 98 \pm 16$ MeV
- $\alpha_s(m_Z) = 0.1176 \pm 0.002$ ($\alpha_s(1\text{ GeV})/\pi = 0.15 \pm 0.01$)
- $\langle \bar{q}q(2\text{ GeV})\rangle = -(0.264^{+0.031}_{-0.020}\text{ GeV})^3$
- $\langle \bar{s}Gs\rangle = m_0^2\langle \bar{s}s\rangle(1\text{ GeV})$ with $m_0^2 = 0.8 \pm 0.2\text{ GeV}^2$



$a_1^K(1\text{ GeV})$ as a function of the Borel parameter (solid); $d = 2$, $d = 4$ and $d = 6$ terms are shown with dashed, dotted and dash-dotted lines; $s_0^K = 1.05\text{ GeV}^2$, s_0^K-dependence is weak

Numerical prediction of the sum rule is

$$a_1^K(1 \text{ GeV}) = 0.100$$

$$\pm 0.003|_{\rm SR} \pm 0.003|_{\alpha_s} \pm 0.035|_{m_s} \pm 0.022|_{m_q} \pm 0.013|_{cond}$$

Adding the individual uncertainties in quadrature we obtain

$$a_1^K(1 \text{ GeV}) = 0.10 \pm 0.04$$

Turn to the pattern of convergence of the perturbative series as it appears in numerical analysis of sum rules.

PT corrections strongly enhance $d = 2$, $O(m_s^2)$ term

$$\Pi^{(m_s^2)} = \frac{m_s^2}{4\pi^2}\left[1 + 3.53\left(\frac{\alpha_s}{\pi}\right) + 33.7\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

For quark condensate contribution ($d = 4$) corrections are smaller

$$\Pi^{(m_s\langle\bar{s}s\rangle)} = m_s\langle\bar{s}s\rangle\left[1 - 3.77\left(\frac{\alpha_s}{\pi}\right) - 10.8\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

Thus, at NNLO the numerical pattern of the sum rule for a_1^K changes: relative weight of $d = 2$ term becomes larger.

The NNLO precision in α_s makes the dependence of the prediction for a_1^K on the renormalization scale μ very week.

4. Conclusion

The NNLO PT corrections to the QCD sum rule for a_1^K are numerically important, they change the relative magnitude of the $d = 2$ (loop diagrams) and $d = 4, 6$ (condensate) terms in the OPE and give

$$a_1^K(1 \text{ GeV}) = 0.10 \pm 0.04$$

while previous result (average)

$$a_1^K(1 \text{ GeV}) = 0.06 \pm 0.03$$

The uncertainty of a_1^K is still large due mainly to the poor precision of the light quark masses: m_s directly entering the sum rule and $m_{u,d}$ determining the quark-condensate densities via Gell-Mann-Oakes-Renner relation.

Our result for a_1^K is larger than two recent lattice determinations:

$$a_1^K(2\ \mathrm{GeV}) = 0.0453 \pm 0.0009 \pm 0.0029$$

V. M. Braun *et al.*, [QCDSF/UKQCD Collaboration]
Phys. Rev. D **74**, 074501 (2006)

and

$$a_1^K(2\ \mathrm{GeV}) = 0.048 \pm 0.003$$

M. A. Donnellan *et al.*, "Lattice Results for Vector Meson Couplings and Parton Distribution Amplitudes," arXiv:0710.0869 [hep-lat];
P. A. Boyle, M. A. Donnellan, J. M. Flynn, A. Juttner, J. Noaki, C. T. Sachrajda and R. J. Tweedie [UKQCD Collaboration], "A lattice computation of the first moment of the kaon's distribution amplitude," Phys. Lett. B **641**, 67 (2006).

By evolving our result to the scale $2\ \mathrm{GeV}$ we find

$$a_1^K(2\ \mathrm{GeV}) = 0.08 \pm 0.04$$