SPIN PHYSICS AND SELF-ORGANIZATION OF PHYSICAL FIELDS

Ivanhoe B.Pestov †

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1 Preliminary

GR and QM show clearly that the physical laws have a more objective character with respect to the known earlier. This is easily seen since coordinates have no physical meaning both in GR and QM (principle of general covariance, uncertainty principle, notion of trajectory disappears in quantum theory of particles).

Thus, GR and QM give a strong grounds for the Theory of Self-Organization (TSO) that formulates physical laws in an absolutely objective form (completely independent of any outer and a priori conditions). The conceptual structure of TSO includes the following foundational notions:
concept of physical space (proper space of the system in question);
concept of really geometrical quantity i.e., quantity connected with the geometrical structure of proper space;
new field concept of time as the cornerstone of any dynamical theory ;
concept of geometrical internal symmetry which comes in after the introduction of really geometrical quantities to make these quantities variable.

2 Introduction

The subject of present consideration is spin as a fundamental manifestation of geometrical structure of physical space in the framework of the Theory of Self-Organization of Physical Fields. Theory of Self-Organization is the new, self-consistent, and integral structure in which geometry, symmetries, and fields are tightly connected and kept inseparable providing the adequate solution of the most difficult conceptual problems. In the dynamical Theory of Self-Organization all manifolds of phenomena are projected on the set of the four fundamental fields: the gravitational field, the temporal field, the general electromagnetic field, the spinning field. The main topic is spin symmetry, which is a fundamental realization of geometrical internal symmetry and spin is a bipolar structure of the group of spin symmetry. Violation of spin symmetry leads to the dynamical theory of spinning field (spindynamics). In the last case, a temporal field enters into the Lagrangian of spinning field as a symplectic structure.

3 Foundational notions

Both in Geometry and in General Relativity a differential manifold is a priori an element and on the same manifold one can consider different metrics and a system of equations. This is not convenient for TSO that is free from any external conditions. Only realization of an abstract manifold as a surface in the Euclidian space of a fairly large number of dimensions permits one to consider a manifold as physical space, i.e., inner element of TSO. The n-dimensional Euclidean space is a linear structure in the set of n-tuples $\mathbf{x} = (x^1, \cdots, x^n)$, which is defined by the natural rules $a\mathbf{x} = (ax^1,\ ax^2, \cdots ax^n)$, $\mathbf{x}+\mathbf{y} = (x^1+y^1, \cdots, x^n+y^n)$, where a is a real number and x^i are independent real variables. The distance function is given by $d(\mathbf{x}, \mathbf{y}) = \sqrt{(x^1-y^1)^2 + \cdots + (x^n-y^n)^2}$.

The physical space is a 4-dimensional manifold that is realized as follows. The region of a 4-dimensional manifold is analytically defined by the equations $x^a = F^a(u^1, u^2, u^3, u^4)$, where the functions $F^a(u^1, u^2, u^3, u^4)$ of four independent variables u^1, u^2, u^3, u^4 (Gauss coordinates) are the solution of the characteristic system of nonlinear equations in partial derivatives

$$g_{ij}(u^1, u^2, u^3, u^4) = \delta_{ab} \frac{\partial F^a}{\partial u^i} \frac{\partial F^b}{\partial u^j}, \quad a,\ b = 1, \cdots, 4+k,\ k \geq 0. \tag{1}$$

The known functions $g_{ij}(u^1, u^2, u^3, u^4)$ in the right-hand side of equations (1) represent the positive- definite Riemann metric

$$ds^2 = g_{ij} du^i du^j. \tag{2}$$

The guide principle of geometrization and self-organization reads: the geometrical structure of physical space (points, curves, congruences of curves, families of curves) determines a very restricted set of really geometrical quantities (fundamental fields) and along with that a geometrical internal symmetry that make these quantities variable. The set of geometrical quantities that are connected with the geometrical structure of the physical space is very restricted since it is defined by the natural functionals and differential equations. Let us enumerate really geometrical quantities: the Reimann metric g_{ij};
the vector field v^i;
the affine (linear) connection P^i_{jk};
the scalar and covariant vector fields, antisymmetric covariant tensor fields. For present purposes it is useful to show the last set of fields as 2^n-tuple

$$(a, a_i, a_{ij}, \cdots a_{ijk\cdots l}).$$

By definition, the geometrical internal transformations come in after the introduction of really geometrical quantities to make them variable. The general role of this symmetry can be outlined as follows: **geometrical internal symmetry makes the really geometrical quantities variable and being broken it leaves a trace in the form of differential equations for these quantities that describe all effects connected with internal symmetry**. Our main goal here is to define and investigate a geometrical internal symmetry that describes such phenomena as spin (spin symmetry). Being broken, spin symmetry designs spinstatics and spindynamics.

The quantum-mechanical causality shows clearly that all dynamical laws of undisturbed systems have the following form: **the rate of change with time of certain quantity equals the result of action of some operator on this quantity. The rate of change with time is the operator of evolution which defines causality in the field theory and the coordinate independent definition of this foundational notion cannot be given without a new field concept of time.** We put forward the idea that **the time is a scalar field** suggesting, by way of justification, a self-consistent theory of fields which does not depend on outer conditions. In the Theory of Self-Organization the properties of time and physical space are not defined by the properties of devices and by the methods of measurements. A temporal field (together with other fields) designs physical space, but it has also other very important functions in spindynamics. The temporal field with respect to the coordinate system u^1, u^2, u^3, u^4 is denoted by $f(u) = f(u^1, u^2, u^3, u^4)$. The gradient of the temporal field (the stream of time) is the vector field $\mathbf{t}$ with the components $t^i = (\nabla f)^i = g^{ij}\frac{\partial f}{\partial u^j} = g^{ij}\partial_j f = g^{ij}t_j$, where g^{ij} are the contravariant components of the Riemann metric (1). The rate of change with time of some quantity is the Lie derivative with respect to the stream of time $\mathbf{t}$ and and the symbol $D_{\mathbf{t}}$ denotes this operation. For the rate of change with time of the temporal field itself we get $D_{\mathbf{t}} f = t^i\partial_i f = g^{ij}\partial_i f\,\partial_j f$. The temporal field obeys the fundamental equation:

$$D_{\mathbf{t}} f = (\nabla f)^2 = g^{ij}\frac{\partial f}{\partial u^j}\frac{\partial f}{\partial u^j} = 1. \tag{3}$$

The other possible operator of evolution has the form of covariant derivative in the direction of the stream of time. This operator is denoted by $\nabla_{\mathbf{t}} = t^i\nabla_i$, where ∇_i is a covariant derivative with respect to the connection that belongs to the Riemann metric g_{ij}. The operator $\nabla_{\mathbf{t}}$ has no sense for the gravitational field since $\nabla_{\mathbf{t}} g_{ij} = 0$. Due to this it was very difficult to recognize that in the spindynamics the operator of evolution takes the form $\nabla_{\mathbf{t}}$.

Bilateral symmetry (the symmetry of the right and left) is tightly connected with the stream of time. A pair of vector fields $\mathbf{v}$ and $\overline{\mathbf{v}}$ has bilateral symmetry with respect to the stream of time if the sum of these fields is collinear to the gradient of a temporal field and their difference is orthogonal to it, $\overline{\mathbf{v}} + \mathbf{v} = \lambda\mathbf{t}, \quad (\overline{\mathbf{v}}, \mathbf{t}) = (\mathbf{v}, \mathbf{t})$, where $(\mathbf{v}, \mathbf{w}) = g_{ij}v^i w^j = v^i w_i$ is a scalar product. In components, we have $\overline{v}^i = 2(\mathbf{v}, \mathbf{t})\,t^i - v^i = (2t^i t_j - \delta^i_j)v^j$. Thus, the bilateral symmetry may be represented as a linear transformation $\overline{v}^i = R^i_j v^j$, where $R^i_j = 2t^i t_j - \delta^i_j$. This transformation is natural to call reflection. For the metric we get $\overline{g}_{ij} = g_{kl}R^k_i R^l_j = g_{ij}$. In the case of antisymmetric tensor fields we have

$$\overline{a}_{i_1\cdots i_p} = R^{j_1}_{i_1}\cdots R^{j_p}_{i_p} a_{j_1\cdots j_p} = (-1)^p(a_{i_1\cdots i_p} - 2pt^k a_{k[i_2\cdots i_p}t_{i_1]}). \tag{4}$$

The scalar product is invariant with respect to reflection $(\overline{\mathbf{v}}, \overline{\mathbf{w}}) = (\mathbf{v}, \mathbf{w})$ since g_{ij} is invariant.

The associated scalar product (defined by the bilateral symmetry) has the form $\langle\mathbf{v}, \mathbf{w}\rangle = (\overline{\mathbf{v}}, \mathbf{w})$ and it is also invariant with respect to reflection. As we see from the formula $(\overline{\mathbf{v}}, \mathbf{v}) = \overline{g}_{ij}v^i v^j$, where

$$\overline{g}_{ij} = 2t_i t_j - g_{ij}, \tag{5}$$

the associated scalar product is tightly connected with auxiliary metric $\overline{g}_{ij}$ as the metric of the normal hyperbolic type. The contravariant components of the tensor field $\overline{g}_{ij}$ are $\overline{g}^{ij} = g^{ij}2t^i t^j - g^{ij}$. Hence, bilateral symmetry defines the causal structure on the physical space and can be identified with it. The auxiliary metric (5) gives the evident method of introduction of temporal field into the Lagrangians of physical fields.

4 Spin symmetry and spin as a bipolar structure

On the physical space there is natural linear space of the dimension $N = 2^n$, $n = 3, 4$ that can be constructed from the really geometrical quantities. The general element $\mathbf{A}$ of this space can be represented as 2^n-tuple

$$\mathbf{A} = (a,\ a_{i_1},\ a_{i_1 i_2},\ \cdots a_{i_1\cdots i_n}). \tag{6}$$

However, from the initial geometrical interpretation it follows that an object like this consists of independent blocks and cannot be considered as a unit. The concept of geometrical internal symmetry makes it possible to remove this discrepancy. To this end, we consider the group of geometrical internal transformations $GL(2^n;\mathbf{R})$ which are defined as follows. Let us consider a set of totally antisymmetrical tensor fields of the type (p,q), $L^{i_1\cdots i_p}_{j_1\cdots j_q}, \quad (p,q = 0,1,\cdots,n)$. A geometrical internal transformation (linear operator) in the space of 2^n-tuples (6) is defined by the equations $\overline{\mathbf{A}} = L\mathbf{A}$, where $(L\mathbf{A})_{i_1\cdots i_p} = \overline{a}_{i_1\cdots i_p}, \quad p = 0,1,\cdots n$ and

$$\overline{a}_{i_1\cdots i_p} = \sum\nolimits_{q=0}^{n} \frac{1}{q!} L^{j_1\cdots j_q}_{i_1\cdots i_p} a_{j_1\cdots j_q}.$$

Here and in what follows it will be convenient to show only a general element $a_{i_1\cdots i_p}$ of 2^n-tuple (6) assuming that p runs from 0 to n, $n = 3,4$. The identical transformation E is defined by the conditions $E^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad E^{i_1\cdots i_p}_{j_1\cdots j_p} = \delta^{i_1\cdots i_p}_{j_1\cdots j_p}$, where the generalized Kronecker delta is used $a_{i_1\cdots i_p} = \frac{1}{p!}\delta^{j_1\cdots j_p}_{i_1\cdots i_p} a_{j_1\cdots j_p}$. So the defined local internal transformations form the group of **spin symmetry** $GL(2^n,\mathbf{R})$ which allows one to consider a 2^n- tuple (6) as a unit. In what follows the 2^n-tuples (6) will be known as a **spinning field**.

The positive definite fundamental bilinear quadratic form in the space in question is defined as follows:

$$(\mathbf{A}|\mathbf{B}) = \sum\nolimits_{p=0}^{n} \frac{1}{p!} a_{i_1\cdots i_p} b_{j_1\cdots j_p} g^{i_1 j_1} \cdots g^{i_p j_p}. \tag{7}$$

We have $(\mathbf{A}|\mathbf{B}) = (\mathbf{B}|\mathbf{A}), \quad (\mathbf{A}|\mathbf{A}) \geq 0$. We consider real spinning fields up to the appearance of a real reason to introduce complexification.

To define a group of spin symmetry, we have introduced a set of tensor fields that are not connected with the geometrical structure of the physical space. Now our goal is twofold. First of all, we would like to represent the transformations of the group $GL(2^n,\mathbf{R})$ in the framework of the really geometrical quantities. To this end, we construct a general covariant basis in the Lee algebra $gl(2^n,\mathbf{R})$ for $GL(2^n,\mathbf{R})$. By means of this we show that spin is the bipolar structure on the group of spin symmetry $GL(2^n,\mathbf{R})$. After that the spin symmetry being broken is visualized as spinstatics and spindynamics. This is a real significance and manifestation of spin symmetry.

To realize this program, let us consider the natural algebraic operators $\overline{\mathbf{A}} = E_{\mathbf{v}}\mathbf{A}$ and $\overline{\mathbf{A}} = I_{\mathbf{v}}\mathbf{A}$ that are defined by the vector field v^i as follows:

$$E_{\mathbf{v}} : \bar{a}_{i_1\cdots i_p} = p v_{[i_1} a_{i_2\cdots i_p]}, \quad I_{\mathbf{v}} : \bar{a}_{i_1\cdots i_p} = v^k a_{k i_1\cdots i_p}, \quad (p = 0,1,\cdots,n),$$

where the square bracket $[\cdots]$ denotes the process of alternation over p indices and $v_i = g_{ij}v^j$. For any vector fields v^i and w^i we have

$$I_{\mathbf{v}}E_{\mathbf{w}} + E_{\mathbf{w}}I_{\mathbf{v}} = (\mathbf{v},\mathbf{w})\cdot E,$$

where $(\mathbf{v},\mathbf{w}) = g_{ij}v^i w^j$. We mention also evident relations

$$E_{\mathbf{v}}E_{\mathbf{w}} + E_{\mathbf{w}}E_{\mathbf{v}} = 0, \quad I_{\mathbf{v}}I_{\mathbf{w}} + I_{\mathbf{w}}I_{\mathbf{v}} = 0.$$

To complete a preliminary, let us introduce one more numerical diagonal operator Z that is defined by the conditions

$$Z^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad Z^{i_1\cdots i_p}_{j_1\cdots j_p} = (-1)^p \delta^{i_1\cdots i_p}_{j_1\cdots j_p}.$$

From the definition of Z, we immediately get the following relations

$$E_{\mathbf{v}} Z + Z E_{\mathbf{v}} = 0, \quad I_{\mathbf{v}} Z + Z I_{\mathbf{v}} = 0, \quad Z^2 = E.$$

Now we introduce the fundamental operators that define the bipolar structure (spin) of the group of spin symmetry $GL(2^n, \mathbf{R})$:

$$Q_{\mathbf{v}} = E_{\mathbf{v}} - I_{\mathbf{v}}, \quad \tilde{Q}_{\mathbf{v}} = (E_{\mathbf{v}} + I_{\mathbf{v}})Z.$$

From the definition, it follows that for any $\mathbf{v}$ and $\mathbf{w}$

$$Q_{\mathbf{v}} Q_{\mathbf{w}} + Q_{\mathbf{w}} Q_{\mathbf{v}} = -2(\mathbf{v}, \mathbf{w}) \cdot E, \quad \tilde{Q}_{\mathbf{v}} \tilde{Q}_{\mathbf{w}} + \tilde{Q}_{\mathbf{w}} \tilde{Q}_{\mathbf{v}} = -2(\mathbf{v}, \mathbf{w}) \cdot E, \quad \tilde{Q}_{\mathbf{v}} Q_{\mathbf{w}} = Q_{\mathbf{w}} \tilde{Q}_{\mathbf{v}}. \quad (8)$$

Further, we take $m, \quad (m = 2, 3, \cdots, n)$ linear independent vector fields $\mathbf{v}, \mathbf{w}, \cdots, \mathbf{z}$ and introduce an alternated product of the operators $Q_{\mathbf{v}}$, $Q_{\mathbf{w}}, \cdots, Q_{\mathbf{z}}$ and $\tilde{Q}_{\mathbf{v}}$, $\tilde{Q}_{\mathbf{w}}, \cdots, \tilde{Q}_{\mathbf{z}}$ $Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}} = \frac{1}{m!}(Q_{\mathbf{v}} Q_{\mathbf{w}} \cdots Q_{\mathbf{z}} - Q_{\mathbf{w}} Q_{\mathbf{v}} \cdots Q_{\mathbf{z}} + \cdots)$, $\tilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}} = \frac{1}{m!}(\tilde{Q}_{\mathbf{v}} \tilde{Q}_{\mathbf{w}} \cdots \tilde{Q}_{\mathbf{z}} - \tilde{Q}_{\mathbf{w}} \tilde{Q}_{\mathbf{v}} \cdots \tilde{Q}_{\mathbf{z}} + \cdots)$. From (8) it follows that operators $Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ and $\tilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ commute with each other and all their possible products constitute the general covariant basis in the Lie algebra $gl(2^n, \mathbf{R})$ for the spin symmetry group $GL(2^n, \mathbf{R})$). The total number of these operators is equal to $2^n \cdot 2^n$. Thus, for the spinning field the transformations of the group $GL(2^n, \mathbf{R})$ can be presented in terms of really geometrical quantities. We see that the Riemann metric looks like a prism and spin symmetry (like light) passing through this prism demonstrates its bipolar structure and becomes apparent in the form of the operators $Q_{\mathbf{v}}, \cdots, \quad Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ on one pole and the operators $\tilde{Q}_{\mathbf{v}}, \cdots \quad \tilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ on the other pole. To represent this bipolar structure more visually, one can consider $n = 3$ orthogonal and unit vector fields with respect to the given metric g_{ij}. Here we have no possibility to discuss this question in more detail.

Thus, spin is a bipolar structure on the group of spin symmetry and is realized in the form of two groups of commuting operators $Q_{\mathbf{v}}, \cdots, Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ and $\tilde{Q}_{\mathbf{v}}, \cdots, \tilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$.

5 Spindynamics

The equations of spindynamics in the Hamiltonian form include four scalar and four vector equations:

$$\begin{aligned}(\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\kappa &= \operatorname{div}\mathbf{K}-m\,\mu\\ (\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\lambda &= \operatorname{div}\mathbf{L}-m\,\nu\\ (\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\mu &= \operatorname{div}\mathbf{M}+m\,\kappa\\ (\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\nu &= \operatorname{div}\mathbf{N}+m\,\lambda\end{aligned} \tag{9}$$

$$\begin{aligned}(\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\mathbf{K} &= -\mathrm{rot}\,\mathbf{L}+\mathrm{grad}\,\kappa+m\,\mathbf{M}\\ (\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\mathbf{L} &= \mathrm{rot}\,\mathbf{K}+\mathrm{grad}\,\lambda+m\,\mathbf{N}\\ (\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\mathbf{M} &= \mathrm{rot}\,\mathbf{N}+\mathrm{grad}\,\mu-m\,\mathbf{K}\\ (\nabla_{\mathbf{t}}+\tfrac{1}{2}\varphi)\mathbf{N} &= -\mathrm{rot}\,\mathbf{M}+\mathrm{grad}\,\nu-m\,\mathbf{L},\end{aligned} \tag{10}$$

where $\nabla_{\mathbf{t}}=t^i\nabla_i, \quad \varphi=\nabla_i t^i$. Some definitions of the vector algebra and vector analysis in the four-dimensional and general covariant form The operator rot is defined for the vector fields as follows:

$$(\mathrm{rot}\mathbf{M})^i = e^{ijkl}t_j\partial_k\mathrm{M}_l = \frac{1}{2}e^{ijkl}t_j(\partial_k\mathrm{M}_l-\partial_l\mathrm{M}_k).$$

It is easy to show that

$$(\mathbf{M},\,\mathrm{rot}\mathbf{N})+\mathrm{div}[\mathbf{MN}]=(\mathrm{rot}\mathbf{M},\,\mathbf{N}),$$

where

$$[\mathbf{MN}]^i = e^{ijkl}t_j\mathrm{M}_k\mathrm{N}_l$$

is a vector product of two vector fields $\mathbf{M}$ and $\mathbf{N}$,

$$\operatorname{div}\mathbf{M}=\nabla_i\mathrm{M}^i.$$

Thus, the operator rot is self-adjoint. We also mention that

$$(\mathrm{grad}\varphi)_i = \triangle_i\,\varphi,\ \triangle_i=\nabla_i-t_i\nabla_{\mathbf{t}}$$

and

$$\mathrm{rot}\,\mathrm{grad}=0,\quad \mathrm{div}\,\mathrm{rot}=0.$$

Equations (9) and (10) describe all phenomena connected with spin symmetry and spin. Since the Theory of Self-Organization is integral structure, spindynamics involves all phenomena connected with spin. Hence, having in our disposal the bipolar structure of spin symmetry we can maintain that observer does not need to consider the artificial concept of weak or strong isotopic spin. In this case, the geometrical and physical nature of the strong interactions can be understood only in the framework of the nontrivial causal structure defining by the temporal field. We can consider in the first approximation that physical space is 4-dimensional Euclidean space E^4. It is easy to find that in this case the basic equation of temporal field has general solution

$$f(u^1,\,u^2,\,u^3,\,u^4)=a_iu^i+a$$

where $(a_1)^2+(a_2)^2+(a_3)^2+(a_4)^2=1$ and singular solution

$$f(u^1,\,u^2,\,u^3,\,u^4)=\sqrt{(u^1)^2+(u^2)^2+(u^3)^2+(u^4)^2}.$$

Thus, in the first case the space sections of E^4 are E^3 and in the second one the space sections are $3D$ spheres S^3.

The first solution defines the causal structure that corresponds to the special relativity. The second solution is considered as the starting point for understanding the strong interactions on the basis of the new causal structure tightly connected with rotations.

Now we introduce the concept of internal spin which is visible in experiment like Stern-Gerlach and after that consider some artificial physical state. The operator

$$\tilde{S}_{\mathbf{E}*\mathbf{H}} = \frac{1}{2}(Q_{\mathbf{E}}\tilde{Q}_{\mathbf{t}} + Q_{\mathbf{H}}Q_{\mathbf{t}}\widetilde{H})Z,$$

where $\mathbf{E}$ is the strength of the electric field and $\mathbf{H}$ is the strength of the magnetic field, represents additional potential energy due to the internal spin.

6 Concluding Remarks

For comparison, let us look at the historical development of Quantum Mechanics. From the geometrical point of view, in the Schrodinger theory the two real scalar fields are introduced and internal symmetry appears at first in the form of the complex scalar field. Here the principle of sufficient cause is substituted by the experiment but the question remain open. In the Dirac theory already the four complex scalar fields are introduced and, hence, it can be considered as the theory of the Higgs fields with nontrivial internal symmetry defined by the Dirac spin matrices. In the electroweak theory and quantum chromodynamics the number of the scalar fields increases again and again and thus, the artificial internal symmetry is extended. This way of development of the theory looks like artificial and oriented on the explanation of the artificial phenomena since it is impossible to derive the theory of elementary particles from the first principles without understanding the essence of time. But nevertheless the final judgment should be leaved to the future development of the theory and experiment.