# Experimental Check of the Born Approximation in elastic *ep* Scattering

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# Outline

Introduction

Form Factors from Rosenbluth and recoil polarization

The proton Form Factor “discrepancy”

Possible $2\gamma$ contribution

Related experiments at JLab

Conclusions

(see also review in “Progress in Particle and Nuclear Physics”, C.F.P., V. Punjabi and M. Vanderhaeghen, Prog. Part. Nucl. Phys. 59, 694-764, 2007)

# Introduction

- At large $Q^2$ electromagnetic Form Factors contain structure information on the many-body system of quarks and gluons of the nucleon. At low $Q^2$ they inform us about the pion cloud.

- When obtained from experiment, the Form Factors are relativistic invariants only to the extent that the probe is a **single** virtual **photon exchanged** between electron and nucleon; higher order contributions destroy this invariance, which one might regain after applying a number of radiative corrections.

- The recent inclusion of $2\gamma$ exchange with two hard photons may help reconciliate the discrepancy between Rosenbluth and Recoil Polarization measurements of $G_{Ep}/G_{Mp}$ , or it may not.

# Rosenbluth Cross Section

The cross section for single photon exchange (Born term) is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_{Mott}} \left( G_{Ep}^2(Q^2) + \frac{\tau}{\varepsilon} G_{Mp}^2(Q^2) \right) / (1+\tau)$$

with:

$$\varepsilon = \left( 1 + 2(1+\tau)\tan^2\left(\frac{\theta_e}{2}\right) \right)^{-1}$$

The reduced cross section used in Rosenbluth separation :

$$\sigma_{reduced} = \left( \varepsilon(1+\tau)\frac{d\sigma}{d\Omega} \right) \Big/ \left( \frac{d\sigma}{d\Omega} \right)_{Mott} = \varepsilon G_{Ep}^2 + \tau G_{Mp}^2$$

# vs. Recoil Polarization

Polarization transfer in $\vec{e}p \rightarrow e\vec{p}$ or spin-target asymmetry $\vec{e}\vec{p} \rightarrow ep$ result in either polarization of the recoil proton, or in parallel-transverse asymmetry, respectively.

For recoil polarization, the two polarization components are:

$$hP_eP_t = -hP_e 2\sqrt{\tau(1+\tau)}G_{Ep}G_{Mp}\tan(\frac{\theta_e}{2})/I_0$$

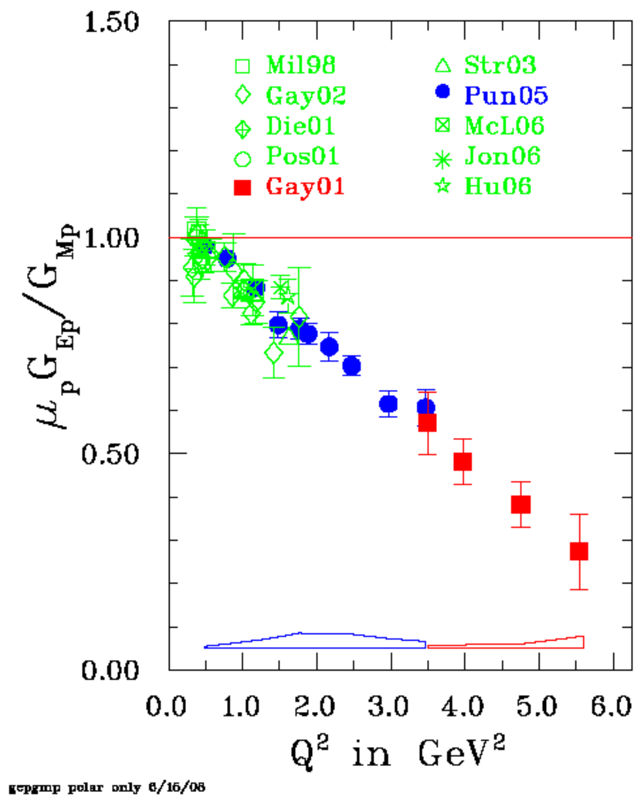
$$hP_eP_\ell = hP_e\frac{(E_e+E_{e'})}{M}G_{Mp}^2\sqrt{\tau(1+\tau)}\tan^2(\frac{\theta_e}{2})/I_0$$

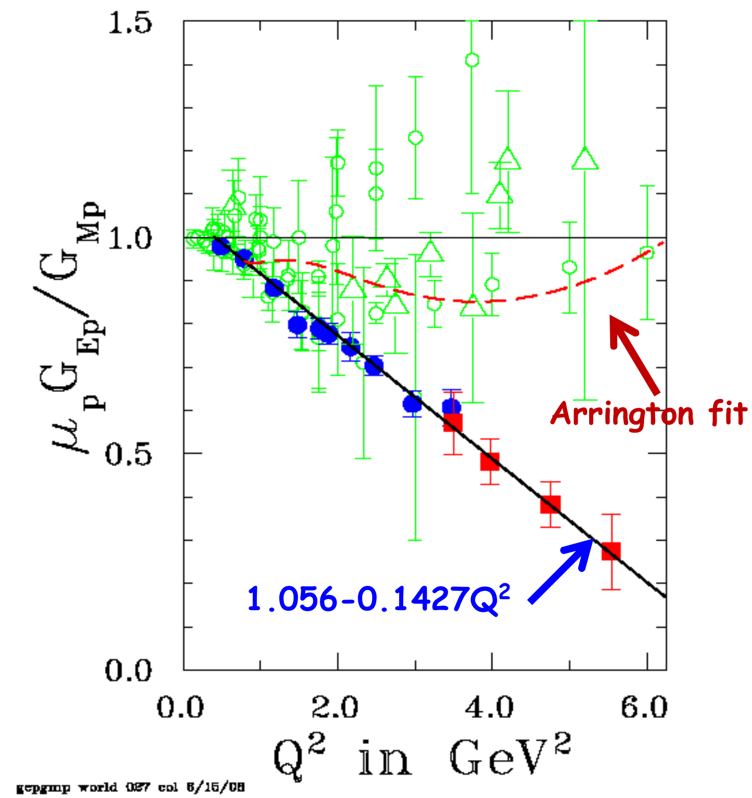
The beauty of the method is that the Form Factor ratio is independent of the electron polarization and of the polarimeter analyzing power:

$$\frac{G_{Ep}}{G_{Mp}} = -\frac{P_t}{P_\ell}\frac{(E_e+E_{e'})}{2M}\tan\left(\frac{\theta_e}{2}\right) \quad or \quad -\frac{P_t}{P_\ell}\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}$$

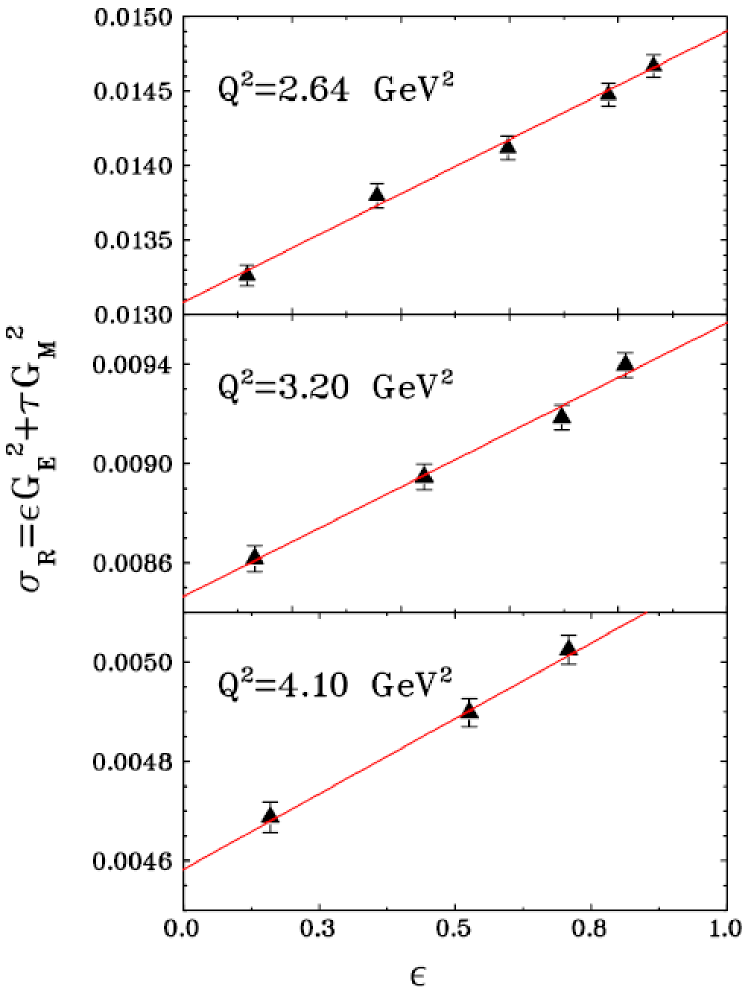
# Recoil Polarization Results

Recoil polarization results are in stark disagreement with Rosenbluth separation data.



# The Rosenbluth data shows no non-linearity so far!




Most recent published data from the (super-) Rosenbluth experiment at Jlab suggest no obvious non-linearity.

I.A. Qattan et al, Phys. Rev. Lett. 94, 142301 (2005)

One more experiment last year (J. Arrington et al) will achieve smaller error bars at several $Q^2$

# So what is the cause for the different results?

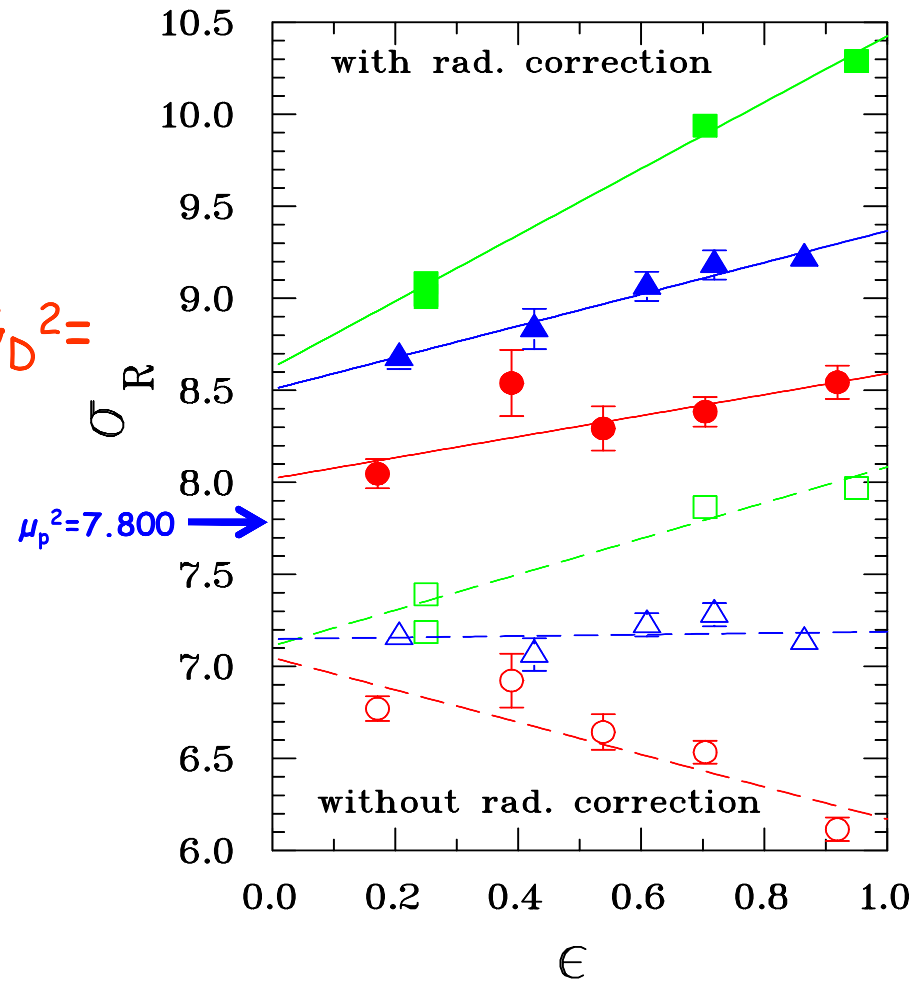
First, radiative corrections at large $Q^2$ are large and strongly ε-dependent.

$$\sigma_R = [\varepsilon(1+\tau)/\tau][\sigma_{exp}/\sigma_{Mott}]/G_D^2 =$$

$$= (G^2_{Mp} + \varepsilon\, G^2_{Ep}/\tau)/G_D^2$$

**green** for 1.75 $GeV^2$
**blue** for 3.75 $GeV^2$
**red** for 5 $GeV^2$

Data from Andivahis et al. (1994)



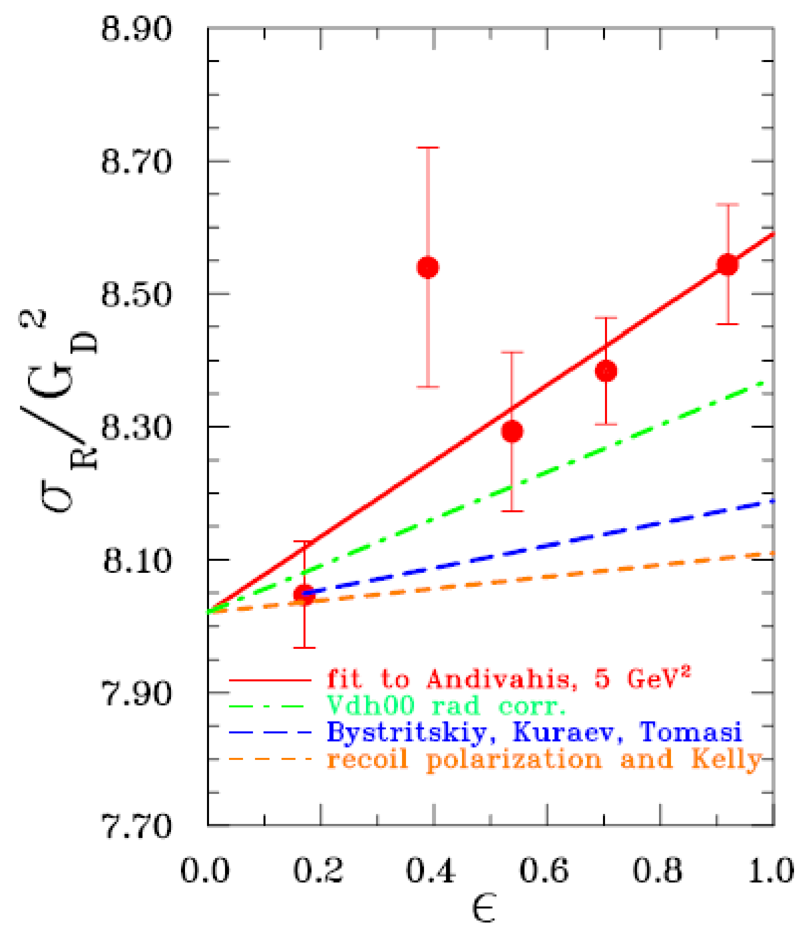
# Second, there is a scatter in size of calculated corrections

Andivahis et al: based on Mo and Tsai, (RMP 41, 205 1969) with improvements from Walker et al. (PR D49, 5671 1994)

Vanderhaeghen et al: code based on Maximon and Tjon (PR C62:054320, 2000): exact soft photon, better vertex and exact box diagram calculations.

Bystritskiy, Kuraev, Tomasi-Gustafsson: (PR C75.015207.2007) with structure function (Drell-Yan parton picture). Radiative correction for electron to all orders.

Interpolation from Hall A recoil polarization, $G_{Mp}$ from Kelly fit (PR C 70: 068202 2004).

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# Is 2γ exchange the Missing Diagram?

Guichon and Vanderhaeghen (2003) were first to suggest that discrepancy may be due to two-photon exchange, with each photon sharing momentum transfer, an interference with the 1γ process.

Adding the next-order diagram make the amplitudes complex and adds a third FF; effect on Rosenbluth cross section large when $G_{Ep}{}^2$ becomes very small. Not known 10 years ago that $G_{Ep}$ decreases faster than $G_{Mp}$ with increasing $Q^2$!

Cross sections, unpolarized and polarized, depend upon Real Part only. The hadronic vertex becomes:

$$\Gamma_\mu(p,p') = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M} F_2(Q^2)$$

$$\Gamma_\mu(p,p') = \gamma_\mu \tilde{G}_{Mp} - \frac{P^\mu}{M}\tilde{F}_2 + \frac{\gamma.KP^\mu}{M^2}\tilde{F}_3$$

Now three elastic amplitudes, $\tilde{G}_{Mp}$, $\tilde{F}_2$, $\tilde{F}_3$, which are not Lorentz invariant and complex, instead of 2 real, invariant FF.

# 2γ Continued

- Cross sections and polarizations in Space-Like region ($q_\mu^2<0$, $Q^2>0$) depend upon real part of Form Factors.
- Imaginary part defines observables forbidden by parity conservation, induced polarization or single spin asymmetries; and affects Form Factors in Time-Like region ($q_\mu^2>0$, $Q^2<0$) .
- To take into account exchange of two hard γ's, can replace Born form factors, by amplitudes as follows:

$$G_{Ep}(Q^2) \Rightarrow \Re e\tilde{G}_{Ep}(Q^2,\varepsilon)$$

$$G_{Mp}(Q^2) \Rightarrow \Re e\tilde{G}_{Mp}(Q^2,\varepsilon)$$

and replace third form factor: $\Re e\tilde{F}_3(Q^2,\varepsilon)$

by:

$$Y_{2\gamma} = \sqrt{\tau(1+\tau)}\sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\frac{\Re e\tilde{F}_3(Q^2,\varepsilon)}{G_{Mp}}, \quad and \quad \tilde{R} = \frac{\Re e\tilde{G}_{Ep}}{\Re e\tilde{G}_{Mp}}$$

As discussed by Rekalo and Tomasi-Gustaffson, (NP A742:322,2004) above is not unique, and may suffer from several defects. We pursue it here because it leads to a solution!

# L. Pentchev's method

Now, we can find values for 3 observables ($Y_{2\gamma}$, $\tilde{G}_M^2$ and $A_y$, the analyzing) from the 3 measured quantities ($d\sigma$, Pt and $A_y \times P_\ell$ ) at 3 values of $\varepsilon$ but fixed $Q^2$, by inverting relations:

$$P_t = \sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}} \frac{\tilde{G}_M^2}{d\sigma_{reduced}} \left( \tilde{R} + Y_{2\gamma} \right)$$

$$P_\ell = \sqrt{(1+\varepsilon)(1-\varepsilon)} \frac{\tilde{G}_M^2}{d\sigma_{reduced}} \left( 1 + \frac{2}{1+\varepsilon} \varepsilon Y_{2\gamma} \right)$$

$$\frac{d\sigma_{reduced}}{\tilde{G}_M^2} = 1 + \frac{\varepsilon}{\tau} \tilde{R}^2 + 2(1 + \frac{R}{\tau}) \varepsilon Y_{2\gamma}$$

For example from $P_\ell$ and $P_t$

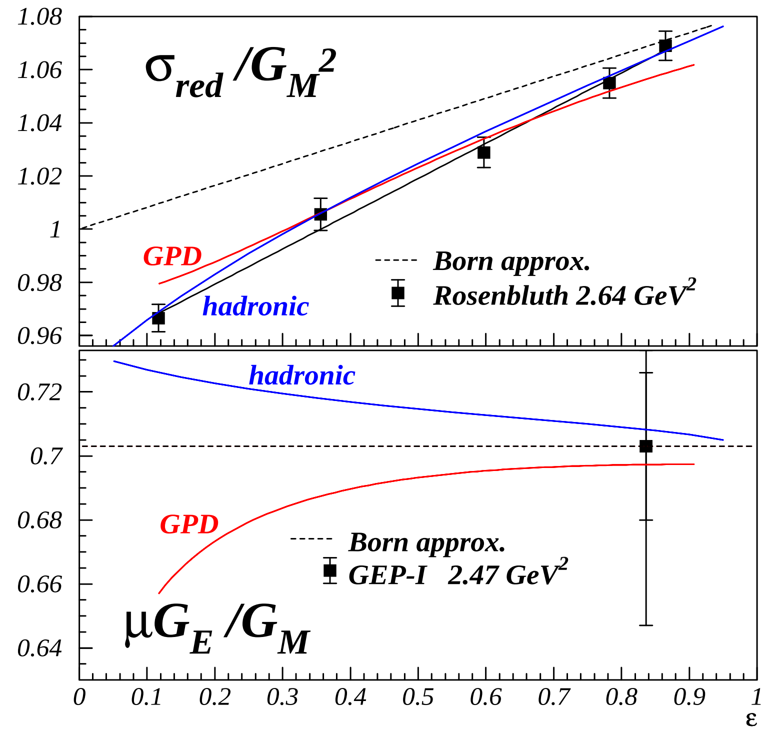
$$Y_{2\gamma} = \frac{\tilde{R} - \sqrt{\tau} F r}{\sqrt{\tau} r / F - 1}$$

where

$$\tilde{R} = \frac{\tilde{G}_{Ep}}{\tilde{G}_{Mp}}, \quad r = \frac{P_t}{P_\ell} \quad and \quad F = \sqrt{\frac{1+\varepsilon}{2\varepsilon}}$$

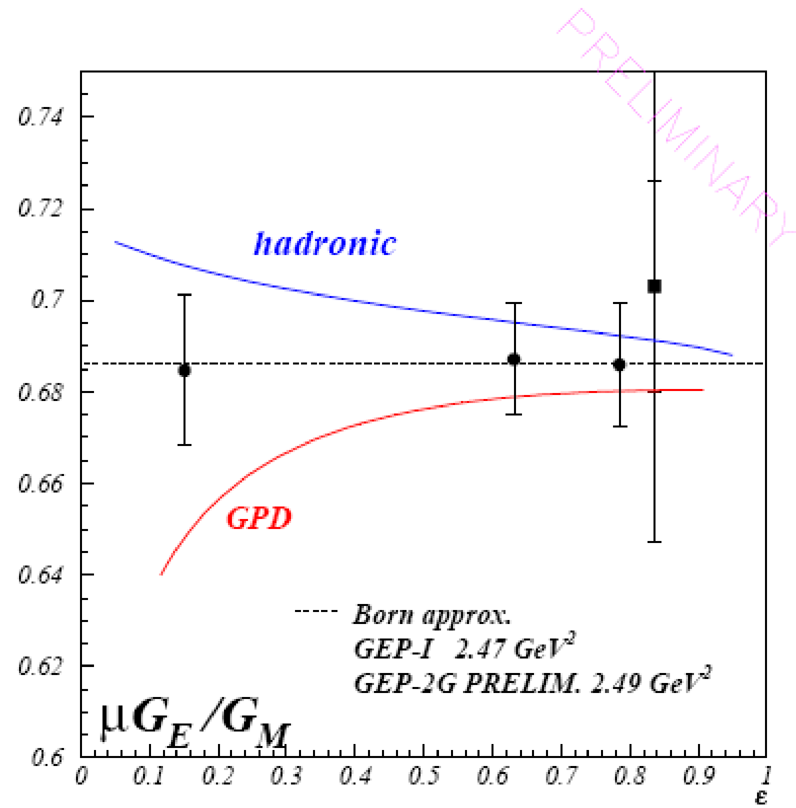
r is measured, so above gives $Y_{2\gamma}$ versus $\tilde{R}$; likewise for $\tilde{G}_M^2$ and $A_y$, the analyzing power.

# 2γ-Gamma Model Prediction




**The inclusion of hard 2γ exchange calculated by Chen et al (2003) with GPDs, and Blunden et al (2003) in hadronic model, creates a very slight non-linearity in Rosenbluth plot (top), but distinct behaviors for $\mu_p G_E/G_M$ from polarization measurements (bottom).**

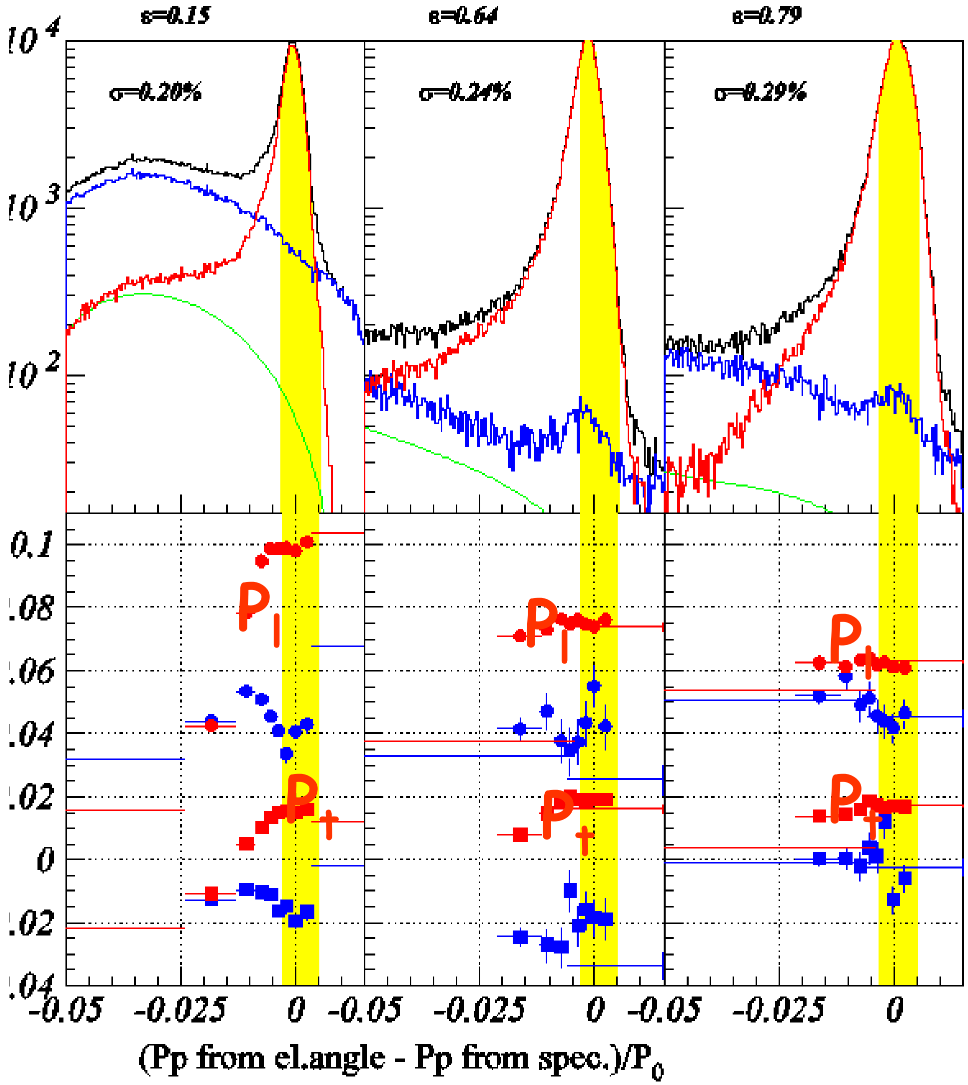
# The Preliminary Results (2008)




The preliminary results from experiment 04-019 at JLab, for $Q^2$=2.49 $GeV^2$ and three values of ε. Error bars not final.

No ε -dependence of $\mu_p G_{Ep}/G_{Mp}$ at the 0.01 level.

# Separated $P_t$ and $P_l$




1) Distribution of relative difference between P from $\theta_p$ and P from spectrometer
2) Physical background
3) Background subtracted
4) Residual background

A full determination of the 2 modified FF and $F_3$ requires additional measurements: e+/e- and triple polarization observables (M.P.Rekalo and E. Tomasi-Gustafsson Nucl.Phys.A740: 271-286,2004)

Presently we can only constrain the contribution from the third non-Born amplitude $Y_{2\gamma}$.

# The L. Pentchev plot

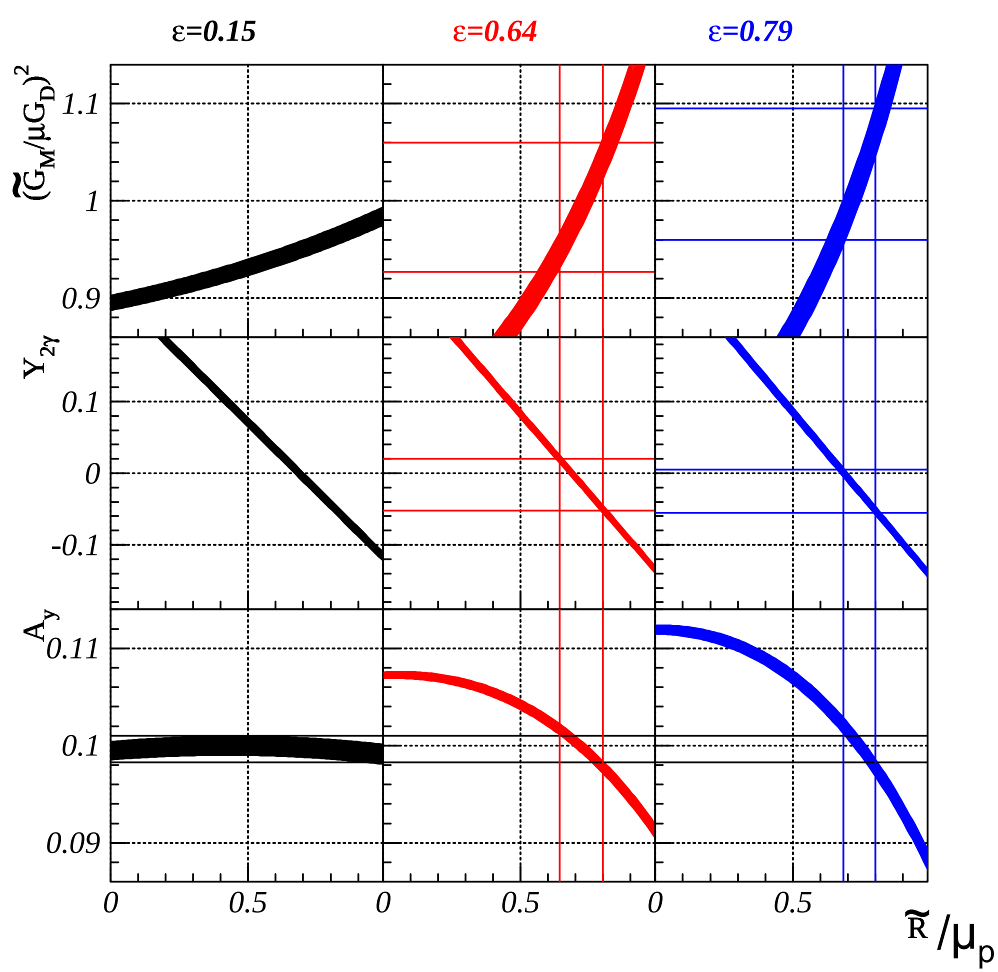
Measured $P_t/P_l$, $A_y{*}P_\ell$, $d\sigma$

Plot $\Re e(\tilde{G}_{Mp})$, $Y_{2\gamma}$ and $A_y$, calculated by inverting $P_t$, $P_\ell$ and $d\sigma$ relations, versus unknown ratio $\tilde{R}$

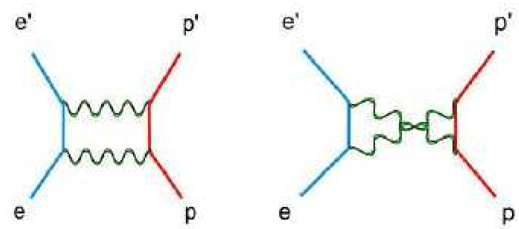
$A_y$ same for all three ε's because same $Q^2$: horizontal lines.

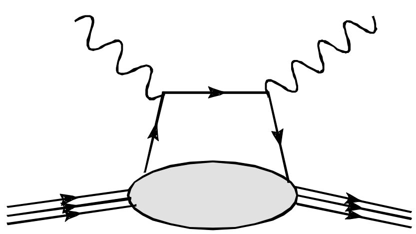
Defines $\tilde{R}$ at 2 other ε's: ~0.7

Intersections of vertical lines with colored bands define $Y_{2\gamma}$ and $\tilde{G}_{Mp}$.



# How to calculate contribution from exchange of two hard photon?



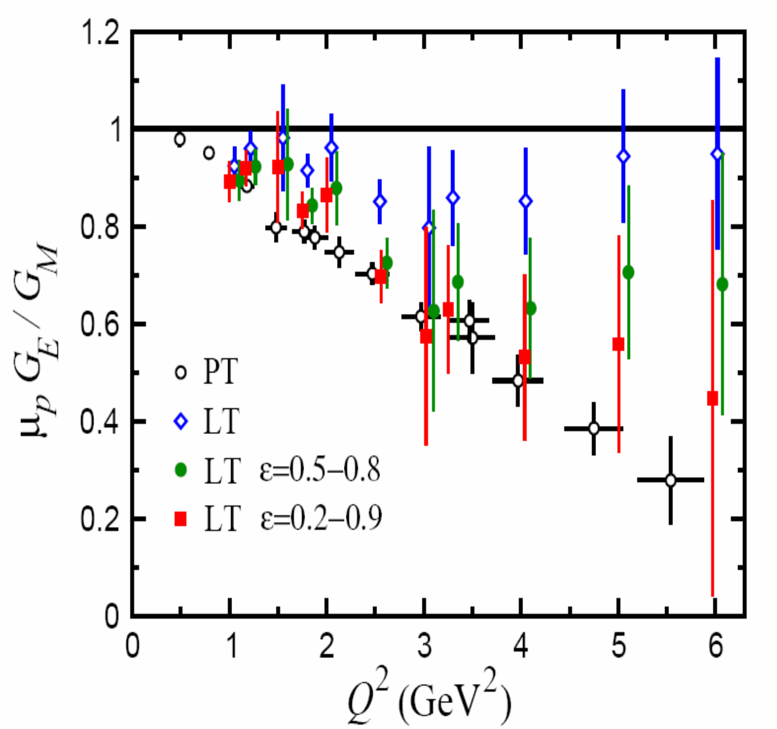



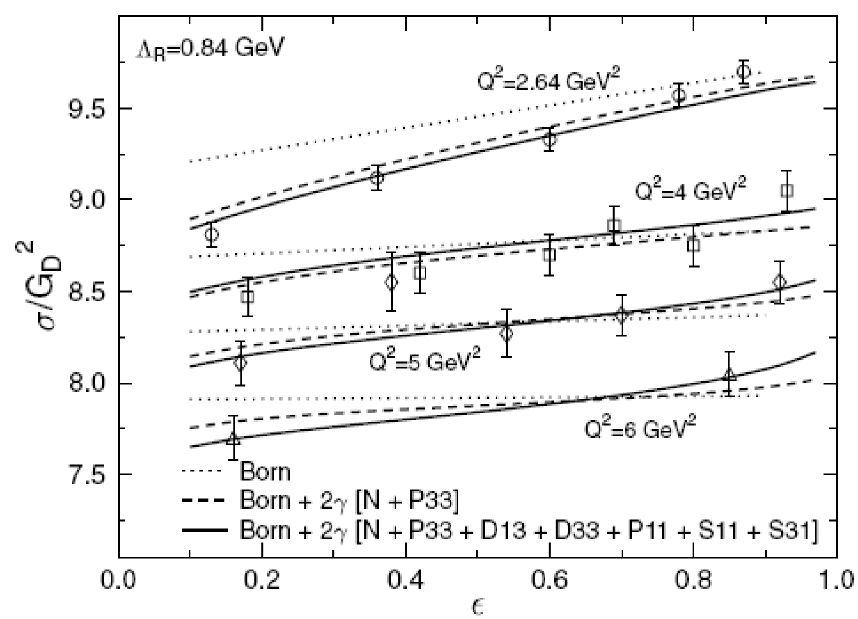
The proton in the intermediate state is virtual, Can be in ground state, or any baryonic state compatible with spin and parity. So far three main approaches to calculate box diagram:

- Generalized parton distribution or GPD, where the photon interacts with a single quark (Afanasev, Brodsky, Carlson, Chen and Vanderhaeghen, PR 72, 013008, 2005)
- Hadronic models (Kondratyuk and Blunden, PR 75, 038201, 2007).
- Include box diagram in K-matrix Drell Young structure function (Bistritskiy, Kuraev, Tomasi-Gustafsson. PR C75.015207.2007)

# Hadronic Evaluations of 2γ

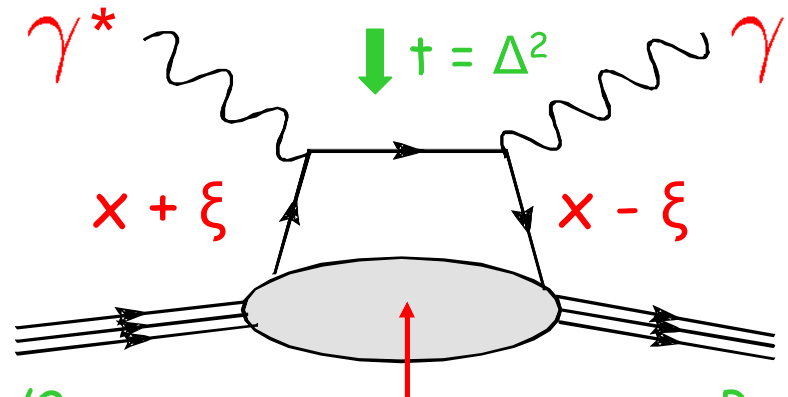
Blunden and Melnitchouk, Blunden, Melnitchouk and Tjon (PR 72, 034612, 2005). First proton contribution only (2003). Later added Δ contribution. Finite size of "Nucleon" included through appropriate FF.




Latest: Kondratyuk and Blunden added 5 low lying resonances to previous calculation with nucleon and Δ, and use polarization data as "Born" FF: higher resonances ~ cancel each other.

# Generalized parton distributions




$(x + \xi)$ and $(x - \xi)$ : longitudinal momentum fractions of quarks

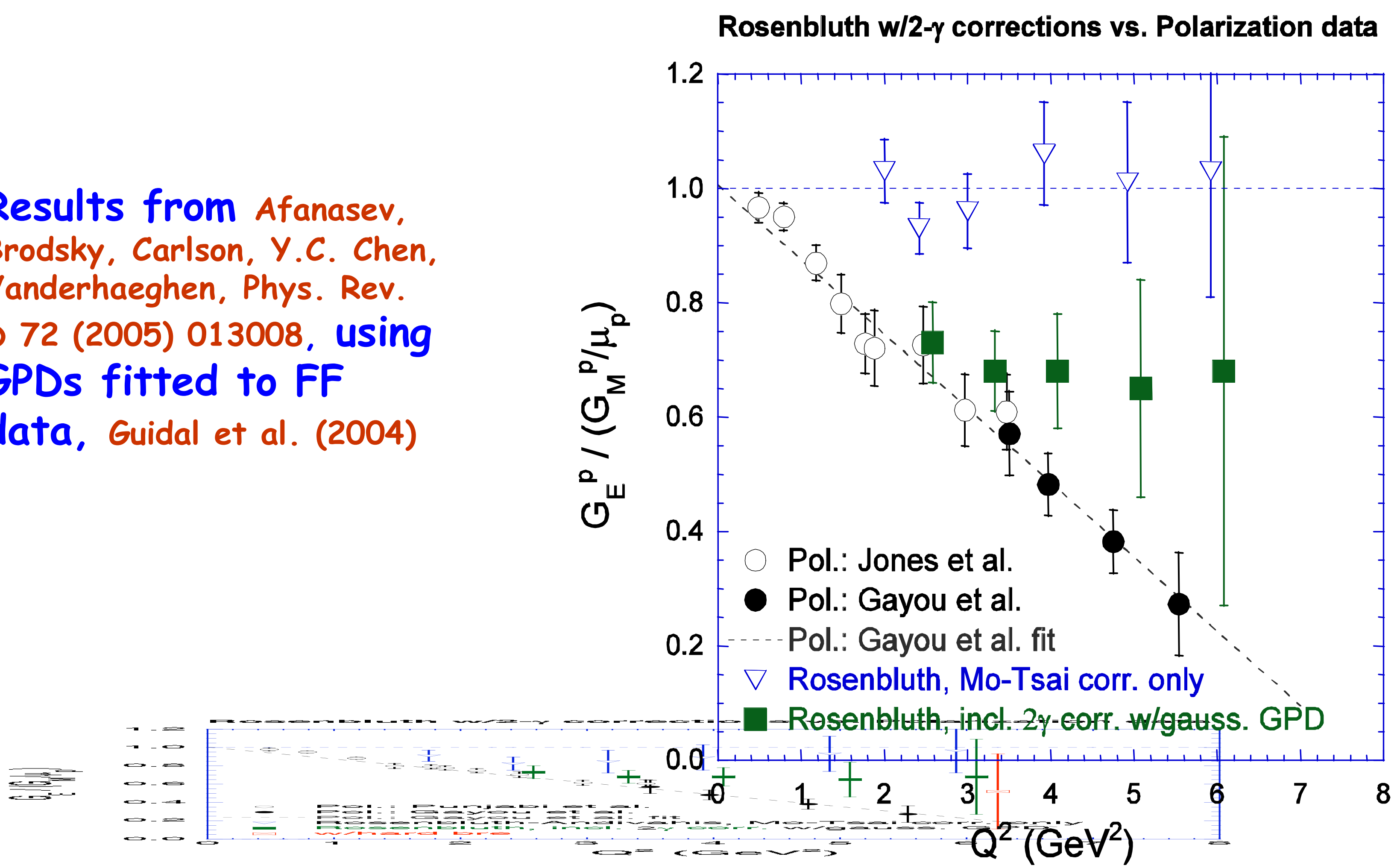
at large $Q^2$ : QCD factorization theorem ➡ hard exclusive process can be described by 4 transitions (GPDs) :

**Vector** : $H(x, \xi, t)$ **Axial-Vector** : $\tilde{H}(x, \xi, t)$

**Tensor** : $E(x, \xi, t)$ **Pseudoscalar** : $\tilde{E}(x, \xi, t)$

# Evaluation of 2γ from GPDs

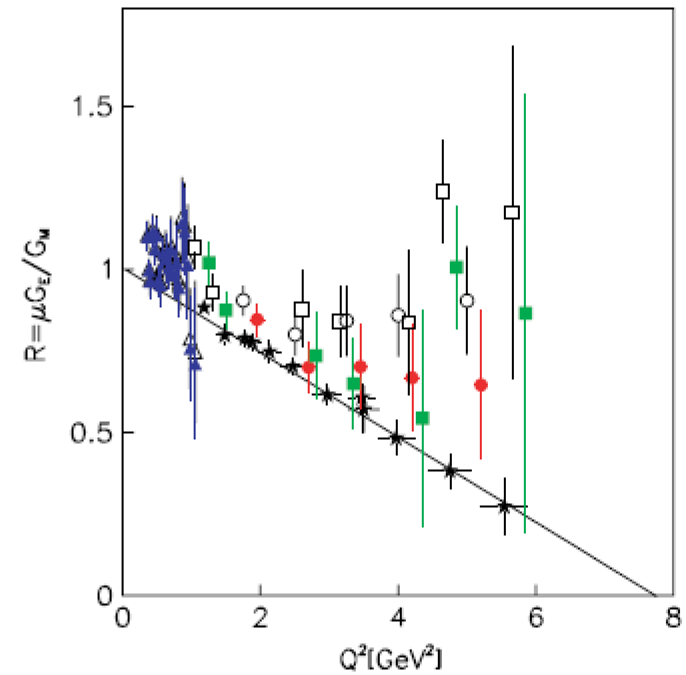
**Results from** Afanasev, Brodsky, Carlson, Y.C. Chen, Vanderhaeghen, Phys. Rev. D 72 (2005) 013008, **using GPDs fitted to FF data,** Guidal et al. (2004)



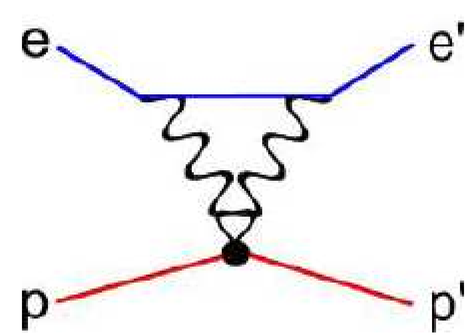
# More calculations

- Bystricky, Kuraev, Tomasi-Gustafsson: structure function (Drell-Yan) method: include 2γ contribution; negligible effect on Cross Section to 5 GeV$^2$ !

- Data from Andivahis et al and Christy et al




- Kuhn & Weigel add one-loop contribution to box diagram in chiral soliton model; contribution from 2 pion diagrams; they find small 2γ effect.



# Whether two-photon exchange is entirely responsible for the FF "crisis" or not at all, is to be determined experimentally

For example: Real part of $Y_{2\gamma}$

1) ε-independence of $G_{Ep}/G_{Mp}$ in recoil polarization → Hall C 04-019; completed
2) cross section difference in $e^+$ and $e^-$ proton scattering → Hall B 04-116; also Novosibisrsk, Olympus/Doris with refurbished BLAST detector
3) non-linearity of Rosenbluth plot → Hall C 05-017; being analyzed

Also imaginary part

4) from induced out-of-plane polarization → by-product of 04-019/04-108?
5) single-spin target asymmetry → Hall A 05-015 ($^3$He↑ )

# Positron/Electron Difference

The $e^+/e^-$ cross section data, J. Mar et al. (1969). Beam energy 4 to 10 GeV, $Q^2$ from 0.2 to 5 GeV$^2$; no systematics in ε-value.

In principle best way to determine 2γ contribution, because sign of 1γ-2γ interference changes with sign of electron charge.
Experiments with positrons will determine the Born values of the FF:

$$\sigma^{(+)} - \sigma^{(-)} = 2\sigma_0 \left[ \varepsilon G_{Ep}{}^2(Q^2) + \tau G_{Mp}{}^2(Q^2) \right]$$

$$\frac{1}{2} I_0 (P_t{}^{(+)} + P_t{}^{(-)}) = P_e \sqrt{2\varepsilon(1-\varepsilon)}\tau G_{Ep}(Q^2) G_{Mp}(Q^2)$$

$$\frac{1}{2} I_0 (P_\ell{}^{(+)} + P_\ell{}^{(-)}) = P_e \tau \sqrt{2\varepsilon(1-\varepsilon)} G_{Mp}{}^2(Q^2)$$

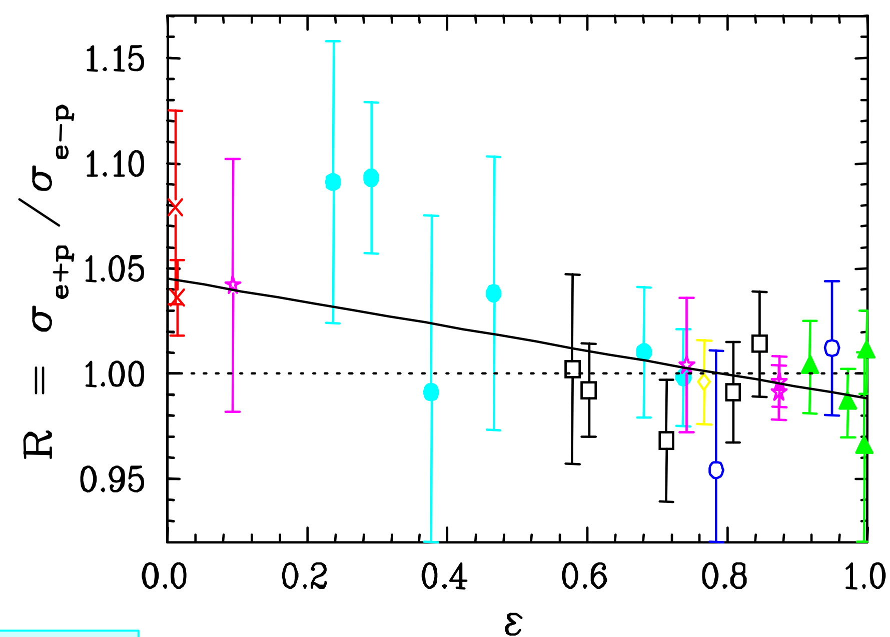



Figure from J. Arrington (2003)

# And why do we need to know if two-photon effects significant?

The Born Form Factors of the nucleon describe the spatial distribution of charge and magnetization in some appropriate reference frame; they are fundamental.

The nucleon form factors are ingredients of the observables of many other important quantities:
Parity violating amplitudes in electroweak interaction
Compton scattering
Virtual Compton scattering
Nucleon radius
Nucleus properties for quasi-elastic e,p scattering
Hyperfine structure of hydrogen

# Conclusion, Perspective

Both experimental characterization and phenomenological understanding of the structure of the proton, have changed drastically since 1998, year of the first recoil polarization experiment in Hall A.

Rapid decrease of $G_{Ep}$ with $Q^2$ not a surprise: predicted in at least 3 papers: Iachello Jackson and Lande (73) with VMD, Frank, Jennings and Miller (96) with CQM, and Holzwarth (96) with chiral soliton.

Currently under development are efforts to get a full understanding of two-photon effects, and revision of standard radiative correction calculation codes.

In my view clear experimental evidence for two-photon exchange as the explanation for the discrepancy between Rosenbluth and recoil polarization, is not in yet.

# The End

Experiments are the only means of knowledge
at our disposal. The rest is poetry, imagination.
Max Planck