# Walecka model with fractional exclusion statistics

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# Outline

- 1. Introduction
- 2. Fractional exclusion statistics
- 3. Walecka model: Relativistic mean-field theory
  - > The quantum field theory
  - > The mean-field approximation
  - Walecka model with fractional exclusion statistics
- 4. Example: The nuclear matter at finite temperature
- 5. Summary

Fractional exclusion statistics (I)

Ideal gas of several kinds of particles

#### Grand canonical ensemble (T,V,{ $\mu_{\lambda}$ })

$$\hat{\rho} = \frac{1}{Z} e^{-\frac{\hat{H} - \sum_{\lambda} \mu_{\lambda} \hat{N}_{\lambda}}{T}}$$
$$Z = Tr \left( e^{-\frac{\hat{H} - \sum_{\lambda} \mu_{\lambda} \hat{N}_{\lambda}}{T}} \right)$$

The second quantization method:

> the partition function:

$$Z = \sum_{\{n_{\bar{p}\lambda}\}} e^{-\frac{1}{T} \sum_{\bar{p}\lambda} (\varepsilon_{\bar{p}\lambda} - \mu_{\lambda}) n_{\bar{p}\lambda}} \lambda - \lambda}$$

 $\lambda$  – species of particles

the total energy and total number of particles of the given microstate:

$$\begin{split} \widehat{H} &= \sum_{\vec{p}\lambda} \varepsilon_{\vec{p}\lambda} n_{\vec{p}\lambda}, \\ \widehat{N}_{\lambda} &= \sum_{\vec{p}} n_{\vec{p}\lambda}, \end{split}$$

The occupation number representation:

(spineless particles)

> the single-particle energies:

$$\varepsilon_{\vec{p}\lambda} = \sqrt{\vec{p}^2 + m_{\lambda}^2}$$

- the single-particle states (momentum):  $p_{\alpha} = \Delta p_{\alpha} n_{\alpha}, n_{\alpha} = 0, \pm 1, ..., \Delta p_{\alpha} = \frac{2\pi\hbar}{L},$   $L = V^{1/3}, \alpha = 1, 2, 3$ -cubic box V
  - -infinite momentum lattice
- > the occupation numbers:

$n_{\vec{p}\lambda} = 0, 1, 2, \dots$	(bosons)
$n_{\vec{p}\lambda} = 0,1$	(fermions)

-the number of spineless particles in the given single-particle state

Fractional exclusion statistics (II)

Representation of the partition function using group of states (standard statistics)

$$Z = \sum_{\{n_{i\lambda}\}} W\{n_{i\lambda}\} e^{-\frac{1}{T}\sum_{i,\lambda} (\varepsilon_{i\lambda} - \mu_{\lambda})n_{i\lambda}}$$

the number of states of the system corresponding to the set  $\{n_{i\lambda}\}$ 

$$W\{n_{i\lambda}\} = \prod_{i,\lambda\in\Lambda_f} \frac{g_{i\lambda}!}{n_{i\lambda}!(g_{i\lambda}-n_{i\lambda})!} \times \prod_{k,\lambda'\in\Lambda_b} \frac{(g_{k\lambda'}+n_{k\lambda'}-1)!}{n_{k\lambda'}!(g_{k\lambda'}-1)!}$$

i – enumerates the sets of single-particle states  $\lambda$  – index of species of particles

 $\Lambda_{\rm f}$  – fermionic species,  $\Lambda_{\rm b}$  – bosonic species

Splitting of the continuum single-particle energy spectrum into sets:

 $V \rightarrow \infty$  (continuum spectrum)

> the set of single-particle states:

 $g_{i\lambda}$  – number of states in the set (i, $\lambda$ )

> The set's average energy:

$$\varepsilon_{i\lambda} \approx \varepsilon_{\vec{p}\lambda}, \vec{p} \in d_{i\lambda}$$

 the number of particles in the set (i,λ) :

$$n_{i\lambda} = \sum_{\vec{p} \in d_{i\lambda}} n_{\vec{p}\lambda},$$

#### **Fractional exclusion statistics**

> The partition function :

$$Z = \sum_{\{n_{i\lambda}\}} W\{n_{i\lambda}\} e^{-\frac{1}{T} \sum_{i,\lambda} (\varepsilon_{i\lambda} - \mu_{i\lambda}) n_{i\lambda}}$$

- the same as standard one!

-But, the number of states of the system corresponding to the set  $\{n_{i\lambda}\}$  is defined as

$$W\{n_{i\lambda}\} = \prod_{i} \prod_{\lambda} \frac{[g_{i\lambda} + n_{i\lambda} - 1 - \sum_{k\lambda'} \alpha_{i\lambda,k\lambda'} (n_{k\lambda'} - \delta_{ik} \delta_{\lambda\lambda'})]!}{n_{i\lambda}! (g_{i\lambda} - 1 - \sum_{k\lambda'} \alpha_{i\lambda,k\lambda'} (n_{k\lambda'} - \delta_{ik} \delta_{\lambda\lambda'}))!}$$

> The diagonal case:

$$W\{n_{i\lambda}\} = \prod_{i} \prod_{\lambda} \frac{[g_{i\lambda} + n_{i\lambda} - 1 - \alpha_{i\lambda}(n_{i\lambda} - 1)]!}{n_{i\lambda}!(g_{i\lambda} - 1 - \alpha_{i\lambda}(n_{i\lambda} - 1))!}, \qquad \alpha_{i\lambda,k\lambda'} = \delta_{ik}\delta_{\lambda\lambda'}\alpha_{i\lambda}$$

Particular cases (limits):

$$W\{n_{i\lambda}\} = \prod_{i} \prod_{\lambda} \frac{(g_{i\lambda} + n_{i\lambda} - 1)!}{n_{i\lambda}!(g_{i\lambda} - 1)!}, \qquad \alpha_{i\lambda} = 0, \qquad \text{(bosons)}$$
$$W\{n_{i\lambda}\} = \prod_{i} \prod_{\lambda} \frac{g_{i\lambda}!}{n_{i\lambda}!(g_{i\lambda} - n_{i\lambda}!)!}, \qquad \alpha_{i\lambda} = 1 \qquad \text{(fermions)}$$

#### **Approximate partition function:**

$$Z = W \{ \overline{n_{i\lambda}} \} e^{-\frac{1}{T} \sum_{i,\lambda} (\varepsilon_{i\lambda} - \mu_{i\lambda}) \overline{n_{i\lambda}}}$$

The mean occupation numbers (general):

$$\prod_{k\lambda'} w_{k\lambda'}^{\alpha_{k\lambda',i\lambda}} \left(1 + w_{k\lambda'}\right)^{\delta_{ik}\delta_{\lambda\lambda'}\alpha_{k\lambda',i\lambda}} = e^{\frac{\varepsilon_{i\lambda} - \mu_{i\lambda}}{T}},$$

$$w_{i\lambda} = \frac{1}{\overline{n}_{i\lambda}} \left( g_{i\lambda} - \sum_{k,\lambda'} \alpha_{i\lambda,k\lambda'} \overline{n}_{k\lambda'} \right)$$

The mean occupation numbers for singleparticle state:

$$\overline{n}_{\vec{p}\,\lambda} = \frac{\overline{n}_{i\lambda}}{g_{i\lambda}}, \ \vec{p} \in d_{i\lambda}$$

#### The mean occupation numbers

The diagonal representation: 
$$\alpha_{i\lambda,k\lambda'} = \delta_{ik}\delta_{\lambda\lambda'}\alpha_{\lambda}$$
  

$$\frac{\overline{n_{i\lambda}}}{g_{i\lambda}} = \frac{1}{w_{i\lambda} + \alpha_{\lambda}},$$

$$w_{i\lambda}^{\alpha_{\lambda}} (1 + w_{i\lambda})^{1 - \alpha_{\lambda}} = e^{\frac{\varepsilon_{i\lambda} - \mu_{i\lambda}}{T}}$$

> Bosons 
$$lpha_{\lambda}=0$$
 ) and Fermions ( $lpha_{\lambda}=1$  ):

$$\overline{n}_{i\lambda} = \frac{g_{i\lambda}}{e^{\frac{\varepsilon_{i\lambda} - \mu_{i\lambda}}{T}} \mp 1}$$

#### Thermodynamic quantities:

> Thermodynamic potential:

$$\Omega = -T \ln Z = -T \sum_{i\lambda} g_{i\lambda} \ln(1 + w_{i\lambda}^{-1})$$

The average energy:

$$\left\langle H \right\rangle = \sum_{i\lambda} \varepsilon_{i\lambda} \overline{n_{i\lambda}}$$

> Pressure:

$$p = \sum_{i\lambda} \left( -\frac{\partial \varepsilon_{i\lambda}}{\partial V} \right) \overline{n}_{i\lambda}$$

> Entropy:

$$S = \frac{1}{T} \left( -\Omega + \left\langle H \right\rangle - \sum_{i\lambda} \mu_{i\lambda} \overline{n}_{i\lambda} \right)$$

Thermodynamic relations:

> The differential relation:

$$d\Omega = -SdT - pdV - \sum_{i\lambda} \overline{n}_{i\lambda} d\mu_{i\lambda}$$

> The Euler theorem:

$$\Omega = \left\langle H \right\rangle - TS - \sum_{i\lambda} \mu_{i\lambda} \overline{n}_{i\lambda}$$

The fundamental equation of thermodynamics:

$$TdS = d\langle H \rangle + pdV - \sum_{i\lambda} \mu_{i\lambda} d\overline{n}_{i\lambda}$$

#### Walecka model (I)

#### The quantum field theory

> The Lagrangian density:

$$L = \overline{\psi} [\gamma_{\mu} (i\partial^{\mu} - g_{\nu}V^{\mu}) - (m - g_{s}\phi)]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{\nu}^{2}V_{\mu}V^{\mu} + \frac{1}{2}(\partial_{\mu}\phi\partial^{\mu}\phi - m_{s}^{2}\phi^{2}),$$
  
$$F_{\mu\nu} = \partial_{\mu}V_{\nu} - \partial_{\nu}V_{\mu}$$
-qauge invariant

The Euler-Lagrange equations:

 $[\gamma^{\mu}(i\partial_{\mu} - g_{\nu}V_{\mu}) - (m - g_{s}\phi)]\psi = 0,$   $(\partial_{\mu}\partial^{\mu} + m_{s}^{2})\phi = g_{s}\overline{\psi}\overline{\psi},$  $\partial_{\mu}F^{\mu\nu} + m_{\nu}^{2}V^{\nu} = g_{\nu}\overline{\psi}\gamma^{\nu}\psi.$ 

> The conserved baryon current: (Noether theorem)

$$B^{\mu} = \overline{\psi} \gamma^{\mu} \psi$$

> The energy-momentum tensor:

$$T_{\mu\nu} = \frac{1}{2} \left( -\partial_{\lambda}\phi \partial^{\lambda}\phi + m_{s}^{2}\phi^{2} + \frac{1}{2}F_{\lambda\sigma}F^{\lambda\sigma} - m_{\nu}^{2}V_{\lambda}V^{\lambda} \right) g_{\mu\nu} + i\overline{\psi}\gamma_{\mu}\partial_{\nu}\psi + \partial_{\mu}\phi\partial_{\nu}\phi + \partial_{\nu}V^{\lambda}F_{\lambda\mu}$$

-gauge invariant -Lorentz invariant -point like particles

≻

#### The mean-field approximation

$$\phi = \phi_0, V_{\mu} = \delta_{\mu 0} V_0$$

> The Lagrangian density:

$$L = \overline{\psi} [i\gamma_{\mu}\partial^{\mu} - g_{\nu}\gamma^{0}V_{0} - (m - g_{s}\phi_{0})]\psi + \frac{1}{2}(m_{\nu}^{2}V_{0}^{2} - m_{s}^{2}\phi_{0}^{2}),$$

> The equations of motion:

$$i\frac{\partial \psi}{\partial t} = \hat{H}_{D}\psi, \hat{H}_{D} = \vec{\alpha}\vec{p} + \beta m^{*} + g_{v}V_{0},$$
  

$$\phi_{0} = \frac{g_{s}}{m_{s}^{2}}\overline{\psi}\psi,$$
  

$$\vec{\mu} = -i\vec{\nabla},$$
  

$$V_{0} = \frac{g_{v}}{m_{v}^{2}}\overline{\psi}\gamma^{0}\psi.$$
  

$$\vec{n}^{*} = m - g_{s}\phi_{0}$$

> The conserved baryon current:

$$B^{\mu} = \overline{\psi} \gamma^{\mu} \psi$$

> The energy-momentum tensor:

$$T_{\mu\nu} = i \overline{\psi} \gamma_{\mu} \partial_{\nu} \psi - \frac{1}{2} \left( m_{\nu}^2 V_0^2 - m_s^2 \phi_0^2 \right) g_{\mu\nu}$$

#### Walecka model (III)

#### The mean-field approximation. The second quantization method

> The plane wave solution for Dirac equation:

$$\psi(x) = \sum_{\bar{p},\sigma} \frac{1}{\sqrt{2E^*V}} \left( a_{\bar{p}\sigma} u_{p,\sigma} e^{-ip^{(+)}x} + b_{\bar{p}\sigma}^+ u_{-p,-\sigma} e^{ip^{(-)}x} \right),$$

$$E^{*} = \sqrt{\vec{p}^{2} + m^{*2}}, p^{\mu} = (E^{*}, \vec{p}), p^{2} = m^{*2}$$
$$p^{(\pm)\mu} = (\mathcal{E}^{(\pm)}, \vec{p}), \mathcal{E}^{(\pm)} = E^{*} \pm g_{\nu} V_{0}$$

> Anti-commutation relations:

$$a_{\vec{p}\sigma}, a^+_{\vec{p}'\sigma'} = \delta_{\vec{p}\vec{p}'}\delta_{\sigma\sigma'}, \{b_{\vec{p}\sigma}, b^+_{\vec{p}'\sigma'}\} = \delta_{\vec{p}\vec{p}'}\delta_{\sigma\sigma'}$$

-create and annihilate particles

> The energy, baryon charge, scalar and vector fields:

$$\begin{split} \widehat{H} &= \int_{V} d^{3}r T_{00} = \sum_{\vec{p},\sigma} E^{*} \Big[ a_{\vec{p}\sigma}^{+} a_{\vec{p}\sigma} + b_{\vec{p}\sigma}^{+} b_{\vec{p}\sigma} - 1 \Big] + g_{v} V_{0} \widehat{B} - \frac{1}{2} \Big( m_{v}^{2} V_{0}^{2} - m_{s}^{2} \phi_{0}^{2} \Big) V, \\ \widehat{B} &= \int_{V} d^{3}r B^{0} = \sum_{\vec{p},\sigma} \Big[ a_{\vec{p}\sigma}^{+} a_{\vec{p}\sigma} - b_{\vec{p}\sigma}^{+} b_{\vec{p}\sigma} + 1 \Big], \quad \text{-vacuum terms} \\ \phi_{0} &= \frac{g_{s}}{m_{s}^{2} V} \int_{V} d^{3}r \overline{\psi} \psi = \frac{g_{s}}{m_{s}^{2} V} \sum_{\vec{p},\sigma} \frac{m^{*}}{E^{*}} \Big[ a_{\vec{p}\sigma}^{+} a_{\vec{p}\sigma} + b_{\vec{p}\sigma}^{+} b_{\vec{p}\sigma} - 1 \Big], \\ V_{0} &= \frac{g_{v}}{m_{v}^{2} V} \int_{V} d^{3}r \overline{\psi} \psi^{0} \psi = \frac{g_{v}}{m_{v}^{2} V} \sum_{\vec{p},\sigma} \Big[ a_{\vec{p}\sigma}^{+} a_{\vec{p}\sigma} - b_{\vec{p}\sigma}^{+} b_{\vec{p}\sigma} + 1 \Big] \quad \text{-The condition of positive energies:} \\ \text{Spin-statistics theorem (spin=1/2-fermi, Spin=0,1,... - bose)} \end{split}$$

#### Walecka model with fractional exclusion statistics (I) (equilibrium statistical mechanics)

# Grand canonical ensemble $(T,V,\mu_B)$ $\hat{\rho} = \frac{1}{z} e^{-\frac{H-\mu_B \hat{B}}{T}},$ $Z = Tr\left(e^{-\frac{\hat{H}-\mu_B\hat{B}}{T}}\right),$ $\langle A \rangle = Tr(\hat{\rho}\hat{A}).$ the occupation number representation: ≻ $\widehat{H} = \sum_{\overline{p}\lambda} \varepsilon_{\overline{p}\lambda} n_{\overline{p}\lambda} - \frac{1}{2} (m_v^2 V_0^2 - m_s^2 \phi_0^2) V,$ $\widehat{B} = \sum_{\vec{x}, \lambda} b_{\lambda} n_{\vec{p}\lambda},$ $\phi_{0} = \frac{1}{m_{s}^{2}V} \sum_{\vec{p}\lambda} g_{s\lambda} \frac{m_{\lambda}^{*}}{\sqrt{\vec{p}^{2} + m_{\lambda}^{*2}}} n_{\vec{p}\lambda},$ $V_0 = \frac{1}{m_{\nu}^2 V} \sum_{\bar{p}\lambda} g_{\nu\lambda} b_{\lambda} n_{\bar{p}\lambda},$ $\varepsilon_{\vec{p}\lambda} = \sqrt{\vec{p}^2 + m_{\lambda}^{*2}} + b_{\lambda}g_{\nu\lambda}V_0.$

**The fractional exclusion statistics**  
(diagonal representation)  

$$Z = e^{\frac{C}{T}} \sum_{\{n_{i\lambda}\}} W\{n_{i\lambda}\} e^{-\frac{1}{T} \sum_{i,\lambda} (\varepsilon_{i\lambda} - b_{\lambda} \mu_{B}) n_{i\lambda}},$$

$$W\{n_{i\lambda}\} = \prod_{i,\lambda} \frac{[\tilde{g}_{i\lambda} + n_{i\lambda} - 1 - \alpha_{\lambda} (n_{i\lambda} - 1)]!}{n_{i\lambda}! [\tilde{g}_{i\lambda} - 1 - \alpha_{\lambda} (n_{i\lambda} - 1)]!},$$

$$C = \frac{1}{2} (m_{\nu}^{2} V_{0}^{2} - m_{s}^{2} \phi_{0}^{2}) V,$$

$$n_{i\lambda} = \sum_{\vec{p} \in d_{i\lambda}} n_{\vec{p}\lambda},$$

$$\varepsilon_{i\lambda} \approx \varepsilon_{\vec{p}\lambda}, \vec{p} \in d_{i\lambda}.$$

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> The mean occupation numbers:

$$\overline{n}_{\vec{p}\lambda} = \frac{1}{w_{\vec{p}\lambda} + \alpha_{\lambda}}, \qquad \overline{n}_{\vec{p}\lambda} = \frac{\overline{n}_{i\lambda}}{\widetilde{g}_{i\lambda}}, \vec{p} \in d_{i\lambda}$$
$$w_{\vec{p}\lambda}^{\alpha_{\lambda}} (1 + w_{\vec{p}\lambda})^{1 - \alpha_{\lambda}} = e^{\frac{\varepsilon_{\vec{p}\lambda} - b_{\lambda}\mu_{B}}{T}}$$

> The thermodynamic quantities:

#### Example: Nuclear matter at finite temperature (I)

## The variables of state: $(\mathbf{T}, \mathbf{V}, \boldsymbol{\mu}_{B})$

> Parameters  $g_s, g_v$  are determined from the characteristics of cold nuclear matter at T=0.

$$c_s = \frac{g_s^2}{m_s^2}, c_v = \frac{g_v^2}{m_v^2}$$

> The densities of the scalar and vector fields are calculated from the system of two equations:

$$\rho_{s} = \frac{\gamma}{V} \sum_{\vec{p}} \frac{m - c_{s} \rho_{s}}{\sqrt{\vec{p}^{2} + (m - c_{s} \rho_{s})^{2}}} \left[ \overline{n}_{\vec{p}}^{(+)} + \overline{n}_{\vec{p}}^{(-)} \right],$$
$$\rho_{v} = \frac{\gamma}{V} \sum_{\vec{p}} \left[ \overline{n}_{\vec{p}}^{(+)} - \overline{n}_{\vec{p}}^{(-)} \right], \qquad \gamma = 4$$

> Single particles energies:

$$\begin{split} \varepsilon_{\vec{p}}^{(+)} &= \sqrt{\vec{p}^{2} + (m - c_{s}\rho_{s})^{2}} + c_{v}\rho_{v}, \\ \varepsilon_{\vec{p}}^{(-)} &= \sqrt{\vec{p}^{2} + (m - c_{s}\rho_{s})^{2}} - c_{v}\rho_{v}, \end{split}$$

> The parameter of fractional exclusion statistics:  $\alpha$ 

Description	Particles	Mass, MeV	$J^{\pi}$	I
Baryon	$p, n, \overline{p}, \overline{n}$	<i>m</i> = 938	1/2	1/2
Neutral scalar meson	σ	$m_{s} = 550$	0+	0
Neutral vector meson	ω	$m_v = 783$	1 <sup>-</sup>	0

$$\rho_{s} = \frac{m_{s}^{2}}{g_{s}}\phi_{0}, \rho_{v} = \frac{m_{v}^{2}}{g_{v}}V_{0},$$

$$\overline{n}_{\vec{p}}^{(+)} = \frac{1}{w_{\vec{p}}^{(+)} + \alpha}, w_{\vec{p}}^{(+)\alpha} (1 + w_{\vec{p}}^{(+)})^{1-\alpha} = e^{\frac{\varepsilon_{\vec{p}}^{(+)} - \mu_B}{T}},$$
$$\overline{n}_{\vec{p}}^{(-)} = \frac{1}{w_{\vec{p}}^{(-)} + \alpha}, w_{\vec{p}}^{(-)\alpha} (1 + w_{\vec{p}}^{(-)})^{1-\alpha} = e^{\frac{\varepsilon_{\vec{p}}^{(-)} + \mu_B}{T}},$$

#### Example: Nuclear matter at finite temperature (II)

The thermodynamic quantities for nuclear matter at finite temperature:  $\frac{\Omega^2}{V} = -T \frac{\gamma}{V} \sum_{\bar{p}} \left[ \ln(1 + w_{\bar{p}}^{(+)^{-1}}) + \ln(1 + w_{\bar{p}}^{(-)^{-1}}) \right] - \frac{1}{2} \left( c_v \rho_v^2 - c_s \rho_s^2 \right),$  $\varepsilon = \frac{\langle H \rangle}{V} = \frac{\gamma}{V} \sum_{\vec{v}} \sqrt{\vec{p}^2 + (m - c_s \rho_s)^2} [n_{\vec{p}}^{(+)} + n_{\vec{p}}^{(-)}] + \frac{1}{2} (c_v \rho_v^2 + c_s \rho_s^2),$  $p = \frac{\gamma}{3V} \sum_{\bar{p}} \frac{\vec{p}^2}{\sqrt{\vec{p}^2 + (m - c_{\mu} \rho_{\mu})^2}} [n_{\bar{p}}^{(+)} + n_{\bar{p}}^{(-)}] + \frac{1}{2} (c_{\nu} \rho_{\nu}^2 - c_{s} \rho_{s}^2)$  $\rho_{B} = \frac{\langle B \rangle}{V} = \frac{\gamma}{V} \sum_{\vec{v}} \left[ \overline{n}_{\vec{p}}^{(+)} - \overline{n}_{\vec{p}}^{(-)} \right] = \rho_{v},$  $\rho = \frac{\gamma}{V} \sum_{\bar{\nu}} \left[ \overline{n}_{\bar{p}}^{(+)} + \overline{n}_{\bar{p}}^{(-)} \right],$  $s = \frac{S}{V} = \frac{1}{T} \left( -\frac{\Omega}{V} + \varepsilon - \mu_B \rho_B \right)$ 

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#### Example: Nuclear matter at finite temperature (III)



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#### Example: Nuclear matter at finite temperature (IV)



## Summary

- 1. The relativistic mean-field theory (Walecka model) was generalized by introducing the fractional exclusion statistics
- 2. The change of statistics, expressed by the fractional exclusion statistics parameter,  $\alpha$ , has a strong influence on the nuclear matter equation of state at low temperatures—for example in the region of phase transition
- 3. The phase transition parameters range depends on the exclusion statistics parameter  $\alpha$ .