Spin correlations of muons produced in the annihilation process $e^+e^- \to \mu^+\mu^-$

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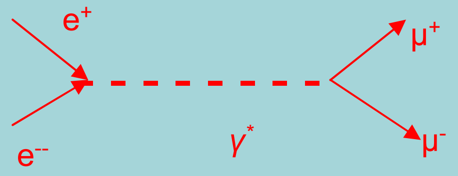
In the first approximation over the constant $e^2/\hbar c$, the process of conversion of the electron-positron pair into the muon one (or $\tau^+\tau^-$) is described by the one-photon diagram :



The virtual photon with time-like momentum transfers angular momentum J=1 and negative parity. The internal parities of μ^+ and μ^- are opposite: the ($\mu^+\mu^-$) pair is generated in triplet states (total spin *S*=1), with total angular momentum J=1 and negative space parity .

Helicity amplitudes:

$$f_{\Lambda'\Lambda}(\theta,\phi) = R_{\Lambda'\Lambda}(E)\, d^{(1)}_{\Lambda'\Lambda}(\theta)\, \exp(i\Lambda\phi)$$

d- functions

θ and φ – polar and azimuthal angles of flight direction of μ^+ with respect to positron momentum in the c.m. frame of the reaction; Λ' – difference of helicities of μ^+ and μ^- , Λ - difference of helicities of e^+ and e^- ; *E* – total energy in the c.m. frame .

Factorization: $$R_{\Lambda'\Lambda}(E) = r_{\Lambda'}^{(\mu)}(E)\, r_{\Lambda}^{(e)}(E)$$

Parity conservation: $$r_{+1}^{(\mu)}(E) = r_{-1}^{(\mu)}(E), \quad r_{+1}^{(e)}(E) = r_{-1}^{(e)}(E)$$

As follows from the structure of electromagnetic current for pairs $(e^+ e^-)$ and $(\mu^+ \mu^-)$:

$$r_0^{(\mu)}(E) = \frac{m_\mu}{E} r_1^{(\mu)}(E) = \sqrt{1-\beta_\mu^2}\, r_1^{(\mu)}(E), \quad r_0^{(e)}(E) = \frac{m_e}{E} r_1^{(e)}(E)$$

m_μ and m_e - muon and electron masses , β_μ - muon velocity .

Since always $E \geq m_\mu >> m_e$, the contribution of states of electron and positron with antiparallel spins (equal helicities) is negligibly small : $r_0^{(e)}(E) \approx 0$, $R_{\Lambda'0}(E) \approx 0$.

At the annihilation of electron and positron being totally polarized in the direction parallel to positron momentum in the reaction c.m. frame, the $(\mu^+ \mu^-)$ system is generated in the triplet state :

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2 \sin^2\theta}} \left(\frac{1+\cos\theta}{2} |+1\rangle - \sqrt{1-\beta_\mu^2}\, \frac{\sin\theta}{\sqrt{2}} |0\rangle + \frac{1-\cos\theta}{2} |-1\rangle \right)$$

$$|+1\rangle = |+1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)}, \quad |-1\rangle = |-1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)},$$

$$|0\rangle = \frac{1}{\sqrt{2}} \left(|+1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)} + |-1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)} \right)$$

- states with projections of total spin of $(\mu^+ \mu^-)$ pair onto the μ^+ momentum direction in the c.m.s. +1, -1 and 0

- If the electron and positron are totally polarized in the direction being antiparallel to the positron momentum in the c.m. frame:

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2 \sin^2\theta}} \left(\frac{1-\cos\theta}{2} |+1\rangle + \sqrt{1-\beta_\mu^2} \frac{\sin\theta}{\sqrt{2}} |0\rangle + \frac{1+\cos\theta}{2} |-1\rangle \right)$$

- When the primary electron and positron are not polarized, the nonfactorizable states $|\psi\rangle^{(+1)}$ and $|\psi\rangle^{(-1)}$ are generated with equal probabilities .

- Spin states of two particles with spin ½ are characterized by the polarization vectors $\vec{P}_1 = \langle \hat{\vec{\sigma}}^{(1)} \rangle$, $\vec{P}_2 = \langle \hat{\vec{\sigma}}^{(2)} \rangle$ and correlation tensor $T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle$;

$\hat{\vec{\sigma}} = \{\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3\} \equiv \{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ - vector Pauli operator ;

$\langle \ldots \rangle$ - sign of averaging .

- In the one-photon approximation, the generated muons are unpolarized but their spins are strongly correlated .
- Correlation tensor components at the choice of axis *z* along the relative momentum of muons in the c.m. frame and axis *y* – along the normal to the reaction plane :

$$T_{zz}^{(\mu^+\mu^-)} = \frac{2\cos^2\theta + \beta_\mu^2 \sin^2\theta}{2 - \beta_\mu^2 \sin^2\theta},$$

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2 - \beta_\mu^2)\sin^2\theta}{2 - \beta_\mu^2 \sin^2\theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2 \sin^2\theta}{2 - \beta_\mu^2 \sin^2\theta},$$

$$T_{xz}^{(\mu^+\mu^-)} = T_{zx}^{(\mu^+\mu^-)} = -\frac{(1 - \beta_\mu^2)^{1/2} \sin 2\theta}{2 - \beta_\mu^2 \sin^2\theta},$$

$$T_{xy}^{(\mu^+\mu^-)} = T_{yx}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = 0.$$

The trace of correlation tensor : $T^{(\mu+\mu^-)} = T_{xx}^{(\mu+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1,$,

just as it should hold for any triplet states .

The trace of correlation tensor *T* determines the angular correlation between the flight directions ($\vec{n}_1$ and $\vec{n}_2$) of products of decay of two unstable particles in the case when space partity is not conserved .

R. Lednicky, V. L. Lyuboshitz (2001) // Phys.Lett., B508, 146
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- Angular distributions at parity nonconservation :

$$dW_1 = \frac{1}{4\pi}(1+\alpha_1\vec{P}_1\vec{n}_1)d\Omega_1, \qquad dW_2 = \frac{1}{4\pi}(1+\alpha_2\vec{P}_2\vec{n}_2)d\Omega_2,$$

$\vec{P}_1, \vec{P}_2$ – polarization vectors, α_1 and α_2 – coefficients of *P* – odd asymmetry ; decay is the spin analyzer for an unstable particle .

Double angular distribution :

$$d^2W(\vec{n}_1,\vec{n}_2) = \frac{1}{16\pi^2}\left(1+\alpha_1\vec{P}_1\vec{n}_1+\alpha_2\vec{P}_2\vec{n}_2+\sum_{i=1}^{3}T_{ik}n_{1i}n_{2k}\right) d\Omega_1 d\Omega_2$$

correlation tensor components

The unit vectors $\vec{n}_1$, $\vec{n}_2$ are defined in the rest frames of the unstable particles 1 and 2 , in the coordinate axes of the c.m. frame of the particle pair .

- Angular correlations between the directions $\vec{n}_1$, $\vec{n}_2$ at the decays of two unstable particles are integrated over all angles except the angle β between $\vec{n}_1$ and $\vec{n}_2$;

$$dN(\beta) = \frac{1}{2}(1 + \frac{\alpha_1\alpha_2}{3} T\cos\beta)\sin\beta\, d\beta$$

irrespective of the polarization vectors $\vec{P}_1$ and $\vec{P}_2$, which may be equal to zero $(\cos\beta = \vec{n}_1\vec{n}_2)$.

$T = \sum_{i=1}^{3} T_{ii} = \rho_t - 3\rho_s$ - trace of the correlation tensor ;

ρ_s and ρ_t - relative fractions of the singlet and triplet states, respectively .

Coefficient of *P* – odd asymmetry of electron emission at the decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, averaged over the electron energy spectrum: α = -⅓ .

At the decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$: α = +⅓ .

- Angular correlations between the flight directions for the electron and positron at the decay of muon pair ($\mu^+\mu^-$) :

$$dN(\beta) = \frac{1}{2}(1 - \frac{1}{27} T\cos\beta)\sin\beta\, d\beta$$

In the process $e^+e^- \to \mu^+\mu^-$ the ($\mu^+\mu^-$) system is produced in the triplet state, thus, *T* =1 : $dN(\beta) = \frac{1}{2}(1 - \frac{1}{27}\cos\beta)\sin\beta\, d\beta$.

- As it was shown previously, in the case of incoherent mixtures of factorizable states of two particles with spin ½ the modulus of sum of any two (and three) diagonal components of the correlation tensor cannot exceed unity :

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$$|T| = |T_{xx} + T_{yy} + T_{zz}| \leq 1 \quad |T_{xx} + T_{yy}| \leq 1 \quad |T_{xx} + T_{zz}| \leq 1 \quad |T_{yy} + T_{zz}| \leq 1$$

In the case of non-factorizable coherent superpositions of two-particle states , the “incoherence” inequalities may be violated .

For the singlet state all the inequalities are violated :

$$T_{xx} + T_{yy} = T_{xx} + T_{zz} = T_{yy} + T_{zz} = -2\,, \quad T = -3 \quad .$$

In the case of the non-factorizable triplet state with zero projection of total spin onto the axis z , one of the restrictions is not satisfied : instead of the inequality $|T_{xx} + T_{yy}| < 1$, the equality $T_{xx} + T_{yy} = 2$ (>1) holds .

In the process of (e^+e^-) annihilation , the ($\mu^+\mu^-$) system is generated in the non-factorizable triplet states : $|\psi\rangle^{(+1)}$ and $|\psi\rangle^{(-1)}$.

- One is the "incoherence" inequalities is always violated in the process $e^+ e^- \to \mu^+ \mu^-$ at $\theta \neq 0$:

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = \frac{2}{2 - \beta_\mu^2 \sin^2\theta} > 1.$$

- Analogous consideration -- for the process $e^+e^- \to \tau^+\tau^-,$ ($m_\mu \to m_\tau, \quad \beta_\mu \to \beta_\tau$) .

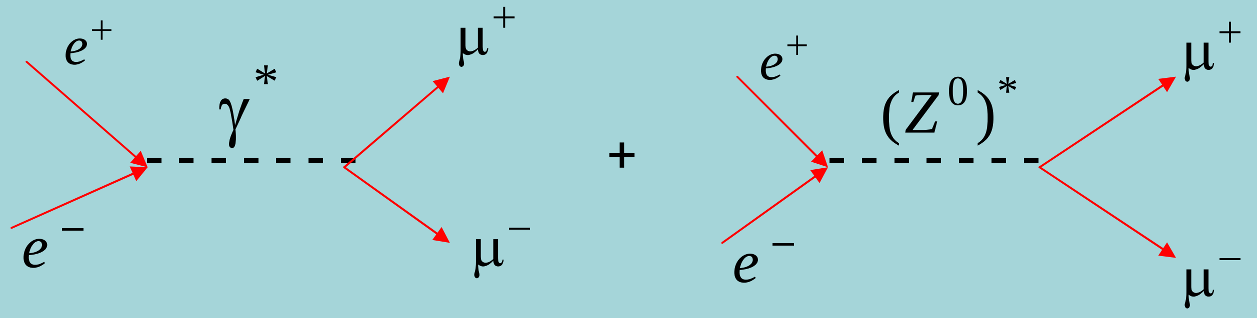
- At very high energies ($\beta_\mu \to 1$, $\beta_\tau \to 1$) , the nonzero components of the correlation tensor are as follows :

$$T_{xx}^{(\mu^+\mu^-)} = -T_{yy}^{(\mu^+\mu^-)} = \frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{zz}^{(\mu^+\mu^-)} = 1.$$

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = \frac{2}{1+\cos^2\theta} > 1$$

- At high energies the annihilation processes

$e^+e^- \to \mu^+\mu^-, \; e^+e^- \to \tau^+\tau^-$ are conditioned not only by electromagnetic interaction but also by the weak interaction of neutral currents through the virtual Z^0 boson :



- Interference of the amplitudes of the purely electromagnetic and weak interaction → leads to the charge asymmetry in lepton emission and to the space parity violation.

$\mu^+\mu^-, \; \tau^+\tau^- \to$ generated in the triplet states : ${}^3S_1, \; {}^3D_1$ and ${}^3P_1$

due to weak interaction

(J = 1, positive *CP* parity)

It follows from the structure of "left" and "right" components of neutral currents that the nonzero helicity amplitudes of the processes

$e^+e^- \to \mu^+\mu^-,\ e^+e^- \to \tau^+\tau^-$ in the c.m. frame take the form :

$$R_{11}(E) = \frac{e^2}{2E}\left[1 + x\left(\xi - \frac{1}{2}\right)^2\right] \ ; \qquad R_{-1-1}(E) = \frac{e^2}{2E}\left[1 + x\,\xi^2\right] \ .$$

$$R_{-11}(E) = R_{1-1}(E) = \frac{e^2}{2E}\left[1 + x\,\xi\left(\xi - \frac{1}{2}\right)\right] \ ;$$

($R_{0\lambda}$, $R_{\lambda'0}$ $\to$ turn practically to zero at high energies) .

Here: $\xi = \sin^2\theta_W$, where θ_W is the Weinberg angle (angle of gauge boson mixing) ; parameter x $\to$ determines the relative contribution of weak interaction : $x = \dfrac{1}{\sin^2\theta_W \cos^2\theta_W} \dfrac{s}{s - (M_{Z^0} - i\frac{\Gamma_{Z^0}}{2})^2}$ $\quad (s = (2E)^2)$

According to the standard model : $$\frac{1}{\sin^2\theta_W \cos^2\theta_W} = \frac{1}{\xi(1-\xi)} \approx 6 = \frac{\sqrt{2}\,G_F M_{Z^0}^2}{\pi\alpha}$$

$G_F = 1.166 \cdot 10^{-5}\ GeV^{-2}$ $\to$ universal Fermi constant of weak interaction, $\alpha = 1/137$

Finally, performing the further analysis, we obtain, in particular, that :

1) Due to the weak interaction through the Z^0 boson, the final leptons, generated at the annihilation of the unpolarized electron and positron, acquire the longitudinal polarization (whereas, if the weak interaction contribution is neglected, the final leptons are correlated but unpolarized) .

At the energies below and above the resonance energy, the average helicities of the final leptons have different signs :

$$E < \frac{M_{Z^0}}{2} \;\rightarrow\; x < 0,\; \overline{\lambda}_+ < 0,\; \overline{\lambda}_- > 0 \;;$$

$$E > \frac{M_{Z^0}}{2} \;\rightarrow\; x > 0,\; \overline{\lambda}_+ > 0,\; \overline{\lambda}_- < 0 \;.$$

2) Structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at high energies .

In doing so, $T_{zz} = 1$ as before ;

the expression for T_{xx} changes , but, as before, $T_{xx} = -T_{yy}$:

$$T_{xx} = -T_{yy} = \sin^2\theta \frac{[1 + x(\xi^2 - \frac{1}{2}\xi + \frac{1}{8})]\,[1 + x\,\xi(\xi - \frac{1}{2})]}{a_+(E)(1+\cos^2\theta) + 2\,a_-(E)\cos\theta}$$

where

$$a_+(E) = 1 + \frac{1}{2}x\left(\frac{1}{2} - 2\xi\right)^2 + \frac{1}{4}x^2\left[\left(\frac{1}{2} - \xi\right)^2 + \xi^2\right]^2 ,$$

$$a_-(E) = \frac{1}{8}x + \frac{1}{4}x^2\left(\frac{1}{4} - \xi\right)^2 .$$

Again, one of the incoherence inequalities for the correlation tensor components is violated : $T_{xx} + T_{zz} > 1$.

Main conclusions

- On the basis of the technique of helicity amplitudes, the process $e^+ e^{--} \rightarrow \mu^+ \mu^{--}$ is theoretically investigated in the one-photon approximation . The structure of triplet states of the $(\mu^+ \mu^{--})$ system is found .
- It is shown that, if the primary electron and positron are not polarized, the final muons μ^+ и μ^{--} are not polarized as well but their spins are strongly correlated . Explicit expressions for the components of correlation tensor of the final $(\mu^+ \mu^{--})$ system are derived .
- The formula for the angular correlation at the decays of muons μ^+ and μ^{--}, generated in the annihilation process $e^+ e^{--} \rightarrow \mu^+ \mu^{--}$, into the channels $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$ and $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$, is obtained .
- It is established that in the process $e^+ e^- \rightarrow \mu^+ \mu^{--}$ one of the "incoherence" inequalities for the correlation tensor components is always violated .
- The obtained results remain qualitatively valid with the account of weak interaction of neutral currents through the exchange by Z^0 boson .

Thank you !