Resonance Chiral Theory and meson production in electron-positron annihilation

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XIX International Baldin Seminar on High Energy Physics Problems, Dubna 29 Sept - 4 Oct, 2008

Plan

- Brief overview of Chiral Perturbation Theory (χPT) and Resonance Chiral Theory (RχT)
- Applications:

  (a) scalar meson production and decay

  (b) $K-$meson production in electron-positron annihilation

Overview of χPT and RχT

Low- and intermediate-energy physics (nonperturbative regime of QCD)

List of references (not complete):

S. Weinberg, Physica **96A** (1979) 327: Effective theories

J. Gasser, H. Leutwyler, Ann. Phys. **158** (1984) 142; Nucl. Phys. **B250** (1985) 465, 517, 539: Chiral Perturbation Theory (χPT)

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. **B321** (1989) 311, *G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael,* Phys. Lett, **B223** (1989) 425: Resonance Chiral Theory (RχT)

V. Cirigliano, G. Ecker, M. Eidenmuller, R. Kaiser, A. Pich, J. Portoles, Nucl. Phys. **B753** (2006) 139: Further developments

....

QCD Lagrangian in presence of external fields

$$\mathcal{L}_{QCD} = \mathcal{L}^0_{QCD} + \mathcal{L}_{ext.\ fields}$$

Lagrangian for massless quarks ($m_u = m_d = m_s = 0$)

$$\begin{aligned}
\mathcal{L}^0_{QCD} &= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} + i\bar{q}\gamma^\mu D_\mu q \\
&= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a} + i\bar{q}_L\gamma^\mu D_\mu q_L + i\bar{q}_R\gamma^\mu D_\mu q_R \\
D_\mu &= \partial_\mu - i g_s \frac{\lambda^a}{2} G^a_\mu \qquad (a = 1, ..., 8)
\end{aligned}$$

G^a_μ – gluon fields, λ^a - Gell-Mann matrices in color space ($a = 1, ..., 8$), g_s - strong interaction constant.

“Right” and “left” components of quark fields

$$q = \frac{1+\gamma_5}{2} q + \frac{1-\gamma_5}{2} q \equiv q_R + q_L \qquad \text{right/left quark field}$$

Interaction with external fields

Introduce external vector, axial-vector, scalar and pseudoscalar fields

$$v^\mu = v_i^\mu \lambda_i/2, \qquad a^\mu = a_i^\mu \lambda_i/2, \qquad s = s_i \lambda_i, \qquad p = p_i \lambda_i$$

matrices λ_i ($i = 0,\ 1...,8$) act in flavor space of light u, d, s quarks

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

The most general interaction with external fields

$$\mathcal{L}_{ext.\ fields} = \bar{q}[\gamma_\mu(v^\mu + \gamma_5 a^\mu) - (s - ip\gamma_5)]q$$

Examples of external fields

- Electromagnetic:

$$v_\mu = -eQB_\mu, \qquad a_\mu = 0,$$

with quark charge matrix

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \equiv \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$$

- Weak neutral and charged:

$$\begin{aligned} r_\mu = v_\mu + a_\mu &= \frac{e\sin\theta_W}{\cos\theta_W} QZ_\mu \\ l_\mu = v_\mu - a_\mu &= \frac{e}{\sin\theta_W\cos\theta_W}(Q\sin^2\theta_W + \frac{1}{6} - Q)Z_\mu \\ &- \frac{e}{\sqrt{2}\sin\theta_W}\begin{pmatrix} 0 & V_{ud}W_\mu^+ & V_{us}W_\mu^+ \\ V_{ud}W_\mu^- & 0 & 0 \\ V_{us}W_\mu^- & 0 & 0 \end{pmatrix} \end{aligned}$$

Examples of external fields

- Z_μ, $W^\pm_\mu$ - weak boson fields,
  CKM matrix elements: $V_{ud} = 0.9739 \pm 0.0003$ from nuclear β-decays, $V_{us} = 0.2248 \pm 0.0016$ from $K_{l3} + K_{\mu 2}$ decays,
  θ_W is Weinberg angle.
- scalar field:

$$s = \mathrm{diag}(m_u, m_d, m_s)(1 + H/v) = \mathrm{diag}(m_u, m_d, m_s) + ...$$

  generates quark mass terms and interaction with Higgs boson H, and $v \approx 250$ GeV (VEV of scalar field).
- pseudoscalar field: additional Higgs bosons in extensions of SM.

Chiral symmetry of QCD Lagrangian

Global $SU(3)_L \times SU(3)_R$ acting on left and right quark fields

$$q_L \to U_L q_L, \qquad q_R \to U_R q_R$$

(of course, in addition to local color symmetry $SU(3)_c$ and $U(1)_V \times U(1)_A$).

Then $\mathcal{L}^0_{QCD} + \mathcal{L}_{ext.\ fields}$ is invariant under local transformations

$$q_L \to U_L(x) q_L, \qquad q_R \to U_R(x) q_R$$

where $U_{R/L}(x) = \exp\{\lambda^a\, \theta^a_{R/L}(x)/2\}$ provided that external fields transform

$$\begin{aligned} r_\mu &\to U_R(x) r_\mu U_R(x)^\dagger + i U_R(x) \partial_\mu U_R(x)^\dagger \\ l_\mu &\to U_L(x) l_\mu U_L(x)^\dagger + i U_L(x) \partial_\mu U_L(x)^\dagger \\ s + ip &\to U_R(x)(s+ip) U_L(x)^\dagger \end{aligned}$$

Effective Lagrangian

Generating functional

$$e^{\,iZ_{QCD}(v,a,s,p)} = \int [dq][d\bar{q}][dG_\mu]\, e^{\,i\int(\mathcal{L}^0_{QCD}+\mathcal{L}_{ext.\ fields})d^4x}$$

at low energies is equivalent to

$$e^{\,iZ_{eff}(v,a,s,p)} = \int [d\ \text{hadronic fields}]\, e^{\,i\int \mathcal{L}_{eff}(v,a,s,p)d^4x}$$

This can be called hadronization: quark currents coupled to external fields at low/intermediate energies are expressed in terms of hadronic degrees of freedom.

Chiral Perturbation Theory (χPT)

Effective meson Lagrangian $\mathcal{L}_{eff}$ accounts for

(a) spontaneous breaking of chiral symmetry
$SU(3)_L \times SU(3)_R \Rightarrow SU(3)_{L+R=V}$

(b) explicit symmetry breaking by quark masses

(c) discrete symmetries P, C

(d) expansion in powers of derivatives $\partial \leftrightarrow p$ (acting on Goldstone fields) and light quark masses:

$$m_q \sim O(p^2) \sim O(M^2)$$

i.e. effective Lagrangian is

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots \qquad \chi\mathrm{PT}$$

with $\mathcal{L}_{2n} \sim O(p^{2n})$.

Chiral Perturbation Theory (χPT)

Expansion scale:

$$\Lambda_\chi \approx M_{resonance}$$

non-Goldstone boson mass, or

$$\Lambda_\chi \approx 4\pi F_\pi$$

$F_\pi = 92.4$ MeV - pion weak decay constant ($\pi^+ \to \mu^+ \nu_\mu$). In any case $\Lambda_\chi \sim 1$ GeV.
$\mathcal{L}_{2n}$ depend on Low Energy Constants (LEC's):

$$2 \quad O(p^2), \quad 10 \quad O(p^4), \quad 90 \quad O(p^6), \ \ldots$$

Should be determined from QCD, however not possible at present...

Lowest-order χPT Lagrangian in $O(p^2)$

$$\mathcal{L}_2 = \frac{F^2}{4} \langle u^\mu u_\mu + \chi_+ \rangle \tag{1}$$

$\langle \ldots \rangle \equiv \mathrm{Tr}(\ldots)$ in flavor space, $F = F_\pi$ (in chiral limit).

Lowest order χPT Lagrangian

$$\begin{aligned} u_\mu &= i[u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i l_\mu)u^\dagger] \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u \end{aligned}$$

Nonlinear representation for pseudoscalar mesons

$$u = \exp(i\Phi/\sqrt{2}F_\pi) = 1 + i\Phi/\sqrt{2}F_\pi - \Phi^2/4F_\pi^2 + \ldots$$

$\Phi = \sum_i \lambda_i \phi_i/\sqrt{2}$ are Nambu-Goldstone bosons ($J^P = 0^-$):

$$\Phi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix}$$

Mass terms in χPT Lagrangian

Explicit breaking of χ symmetry and mass terms

$$\chi = -\frac{2}{F_\pi^2}(s + ip)\langle 0|\bar{q}q|0\rangle \overset{\text{isospin symmetry } SU(2)_I}{=} \mathrm{diag}(M_\pi^2, M_\pi^2, 2M_K^2 - M_\pi^2)$$

in terms of masses of physical mesons π and K.

Value of quark condensate

$$\langle 0|\bar{q}q|0\rangle \approx (-240 \pm 10\ \mathrm{MeV})^3 \quad (\text{at scale } \mu = 1\ \mathrm{GeV})$$

is of the order of QCD scale parameter

$$\Lambda_{QCD} \approx -\langle 0|\bar{q}q|0\rangle^{1/3} = 200 - 300\ \mathrm{MeV}$$

Chiral symmetry of effective Lagrangian

Goldstone fields ϕ_i parameterize $u = u(\phi)$ of coset space $SU(3)_L \times SU(3)_R / SU(3)_{V=R+L}$ transforming as

$$u(\phi) \to u(\phi') = U_R u(\phi) h(U,\phi)^{-1} = h(U,\phi) u(\phi) U_L^{-1}$$

for general chiral transformation $U = (U_R, U_L)$.

Other fields (massive mesons, or resonances) transform under $SU(3)_V$

$$R \to R' = h(U,\phi) R h(U,\phi)^{-1}, \quad or \quad R \to R' = R.$$

Building blocks are labeled according to chiral power counting:

$$\begin{aligned} u \quad &\sim \quad R \sim O(1), \qquad \partial^\mu \sim l^\mu \sim r^\mu \sim O(p), \\ s, p \quad &\sim \quad O(p^2), \qquad \chi_\pm \sim O(p^2)\ldots \end{aligned}$$

Resonance chiral theory RχT

LECs are saturated by low-lying multiplets of resonances: vector $V(1^-)$, axial-vector $A(1^+)$, scalar $S(0^+)$ and pseudo-scalar $P(0^-)$, which successfully describe phenomenology.

$$\mathcal{L}_{eff} \approx \mathcal{L}_2^{GB} + \mathcal{L}_4^{GB} + \mathcal{L}_6^{GB} + \mathcal{L}_2^{R} + \mathcal{L}_4^{R} + \mathcal{L}_2^{RR'} + \mathcal{L}_0^{RR'R''} \qquad R\chi T$$

reproduces χPT Lagrangian $\mathcal{L}_{eff}$ in $O(p^6)$.

In the lowest order (and large N_c: octet + singlet = nonet)

$$\begin{aligned}
\mathcal{L}_2^R &= \mathcal{L}_{R,\ kin} + \mathcal{L}_{R,\ int}, \\
\mathcal{L}_{R,\ kin} &= -\frac{1}{2}\sum_{R=V,A}\left\langle \nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{M_R^2}{2} R_{\nu\mu}R^{\nu\mu}\right\rangle \\
&\quad +\frac{1}{2}\sum_{R=S,P}\left\langle \nabla^\nu R \nabla_\nu R - M_R^2 R^2\right\rangle
\end{aligned}$$

Lowest-order resonance Lagrangian

and interaction of resonances with Goldstone bosons

$$\mathcal{L}_{R,\ int} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle + \frac{iF_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle$$
$$+ c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle + i d_m \langle P \chi_- \rangle$$

$\mathcal{L}_2^{GB} = \mathcal{L}_2$ (lowest-order GB Lagrangian)
$\mathcal{L}_4^{GB} = 0$ in *antisymmetric tensor* representation for spin-1 fields $V^{\mu\nu},\ A^{\mu\nu}$
not known whether $\mathcal{L}_6^{GB} = 0$ or not

Some other definitions

$$
\begin{aligned}
f_{\pm}^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu} u \\
F_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu] \\
F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]
\end{aligned}
$$

$$
\nabla_\mu ... = \partial_\mu ... + [\Gamma_\mu, \ ...], \qquad \Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]
$$

LECs in order $O(p^4)$ are expressed through coupling constants of lowest-order resonance Lagrangian $\mathcal{L}_2^R$

$$
\text{LECs} \qquad L_1, \ ... \ , L_{10} = \text{Functions}\ (F_V, G_V, F_A, c_d, c_m, d_m)
$$

and masses of resonances $M_V, \ M_A, \ M_S, \ M_P$.

Average values of LECs suggest chiral symmetry breaking scale:

$$
L_i \sim (F_\pi / 2\Lambda_\chi)^2 \sim 10^{-3} \quad \Longrightarrow \quad \Lambda_\chi \sim 1 \ \text{GeV}
$$

Resonance multiplets

Scalar nonet ($J^P = 0^+$) is

$$S = \begin{pmatrix} a_0^0/\sqrt{2} + S_8/\sqrt{6} & a_0^+ & \kappa^+ \\ a_0^- & -a_0^0/\sqrt{2} + S_8/\sqrt{6} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -2S_8/\sqrt{6} \end{pmatrix} + \frac{S_0}{\sqrt{3}}$$

$V_{\mu\nu}$ is nonet of vector mesons ($J^P = 1^-$)

$$V_{\mu\nu} = \begin{pmatrix} \rho^0/\sqrt{2} + \omega_8/\sqrt{6} & \rho^+ & K^{*+} \\ \rho^- & -\rho^0/\sqrt{2} + \omega_8/\sqrt{6} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\omega_8/\sqrt{6} \end{pmatrix}_{\mu\nu} + \frac{(\omega_0)_{\mu\nu}}{\sqrt{3}}$$

and similarly for others.

Mixing of mesons

Physical mesons with zero isospin are mixture octet and singlet.

For scalars

$$\begin{pmatrix} f_0(600) \equiv \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos\theta_s & \sin\theta_s \\ -\sin\theta_s & \cos\theta_s \end{pmatrix} \begin{pmatrix} S_8 \\ S_0 \end{pmatrix}$$

with unknown mixing angle.

For pseudoscalars

$$\begin{pmatrix} \eta(547) \\ \eta'(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_{ps,8} & \sin\theta_{ps,0} \\ -\sin\theta_{ps,8} & \cos\theta_{ps,0} \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

with $\theta_{ps,0} = -9.2^\circ$ and $\theta_{ps,8} = -21.2^\circ$ from experiment [Feldmann et al., Phys.Rev. D58 (1998) 114006].

Comment: η' needs a special consideration because of $U(1)_A$ anomaly.

Odd-intrinsic-parity sector

Interactions above couple only even number of pseudoscalar fields (not a symmetry of QCD).

One needs to add terms with odd number of pseudoscalar fields (so-called “anomalous”). The known example is Wess-Zumino-Witten Lagrangian which in particular describes $\pi^0 \to \gamma\gamma$

$$\mathcal{L}_{\pi^0\gamma\gamma} = -\frac{e^2 N_c}{24\pi^2 F_\pi}\epsilon^{\mu\nu\alpha\beta}\pi^0 \partial_\mu B_\nu \partial_\alpha B_\beta$$

The lowest-order effective $\mathrm{R}\chi\mathrm{T}$ Lagrangian contributing to $O(p^6)$

$$\mathcal{L}_3^R = \epsilon^{\mu\nu\alpha\beta}\Big(i\theta_V \left\langle V^\mu u^\nu u^\alpha u^\beta \right\rangle + h_V \left\langle V^\mu \{u^\nu, f_+^{\alpha\beta}\} \right\rangle + h_A \left\langle A^\mu \{u^\nu, f_-^{\alpha\beta}\} \right\rangle \Big)$$

with free parameters h_V, h_A, θ_V.

Inclusion of baryons

How to add baryons p, n, Λ, $\Sigma^{\pm,0}$, ...?

In $SU(3)_L \times SU(3)_R$ case (active quarks u, d, s)

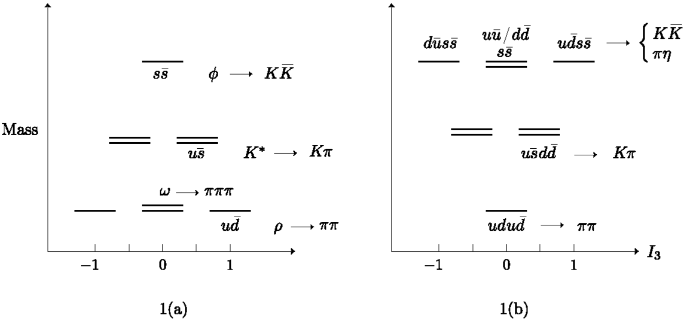
$$\mathcal{L}_{BM} = \langle \bar{B}(i\gamma_\mu \mathcal{D}^\mu - M_0)B \rangle - \frac{D}{2} \langle \bar{B}\gamma_\mu\gamma_5 [u^\mu, B]_+ \rangle - \frac{F}{2} \langle \bar{B}\gamma_\mu\gamma_5 [u^\mu, B]_- \rangle$$

with a “covariant” derivative

$$\mathcal{D}^\mu B = \partial^\mu B + [\Gamma^\mu,\ B]_-$$

B is baryon $SU(3)_f$ octet matrix and Dirac spinor $B_\alpha,\ \alpha = 1, ..., 4$.
Experiments give $D \approx 0.80$, $F \approx 0.46$.
This Lagrangian gives rise to various interactions $\bar{N}N\Phi$, $\bar{N}N\Phi\Phi$, ...

Mass spectrum and quark content



Vector mesons $q\bar{q}$ (left), scalar mesons $(qq)(\bar{q}\bar{q})$ (right)

R.L. Jaffe, F. Wilczek 2003; L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer 2004; G. 't Hooft, G. Isidori, L. Maiani, A.D. Polosa, V. Riquer 2008

Motivation:

- Radiative decays featured by scalars $f_0(980)$ ($I^G(J^{PC}) = 0^+(0^{++})$) and $a_0(980)$ ($1^-(0^{++})$):

$$
\begin{array}{rr}
a_0 \to \gamma\gamma, & f_0 \to \gamma\gamma \\
\phi(1020) \to \gamma a_0, & \phi(1020) \to \gamma f_0 \\
f_0/a_0 \to \gamma\rho(770) & f_0/a_0 \to \gamma\omega(782)
\end{array}
$$

- Invariant mass spectra of $\pi\pi$ (or $\pi\eta$) in $e^+e^- \to \gamma^* \to \gamma f_0$ (a_0) $\to \pi\pi\gamma$ ($\pi\eta\gamma$) reactions
- Comparison with experiments SND and CMD (Novosibirsk) and KLOE (Frascati).

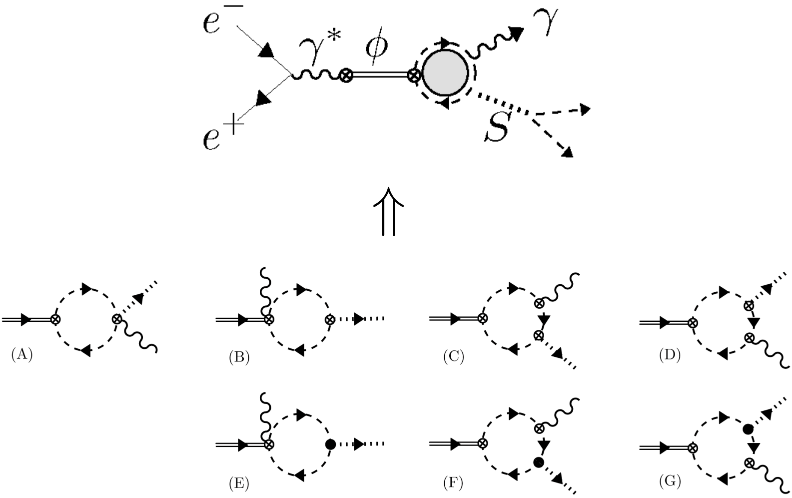
General features, decay channels of scalars

Calculated decay widths:

- Two-photon decays $a_0 \to \gamma\gamma$ and $f_0 \to \gamma\gamma$
  $\phi(1020)$ decays $\phi \to \gamma a_0$ and $\phi \to \gamma f_0$
  decays of scalars $a_0 \to \gamma\rho$, $a_0 \to \gamma\omega$, $f_0 \to \gamma\rho$, $f_0 \to \gamma\omega$
- Divergent part of $\phi \to \gamma f_0/a_0$ amplitude is proportional to $F_V - 2\,G_V$, therefore $F_V = 2\,G_V$.

  Experiment [Particle Data Group] gives: $G_V = 65.2$ MeV and $F_V = 156.16$ MeV from widths $\Gamma_{\rho\to\pi\pi} = 146.4 \pm 1.5$ MeV and $\Gamma_{\rho\to e^+e^-} = 7.02 \pm 0.11$ keV

One-loop mechanism for ϕ decays to f_0 and a_0



(momentum-dependent ⊗ and momentum-independent ● vertices for interactions with scalar meson)

Invariant-mass distributions of $\pi^0\pi^0$ and $\pi^0\eta$

- Charged-meson final states in

$$e^+e^- \to \ \dots \ \to \pi^+\pi^-\gamma$$

are used for scanning cross section $\sigma(e^+e^- \to \pi^+\pi^-)$ via **radiative return method** which is based on dominance of initial-state radiation (photon is emitted from e^- or e^+)

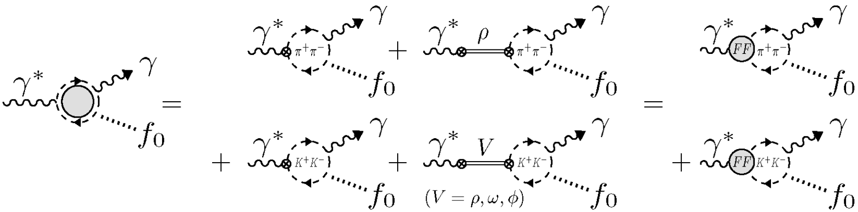
- Neutral-meson final states in

$$e^+e^- \to \ (\ f_0\gamma,\ \dots\ ) \to \pi^0\pi^0\gamma, \qquad e^+e^- \to (\ a_0\gamma,\ \dots\ ) \to \pi^0\eta\gamma$$

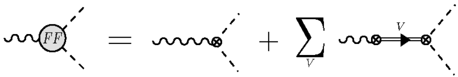
are convenient for studying **chiral dynamics** as there is no initial-state radiation ($\gamma^* \to \pi^0\pi^0$ and $\gamma^* \to \pi^0\eta$ are forbidden)

Invariant-mass distributions

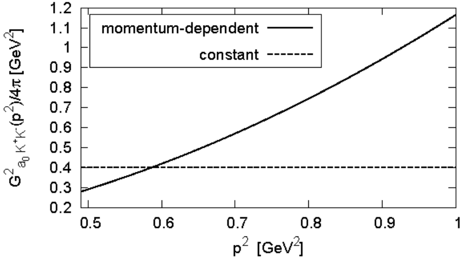
Transition $\gamma^* \to \gamma f_0$



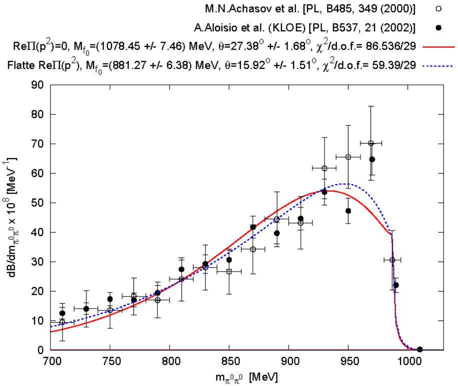
in terms of pion and kaon EM form factors $F^{\pi}_{em}(Q^2)$ and $F^{K}_{em}(Q^2)$



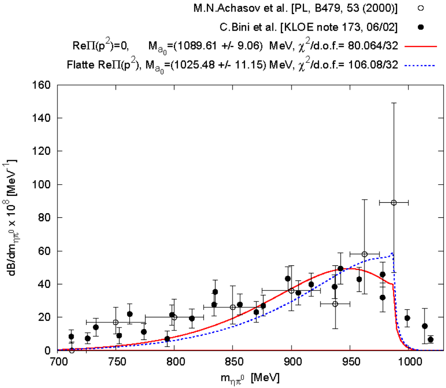
Similarly for $\gamma^* \to \gamma a_0$ transition.



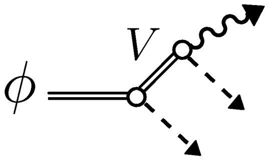
Interaction vertices a_0KK, $a_0\pi\pi$, f_0KK, $f_0\pi\pi$ are strongly momentum-dependent



Invariant mass distribution in $\phi \to \pi^o \pi^o \gamma$. Parameters $c_m = c_d = F_\pi/2 \approx 46.2$ MeV. Data: Frascati (KLOE) and Novosibirsk.



Invariant mass distribution in $\phi \to \pi^o \eta\gamma$. Parameters $c_m = c_d = F_\pi/2 \approx 46.2$ MeV. Data: Frascati (KLOE) and Novosibirsk.



Double vector-meson contributions

- $\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma$
- $\phi \to \omega\eta \to \eta\pi^0\gamma$
- $\phi \to \rho^0\pi^0 \to \eta\pi^0\gamma$
- and other channels, if energy is aside of ϕ-meson pole

Interference of $f_0(980)$ and $f_0(600) \equiv \sigma$.

Inclusion of background is in progress

Calculations vs. PDG values

widths	our fit	[Kalashnikova et al. (2006)]	PDG average
$\frac{\Gamma_{\phi\to\gamma a_0}}{\Gamma_\phi}$	1.7×10^{-4}	1.4×10^{-4}	$(7.6\pm 0.6)\times 10^{-5}$
$\frac{\Gamma_{\phi\to\gamma f_0}}{\Gamma_\phi}$	$4.4\times 10^{-4*}$	1.4×10^{-4}	$(4.40\pm 0.21)\times 10^{-4}$
$\frac{\Gamma_{\phi\to\gamma f_0}}{\Gamma_{\phi\to\gamma a_0}}$	2.6	1	6.1 ± 0.6
$\Gamma_{a_0\to\pi\eta}$	14.2 **MeV**	—	—
$\Gamma_{f_0\to\pi\pi}$	41.8 **MeV**	—	$34.2^{+22.7}_{-14.3}$ MeV
$\Gamma_{a_0,\,total}$	17.8 **MeV**	—	50 – 100 MeV
$\Gamma_{a_0\to\gamma\gamma}$	0.30 keV*	0.24 keV	0.30 ± 0.10 keV
$\Gamma_{f_0\to\gamma\gamma}$	0.31 keV*	0.24 keV	$0.31^{+0.08}_{-0.11}$ keV
$\Gamma_{a_0\to\gamma\rho}$	9.1 **keV**	3.4 keV	—
$\Gamma_{f_0\to\gamma\rho}$	9.6 **keV**	3.4 keV	—
$\Gamma_{a_0\to\gamma\omega}$	8.7 **keV**	3.4 keV	—
$\Gamma_{f_0\to\gamma\omega}$	15.0 **keV**	3.4 keV	—

(* marks input values)

Electromagnetic form factors of K-mesons

Quark content $K^+ = u\bar{s},\ K^0 = d\bar{s},\ \bar{K}^0 = \bar{d}s,\ K^- = \bar{u}s$, mass about 494 MeV.

Time-like region $q^2 \equiv s \geq 4m_K^2$:

$$\sigma(e^+e^- \to K\bar{K}) = \frac{\pi\alpha^2}{3q^2}\left(1 - \frac{4m_K^2}{q^2}\right)^{3/2}|F_K(q^2)|^2 \qquad (2)$$

data: CMD-2 and SND (Novosibirsk, Budker INP) and KLOE (Frascati, INFN).

Space-like region $q^2 \equiv s < 0$:

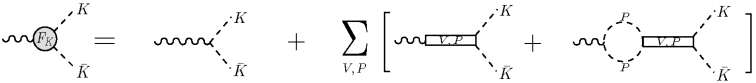
(i) $K^\pm$ scattering on atomic electrons at CERN, SPS ($-s < 0.16$ GeV^2)

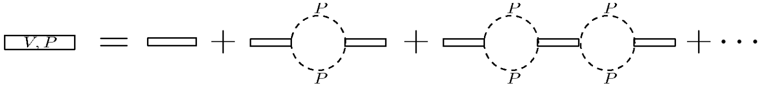
(ii) $e^-p \to e^-\Lambda K^+$ and $e^p \to e^-\Sigma^0 K^+$) at JLab (large $-s \sim 3$ GeV^2), data are analyzed

Motivations:

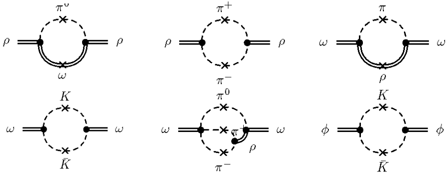
Comparison with experiment ; radial excitations $\rho(1450)$, $\omega(1420)$, $\phi(1680)$; contribution of $K\bar{K}$ to muon magnetic moment

EM form factors in RχT [S. Ivashyn, A. Korchin, Eur.Phys.J. C49, 697 (2007)]

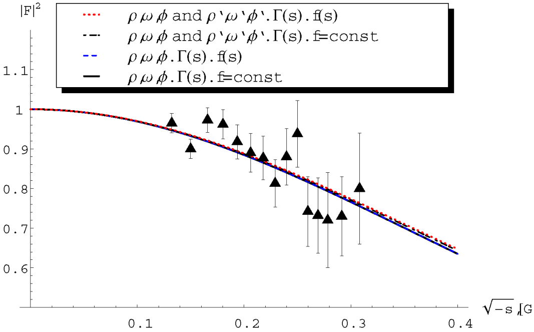




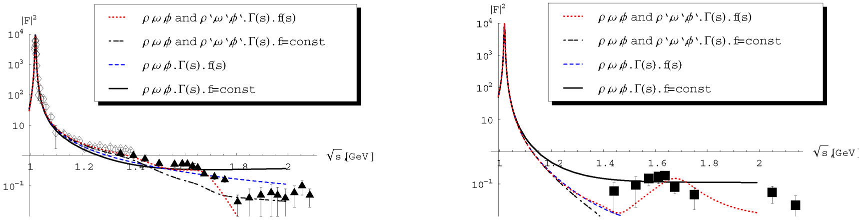
Electromagnetic form factor of K^+ (or K^-)



Self-energy of vector mesons

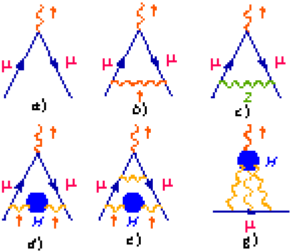


Form factor of $K^{\pm}$ in space-like region $s < 0$. Data: NA7 (CERN, SPS).

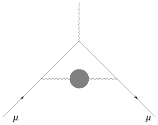


Form factor in time-like region $s > 4m_K^2$ for charged kaons $K^{\pm}$ (left) and neutral kaons K^0 (right).
Data: Frascati (KLOE) and Novosibirsk.

Contribution of kaon loops to muon anomalous MM



Typical diagrams contributing to muon magnetic moment



Hadronic contributions to $\Pi_\gamma(k^2)$ from $\pi^+\pi^-$, K^+K^-, $K^0\bar{K}^0$, 3π, 4π, ...

Gyromagnetic ratio

$$\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{s}$$

$a_\mu \equiv g_\mu/2 - 1.$

$K\bar{K}$ contribution to a_μ is determined (in lowest order) via dispersion integral [Gourdin, de Rafael 1969]

$$a_\mu^{had,K\bar{K}} = \frac{\alpha^2}{3\pi^2}\int_{4m_K^2}^{\infty} W(s)R(s)\frac{\mathrm{d}s}{s}$$

$$R(s) = \frac{\sigma(e^+e^- \to K\bar{K})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{(1-\frac{4m_K^2}{s})^{3/2}}{4(1+2\frac{m_\mu^2}{s})(1-\frac{4m_\mu^2}{s})^{1/2}}|F_K(s)|^2.$$

- $W(s) \sim 1/s$ enhances low-energy spectrum
- $\sqrt{s} < 1.8$ GeV gives 91% of total hadronic
- $\pi\pi$ through $\rho(770)$ yields 73% of total hadronic contribution.

	K^+K^-	$K^0\bar{K}^0$	total $K\bar{K}$
$a_\mu^{had,K\bar{K}}, \quad 10^{-10}$	19.06 ± 0.57	15.64 ± 0.44	34.70 ± 1.01

Value $(34.70 \pm 1.01) \times 10^{-10}$ is close (within 1.5%) to results from $e^+e^- \to K\bar{K}$ experimental cross sections (Novosibirsk). $K\bar{K}$ contributes $\sim 0.5\%$ to total hadronic contribution. The latter is (in lowest order)

$$\begin{aligned} a_\mu^{had}(e^+e^-) &= (696.3 \pm 6.2_{exp} \pm 3.6_{rad}) \times 10^{-10} \\ a_\mu^{had}(\tau\ decay\ +\ CVC) &= (711.0 \pm 5.0_{exp} \pm 2.8_{SU(2)}) \times 10^{-10} \end{aligned}$$

Difference between experiment and theory based on e^+e^- data

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (25.2 \pm 9.2) \times 10^{-10}$$

is 2.7 “standard deviations” $\Rightarrow$ “new physics” ? Though there is no difference between experiment and theory based on τ decay.

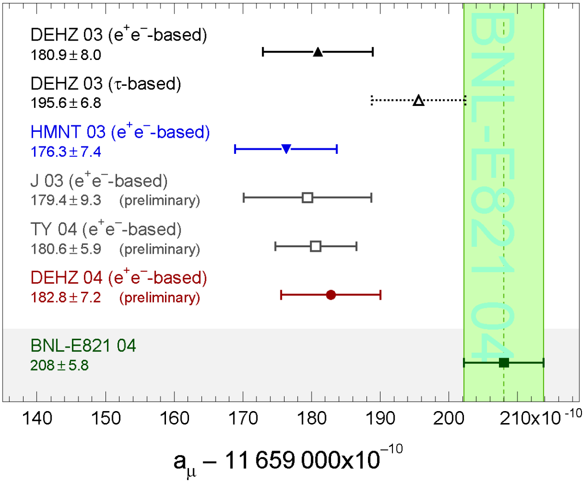
Some other applications of RχT :

S. Dubinsky, A. Korchin, N. Merenkov, G. Pancheri, O. Shekhovtsova, Eur.Phys.J. C40, 41 (2005);
G. Pancheri, O. Shekhovtsova, G. Venanzoni, Eur.Phys.J. A31, 458 (2007), JETP 106, 470 (2008), Acta Phys.Polon. B38, 2999 (2007)
$e^+e^- \to \pi^+\pi^-\gamma$ and interference of initial- and final-state radiation

A. Korchin et al, decays of τ-lepton: $\tau^- \to \nu_\tau +$ mesons,
mesons $= \pi^-, K^-, \pi^-\pi^0, \pi^-\bar{K}^0, \pi^0 K^-, \pi^-\pi^+\pi^-, ...$ (in progress)

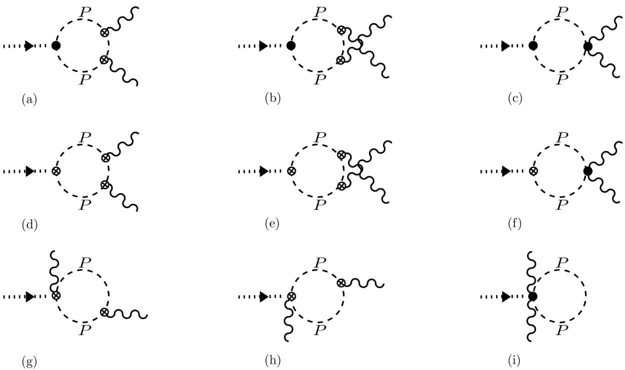
Thank you for attention !

Spare slides



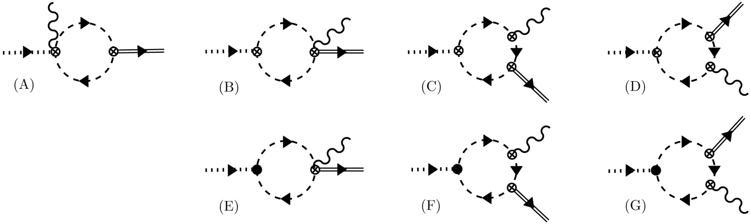
Experiment vs. theory for muon anomalous MM [Davier, Hocker, Zhang, Rev.Mod.Phys. 78, 1043 (2006)]

Two-photon decay amplitude for scalar meson



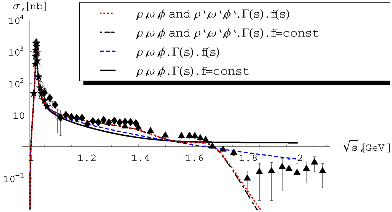
(momentum-dependent ⊗ and momentum-independent ● vertices)

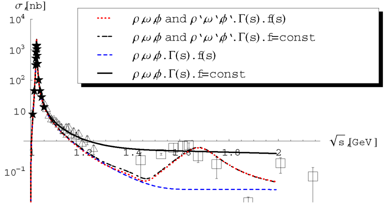
Scalar-meson radiative decay amplitude



(momentum-dependent ⊗ and momentum-independent ● vertices)

All amplitudes are expressed via a generic 3-point γVS loop integral $\Psi(m_P^2, M_S^2, M_V^2)$ which is finite.





Total cross section of e^+e^- annihilation into charged kaons K^+K^- (left) and neutral kaons $K^0\bar{K}^0$ (right).
Data: Frascati (KLOE) and Novosibirsk.

Physical states of scalars and pseudoscalars

- Physical states of scalar mesons with isospin zero, $f_0(980)$ and $\sigma \equiv f_0(600)$

$$
\begin{aligned}
f_0 &= S_0 \cos\theta - S_8 \sin\theta \\
\sigma &= S_0 \sin\theta + S_8 \cos\theta
\end{aligned}
$$

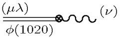
  with *unknown* octet-singlet mixing angle θ.

- For pseudoscalar meson η two-parameter mixing scheme is applied

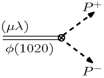
$$
\begin{aligned}
\eta &= \cos\theta_8\, \eta_8 - \sin\theta_0\, \eta_0 \\
\eta' &= \sin\theta_8\, \eta_8 + \cos\theta_0\, \eta_0
\end{aligned}
$$

  with $\theta_0 = -9.2^\circ$ and $\theta_8 = -21.2^\circ$ from experiment [Feldmann et al., Phys.Rev. D58 (1998) 114006].
  Comment: η' needs a special consideration because of $U(1)_A$ anomaly.

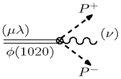
Examples of interaction vertices in RχT for pseudoscalars, scalars, vectors and photon.



$-\frac{eF_V}{3\sqrt{2}}[g_{\nu\lambda}q_\mu - g_{\nu\mu}q_\lambda]$

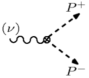


$\frac{-G_V}{\sqrt{2}f_\pi^2}[l^-_\mu l^+_\lambda - l^+_\mu l^-_\lambda]$

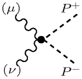


$\frac{-eG_V}{\sqrt{2}f_\pi^2}[g_{\nu\lambda}(l^- + l^+)_\mu - g_{\nu\mu}(l^- + l^+)_\lambda]$

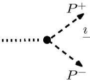
$-\frac{eF_V}{2\sqrt{2}f_\pi^2}[g_{\nu\lambda}q_\mu - g_{\nu\mu}q_\lambda]$



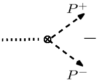
$\imath e(l^+ - l^-)_\nu$



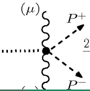
$2\imath e^2 g_{\mu\nu}$



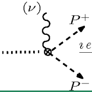
$\frac{\imath\, g_{SPP}}{f_\pi^2}$



$-\frac{\imath\, \hat{g}_{SPP}}{f_\pi^2}\, l^+ \cdot l^-$



$\frac{2e^2 \imath\, g_{SPP}}{f_\pi^2}\, g_{\mu\nu}$



$\frac{\imath\, e\, \hat{g}_{SPP}}{f_\pi^2}(l^+ - l^-)_\nu$