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ON HIGH ENERGY PROBLEMS

RELATIVISTIC NUCLEAR PHYSICS & QUANTUM CHROMODYNAMICS

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TWO-PHOTON EXCHANGE AND POLARIZATION PHYSICS IN ELECTRON-PROTON SCATTERING

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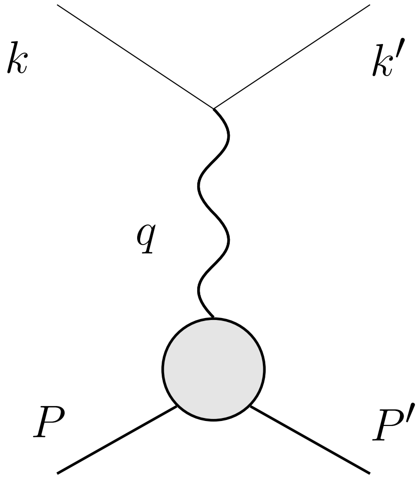


$$\alpha \approx \frac{1}{137}$$

$$\Gamma_\mu = 2M(G_E - G_M)\frac{P_\mu + P'_\mu}{(P+P')^2} + G_M\gamma_\mu$$

$G_E \equiv G_E(q^2)$ and $G_M \equiv G_M(q^2)$ - form factors

$$\boxed{eN \to eN}$$

The nucleon form factors (NFFs) are fundamental observables

- **NFFs are used in calculations of the e.m. properties of more complicated objects (the deuteron, ^{3}He, ^{4}He, etc.)**
- **NFFs give information about structure of the nucleon.**
  - **Size of the nucleon** $\quad Q^2 \equiv -q^2 \ll m_N^2$
  - **Quark counting rules, pQCD, etc.** $\quad Q^2 \equiv -q^2 \gg m_N^2$

Rosenbluth separation method

$$\frac{d\sigma}{d\Omega} \sim G_E^2 + \frac{\tau}{\epsilon} G_M^2$$

$$\tau = \frac{Q^2}{4M^2}$$

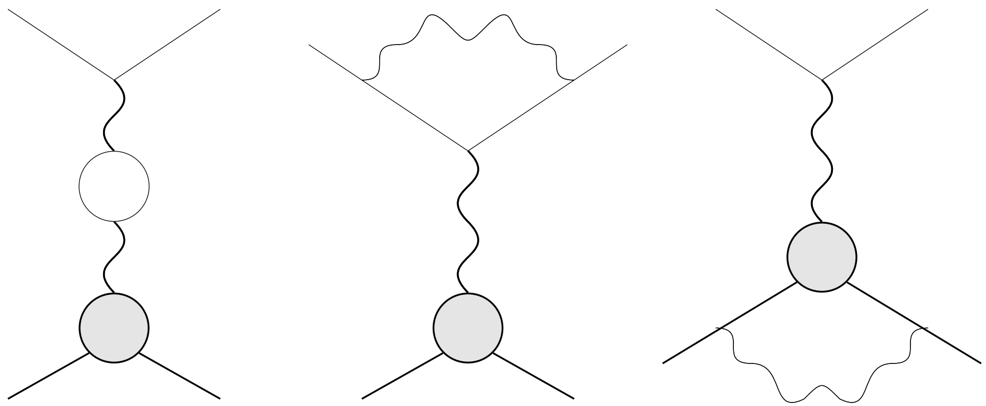
$$\epsilon = [1 + 2(1+\tau)\tan^2\frac{\theta_e}{2}]^{-1}$$

Recoil polarization

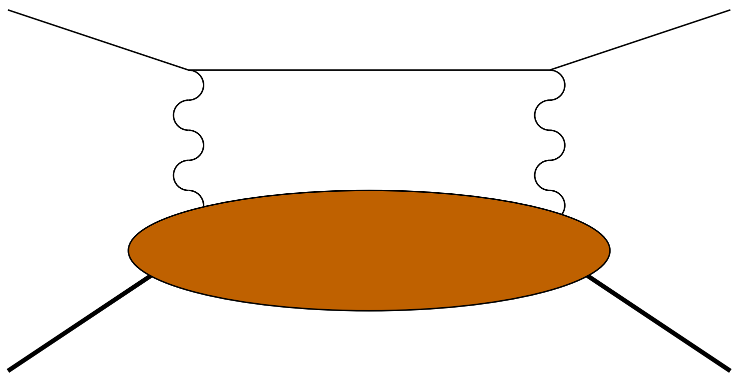
A.I.Akhiezer and M.P.Rekalo, Dokl. Akad. Nauk SSSR (1968)

$$\frac{\mathbf{G_E}}{\mathbf{G_M}} = -\frac{\mathbf{P_t}}{\mathbf{P_l}} \times \frac{(\mathbf{E_e} + \mathbf{E_{e'}})}{\mathbf{2M}} \mathbf{tan}(\theta_\mathbf{e}/\mathbf{2}) \quad \begin{cases} \vec{\mathbf{e}} + \mathbf{p} \rightarrow \mathbf{e} + \mathbf{p}^{\uparrow} \\ \vec{\mathbf{e}} + \mathbf{p} \rightarrow \mathbf{e} + \vec{\mathbf{p}} \end{cases}$$

The precision level of present-day electron-proton scattering experiments makes it necessary to take into account effects beyond Born approximation



Are factorized out $d\sigma = d\sigma^{(1)}[1 + \delta(q^2)]$



Destroy the Rosenbluth formula

Second order diagrams

- **Scattering amplitude**

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\bar{u}'\gamma_\mu u \cdot \bar{U}' \left(\gamma^\mu \tilde{F}_1 - \frac{1}{4M}[\gamma^\mu, \hat{q}]\tilde{F}_2 + \frac{P^\mu}{M^2}\hat{K}\tilde{F}_3\right) U$$

depends on three amplitudes (form factors) $\tilde{F}_1$, $\tilde{F}_2$. and $\tilde{F}_3$

- **The amplitudes are functions of two variables,**

$$q^2 \qquad \text{and} \qquad \varepsilon = [\nu^2 + t(4M^2 - t)]/[\nu^2 - t(4M^2 - t)].$$

- **The form factors are complex.**

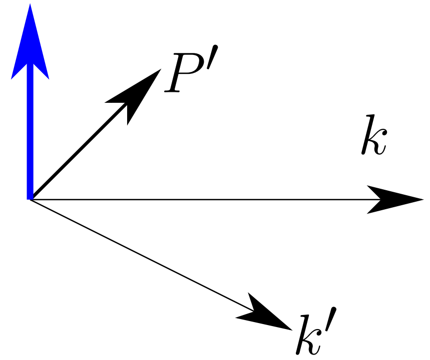
The real part of the amplitude contributes to
the reaction cross section

The imagine part of the amplitude determines
SINGLE SPIN ASYMMETRIES.

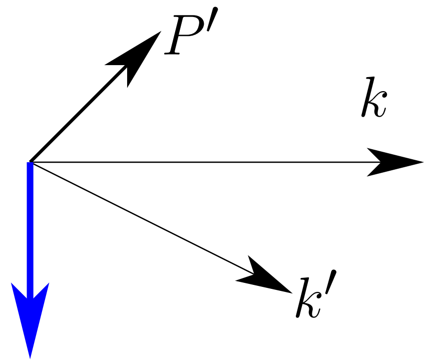
Single spin asymmetry

$eN \to eN$

$\sigma_\uparrow \quad =$



$\sigma_\downarrow \quad =$



Normal spin asymmetry

$$\mathcal{A}_n = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

There are two types of the normal spin asymmetries, related to spins of the four particles in the reaction

- **Target spin asymmetry, A_n.**
- **Beam spin asymmetry, B_n.**

It has been known for a long time that the 2γ exch. can generate single-spin normal asymmetry

- **N.F. Mott, Proc. R. Soc. London, Ser.A124, 425 (1929)** *Noted that asymmetry is due to spin-orbit coupling in the Coulomb scattering of electrons*
- **N.F. Mott, Proc. R. Soc. London, Ser.A135, 429 (1935)** *Spin asymmetry of beam electron*
- **A.O. Barut and C. Fronsdal, 1960**
- **F.Guerin and C.D. Picketty, 1964**
- **... 70-th**

1965—70: Attempts to measure such effects were done, but only upper limit for the target and recoil proton spin asymmetry were reported.

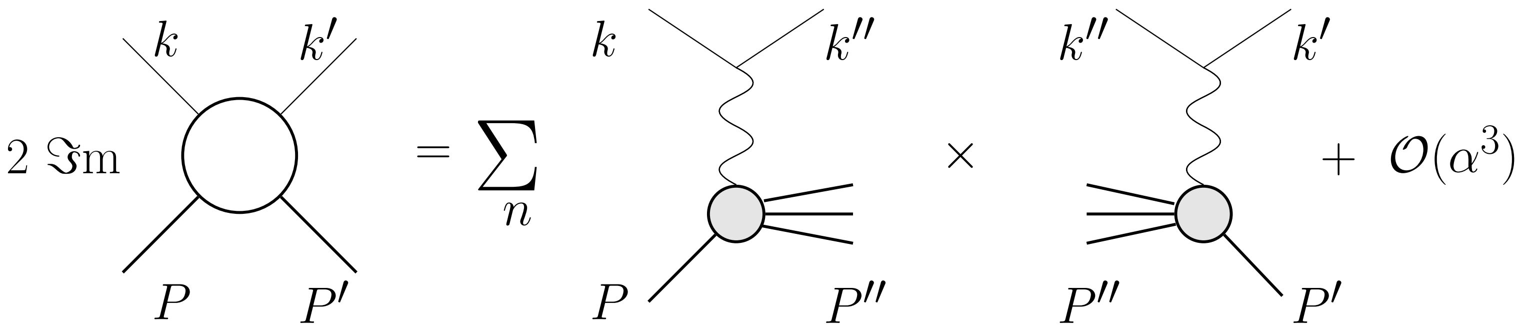
2001—04: First measurements of the B_n were done by SAMPLE Coll. (MIT/Bates) and MAMI/A4 Coll. (Mainz).

$$\sigma_\uparrow \sim |\langle k'P'|T|kP\uparrow\rangle|^2, \quad \sigma_\downarrow \sim |\langle k'P'|T|kP\downarrow\rangle|^2$$

$$|\langle k'P'|T|kP\downarrow\rangle|^2 = |\langle kP\uparrow|T|k'P'\rangle|^2 = |\langle k'P'|T^+|kP\uparrow\rangle|^2$$

$$\boxed{\sigma_\uparrow - \sigma_\downarrow \sim \tfrac{1}{2}\langle k'P'|T-T^+|kP\uparrow\rangle\langle kP\uparrow|T+T^+|k'P'\rangle + \text{c.c.} \sim \Im\text{m}T_{2\gamma}}$$

$$i(T_{fi} - \overset{*}{T}_{if}) = \sum_n T_{fn}\overset{*}{T}_{in}$$



- **Single asymmetries A_n and B_n disappear in the 1γ-exch.**
- **The asymmetries are proportional to the Im part of the 2γ-exch., which is simpler to theoretical analysis that the Re part of the 2γ-exch.**
  - **From the unitarity it is related to the electro-production amplitudes**
  - **All intermediate particles are on the mass shell**
  - **Electroproduction amplitudes can be taken from experiment**

Target Spin Normal Asymmetry

$\mathbf{A_n}$

$$A = \frac{i\alpha q^2}{2\pi^2 D} \int\limits_{M^2}^{s} \frac{s - W^2}{8s} dW^2 \int d\Omega'' \frac{1}{q_1^2 q_2^2} L^{\alpha\mu\nu} \sum_{\lambda_p, \lambda'_p} W_{\mu\nu}(P'\lambda'_p; P\lambda_p) \bar{u}_{\lambda_p}(P)(-\gamma^5 \hat{S} \Gamma_\alpha) u_{\lambda'_p}(P')$$

where $\gamma^5 \hat{S} \equiv \gamma^5 \gamma^\mu S_\mu$ **is the operator of spin projection**

$$S_\mu = A\epsilon_{\mu\nu\sigma\tau} k^\nu P^\sigma P'^\tau \qquad S^2 = -1$$

$$D = \frac{16((s - M^2)^2 + sq^2)}{4M^2 - q^2} (4M^2 G_E^2 - q^2 G_M^2) + 8q^4 G_M^2$$

$$L^{\alpha\mu\nu} = \mathrm{Tr}(\hat{k}'\gamma^\mu \hat{k}''\gamma^\nu \hat{k}\gamma^\alpha)$$

The electron mass $m = 0$

$$W_{\mu\nu}(P'\lambda'_p; P\lambda_p) = (2\pi)^4 \sum_h \delta(P + k - P'' - k'') \langle P'\lambda'_p | J_\mu | h\rangle \langle h | J_\nu | P\lambda_p\rangle$$

Models for $W_{\mu\nu}$

- **h = proton, elastic contribution** [*A.J.G Hey, 1971*]
- **C.-B. inequality estimate**

$$\sum_h |\langle P'\lambda_p'|J_\mu|h\rangle\langle h|J_\nu|P\lambda_p\rangle| \leq \left(\sum_h |\langle P'\lambda_p'|J_\mu|h\rangle|^2 \sum_h |\langle h|J_\nu|P\lambda_p\rangle|^2\right)^{1/2}$$

[*A. De Rujula, J.M. Kaplan and E. de Rafael, 1972*]

- **partonic calculations at large** Q^2 [*A. Afanasev et al., 2003*]
- **h = N and** π**N** [*B. Pasquini and M. Vanderhaegen, 2004*]

D.Borisyuk and A.K. Phys. Rev. C **72** *035207 (2005):*

contribution of resonances

1st res. region - $P_{33}(1232)$

2nd res. region - $D_{13}(1520)$**,** $S_{11}(1535)$

3rd res. region - many resonances. We include $F_{15}(1680)$ **only**

And also $P_{11}(1440)$

Electroproduction amplitudes

$$f_\lambda^{(h)}(q^2) = \varepsilon_\mu^{(\lambda)} \langle h, \lambda + {}^1/{}_2 | J^\mu | P\, {}^1/{}_2 \rangle, \qquad \lambda = \pm 1, 0$$

$$W_{\mu\nu}(P'\lambda'_p; P\lambda_p) = \sum_{\lambda,\lambda'} (-1)^{\lambda+\lambda'} \varepsilon_{1\nu}^{(2\lambda_p\lambda)} \overset{*}{\varepsilon}{}_{2\mu}^{(2\lambda'_p\lambda')} \times$$

$$\times \sum_h{}' (2\pi)^4 \delta(P + q_1 - P'') f_\lambda^{(h)}(q_1^2) \overset{*}{f}{}_{\lambda'}^{(h)}(q_2^2) \ \cdot \ \eta_h^{\lambda_p - \lambda'_p} \mathcal{D}^{(s_h)}_{\lambda_p(2\lambda+1), \lambda'_p(2\lambda'+1)}(0, \beta, 0)$$

$$A = \frac{\alpha q^2}{\pi D} \int_{M^2}^{s} \frac{s - W^2}{8s} dW^2 \int d\Omega'' \frac{1}{q_1^2 q_2^2} \times$$

$$\times \sum_h{}' (2\pi)^3 \delta(P + k - P'' - k'') \sum_{\lambda,\lambda'} f_\lambda^{(h)}(q_1^2) \overset{*}{f}{}_{\lambda'}^{(h)}(q_2^2) X_{\lambda\lambda'}^{(h)}(W, q_1^2, q_2^2)$$

For a stable particle (e.g. proton): $\sum_h{}'(2\pi)^3\delta(P+q-P'')=\delta(W^2-M^2)$

For the resonance: $\delta(W^2-M_R^2)\rightarrow\frac{\Gamma_R M_R}{\pi}\frac{1}{(W^2-M_R^2)^2+M_R^2\Gamma_R^2}$

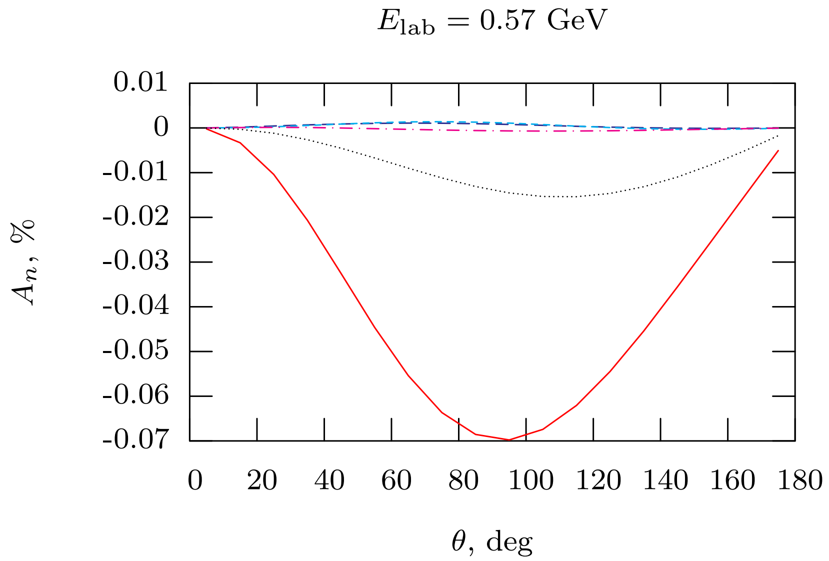
$$\sum_h{}'(2\pi)^3\delta(P+q-P'')f^{(h)}(q_1^2)\overset{*}{f}{}^{(h)}(q_2^2)\rightarrow$$

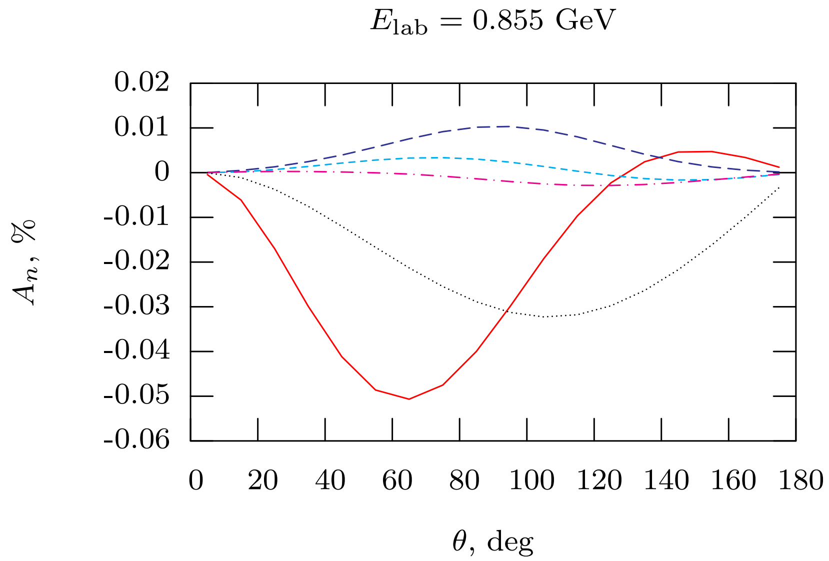
$$f^{(p)}(q_1^2)\overset{*}{f}{}^{(p)}(q_2^2)\delta(W^2-M^2)+\sum_R f^{(R)}(q_1^2)\overset{*}{f}{}^{(R)}(q_2^2)\frac{\Gamma_R M_R}{\pi}\frac{1}{(W^2-M_R^2)^2+M_R^2\Gamma_R^2}$$

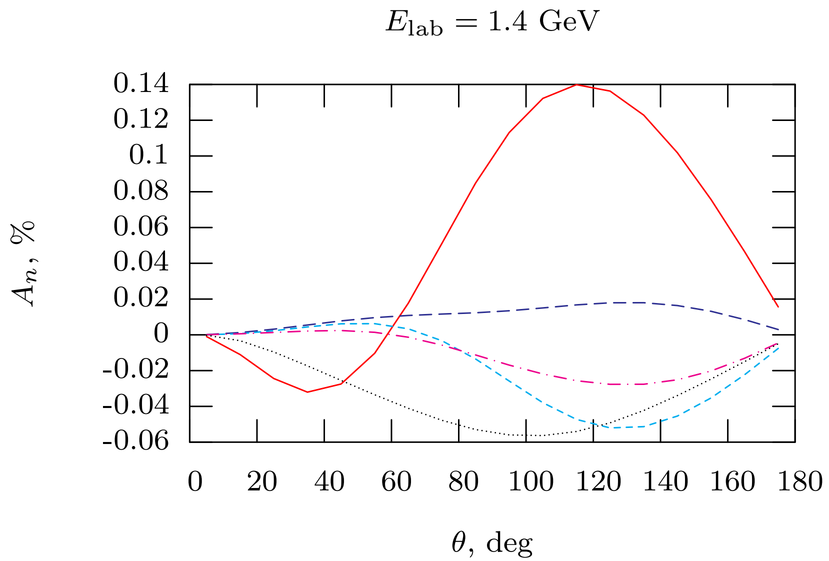
$$f_1^{(p)}(q^2)\equiv 0,\qquad f_0^{(p)}(q^2)=2MG_E(q^2),\qquad f_{-1}^{(p)}(q^2)=-G_M(q^2)\sqrt{-2q^2}$$

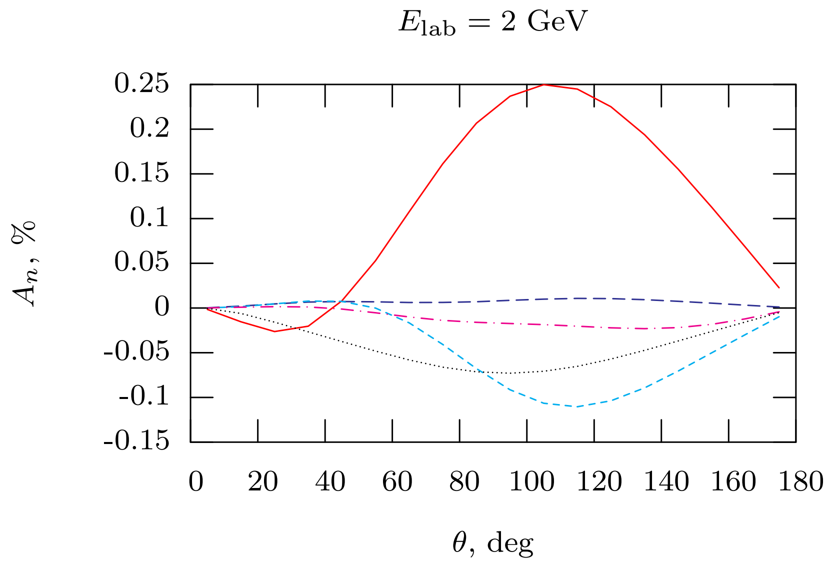
$$f_1\sim A_{3/2},\qquad f_{-1}\sim A_{1/2},\qquad f_0\sim S_{1/2}$$

Numerical results, resonances contribution

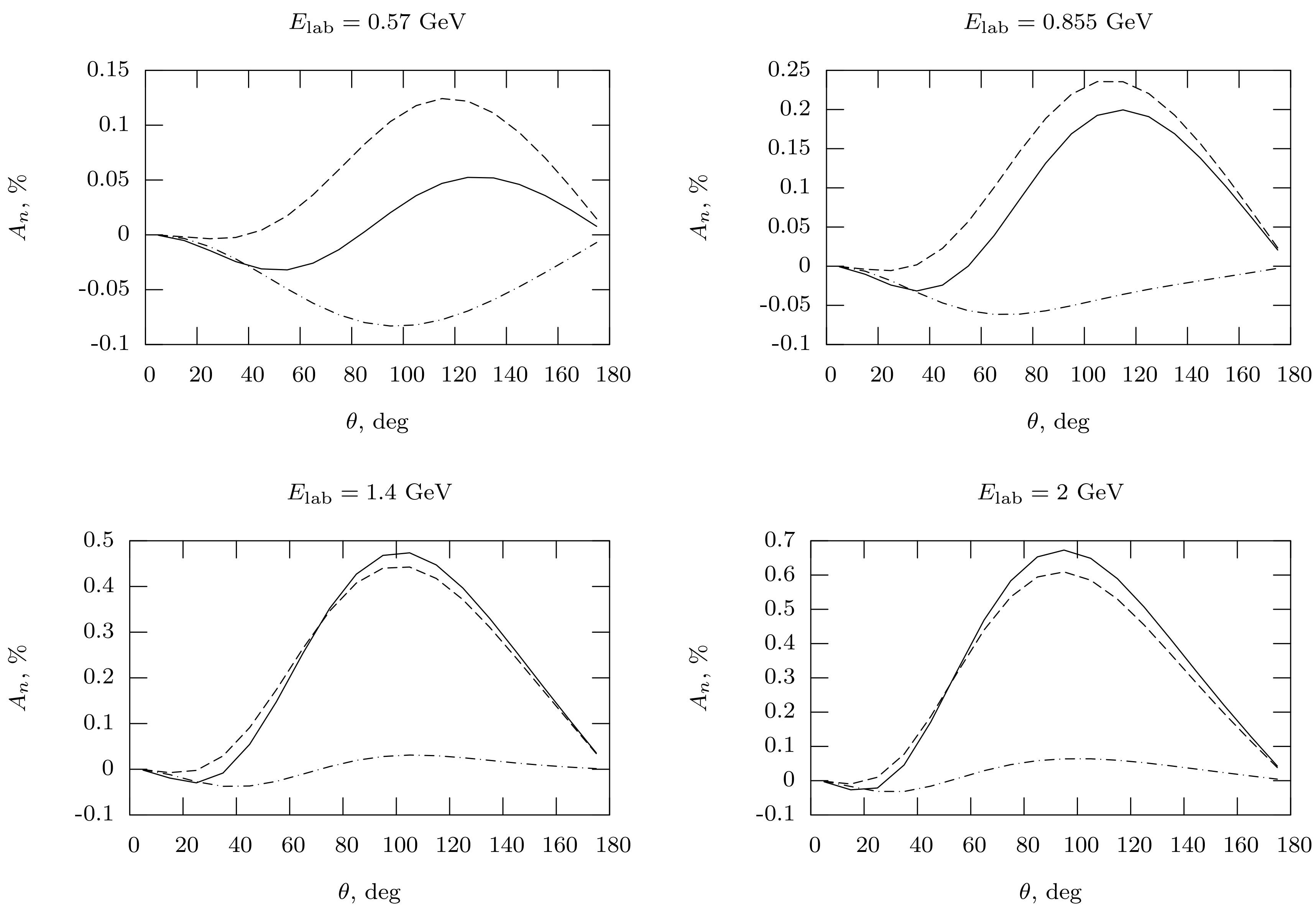








Numerical results

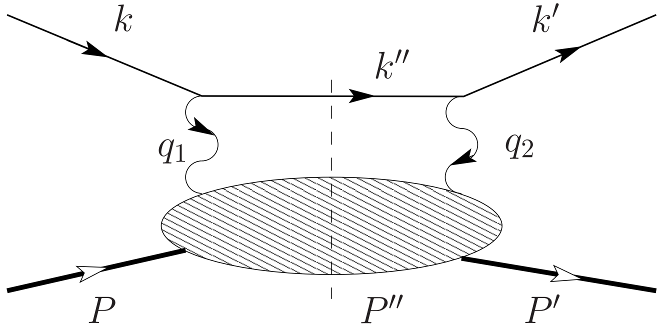


- **The $\Delta(1232)$ contribution is dominant among other resonances**
- **The contribution of the Roper resonance was obtained to be not negligible**
- **The contributions from the Δ and other resonances have mostly opposite sign and tend to cancel each other, especially at high beam energy, so the asymmetry is defined mostly by proton contribution**

Beam Spin Normal Asymmetry

B_n

D.Borisyuk and A.K., Phys. Rev. **C** *045210 (2006)*



$$B_n = \frac{i\alpha q^2}{2\pi^2 D}\int \frac{d^3k''}{2\epsilon''}\frac{1}{q_1^2 q_2^2} L^{\alpha\mu\nu}\sum_{\lambda_p,\lambda_p'} W_{\mu\nu}(P'\lambda_p'; P\lambda_p)\,\langle P\lambda_p|\Gamma_\alpha|P'\lambda_p'\rangle$$

Hadronic tensor: $W_{\mu\nu}(P'\lambda_p'; P\lambda_p) = \sum_h (2\pi)^4\delta(P+k-P''-k'')\langle P'\lambda_p'|J_\mu|h\rangle\langle h|J_\nu|P\lambda_p\rangle$

Leptonic tensor: $L^{\alpha\mu\nu} = -\mathrm{Tr}\,\left\{(\hat{k}'+m)\gamma^\mu(\hat{k}''+m)\gamma^\nu(\hat{k}+m)\gamma^5\hat{S}\gamma^\alpha\right\} \sim m$ **!!!**

- **Contrary to the TNSA A_n, the BNSA B_n vanishes in the $m\to 0$ limit.**
  - **We cannot neglect m completely**
  - **Instead we will systematically neglect $o(m)$ terms.**
- **B_n contains logarithmic and double-logarithmic terms**

$$\sim m\ln\frac{Q^2}{m^2} \qquad \textbf{and} \qquad \sim m\ln^2\frac{Q^2}{m^2},$$

which is not the case for A_n.

$$I = m \int \frac{d^3k''}{2\epsilon''} \frac{1}{q_1^2 q_2^2} Y(W, q_1^2, q_2^2) + o(m).$$

- **If we put $m = 0$ in the integrand, the integral will have two types of singularities:**
  - **When $q_1^2 \to 0$, but q_2^2 is finite or vice versa, $q_2^2 \to 0$, but q_1^2 is finite**
  - **When $W \to W_{\max}$, i.e. both q_1^2 and $q_2^2 \to 0$**
- **For $m \neq 0$ the integral is nonsingular, $|q_1^2|_{\max}$ $= \widetilde{Y} = Y - Y_0$. After $|q_2^2|_{\max} \sim m$. Those "singularities" result in the above-mentioned $\ln \frac{Q^2}{m^2}$ and $\ln^2 \frac{Q^2}{m^2}$ terms.**

If $Q^2 = 0.25$ GeV^2, then $\ln^2 \frac{Q^2}{m^2} \approx 200$

that we integrate each addendum separately, neglecting the terms which are zero in the $m \to 0$ limit.

$$\boxed{\int \frac{d^3k''}{2\epsilon''}\frac{1}{q_1^2 q_2^2} Y(W, q_1^2, q_2^2) \approx \frac{\pi Y_0(\sqrt{s})}{4Q^2} \ln^2 \frac{Q^2}{m^2}}$$

Approx. of leading Logs

$$Y_0(\sqrt{s}) = Y(\sqrt{s}, \tilde{q}_1^2, \tilde{q}_1^2)$$

where $\tilde{q}_1^2 = \tilde{q}_2^2 = -2m(\epsilon - m) \approx 0$**.**

The photons are very close to real photons

$$W_{\mu\nu}(P'\lambda'_p; P\lambda_p) = \sum_{\lambda,\lambda'=\pm 1} \varepsilon_{1\nu}^{(2\lambda_p\lambda)} \overset{*}{\varepsilon}{}_{2\mu}^{(2\lambda'_p\lambda')} {\sum_h}' (2\pi)^4 \delta(P + k - P'') f_\lambda^{(h)}(0)\ \overset{*}{f}{}_{\lambda'}^{(h)}(0) \times$$

$$\times\ \eta_h^{\lambda_p - \lambda'_p} \mathcal{D}^{(s_h)}_{\lambda_p(2\lambda+1),\lambda'_p(2\lambda'+1)}(0, \theta, 0)$$

$${\sum_h}' (2\pi)^3 \delta(P + k - P'') f_\lambda^{(h)}(0)\ \overset{*}{f}{}_{\lambda'}^{(h)}(0) \rightarrow$$

$$\rightarrow \frac{4W|\vec{k}_\pi|}{\pi\alpha} |E_{0+}(W)|^2 \delta_{\lambda,-1}\delta_{\lambda',-1} + \sum_R f_\lambda^{(R)}(0)\ \overset{*}{f}{}_{\lambda'}^{(R)}(0) \frac{\Gamma_R M_R}{\pi} \frac{1}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2}$$

VALIDITY OF THE APPROXIMATION

Taking into account

$$\mathcal{D}_{\lambda'\lambda}(0,\theta,0) \sim \left(\sin\frac{\theta}{2}\right)^{|\lambda-\lambda'|} \sim Q^{|\lambda-\lambda'|}, \quad \textbf{at} \quad \theta \to 0,$$

one gets

$$B_n^{(\ln^2)} \sim Q^3 \ln^2 \frac{Q^2}{m^2}$$

This is valid if

$$\sin^2\frac{\theta}{2}\,\ln\frac{Q^2}{m^2} \gg 1$$

ALTERNATIVELY

For the condition

$$\sin^2\frac{\theta}{2}\,\ln\frac{Q^2}{m^2}\ll 1$$

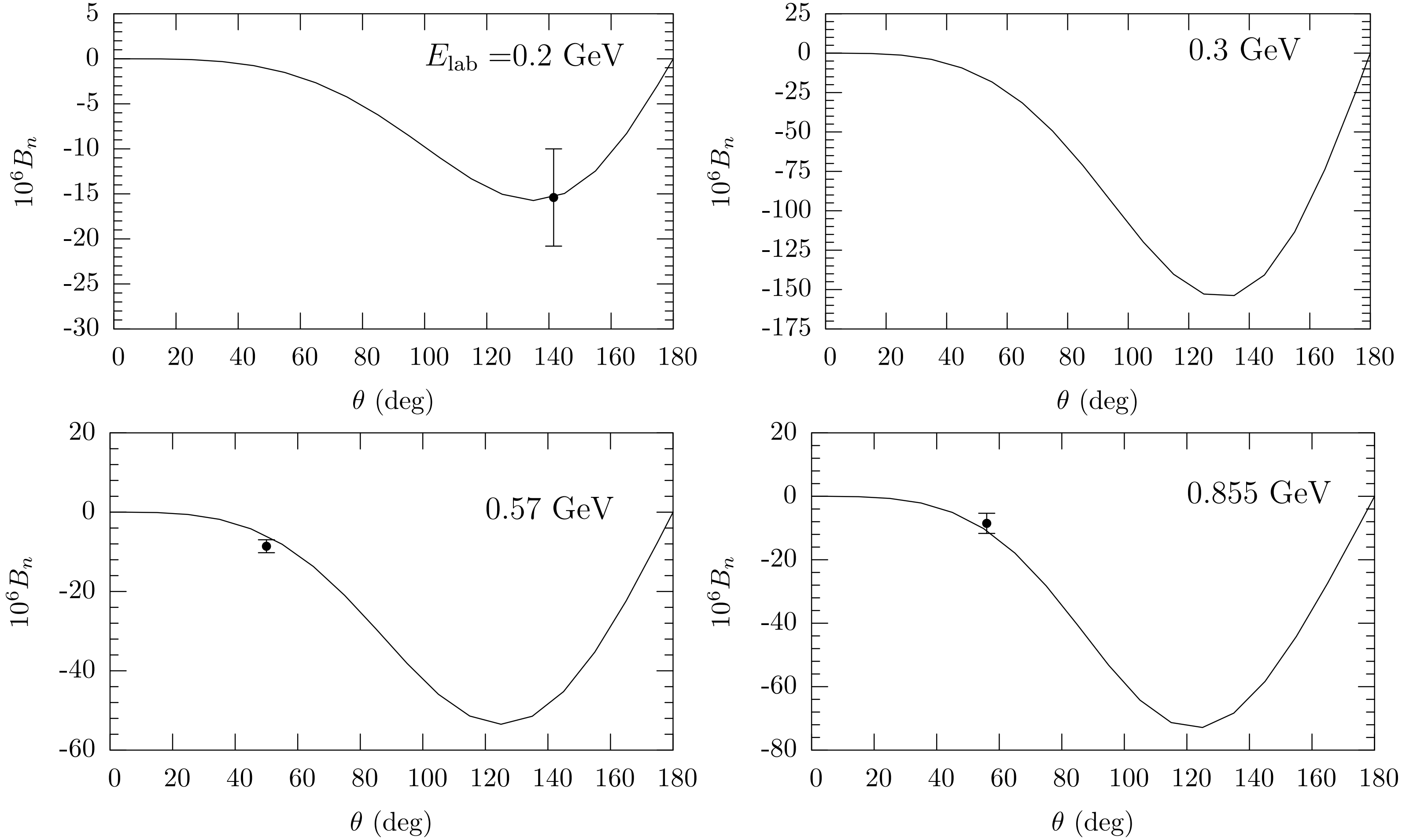
one has to look for the terms with the lowest power of Q in the limit of forward scattering. As a result one has

$$B_n \approx B_n^{(\ln^1)} = -\frac{2m(s-M^2)^2}{\pi^2 D}\left(G_E+\tfrac{Q^2}{4M^2}G_M\right)Q\ln\frac{Q^2}{m^2}\,\sigma_{\rm tot},$$

where $\sigma_{\rm tot}$ is the total photoabsorbtion cross-section.

(*Afanasev and Merenkov; Borislyuk and Kobushkin*)

$$e^- + p \rightarrow e^- + p$$



The following resonances were taken into account:

$\Delta(1232)$, $D_{13}(1520)$, $S_{11}(1535)$

THE REAL PART

The problem

One does not know the $\gamma^* NN$-vertex if one of the nucleon is out of mass shell

$$\Gamma_\mu(q^2; p^2 = M^2, p'^2 \neq M^2) \neq \gamma_\mu F_1(q^2) - \frac{1}{4M} F_2(q^2)[\gamma_\mu, \hat{q}]$$

The uncertainty of these FFs is believed to be the main source of theoretical uncertainty in TPE calculations.

We develop the approach which is based on the dispersion relations.

—At first, the absorptive part of the amplitude is calculated using unitarity. Thus only "on-shell" FFs are needed to evaluate it.
—Then the whole amplitude is reconstructed by dispersion relations.

D.Borisyuk and A.K., Phys. Rev. C **78** *025208 (2008)*
C **75** *038202 (2007)*

THE MAIN STEPS OF THE CALCULATIONS

The zero step

The general expression for the scattering amplitude

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\bar{u}'\gamma_\mu u \cdot \bar{U}' \left(\gamma^\mu \tilde{F}_1 - \frac{1}{4M}[\gamma^\mu, \hat{q}]\tilde{F}_2 + \frac{P^\mu}{M^2}\hat{K}\tilde{F}_3\right) U$$

We introduce the new set of amplitudes

$$\begin{aligned}
\mathcal{G}_E &= \tilde{F}_1 - \tau\tilde{F}_2 + \nu\tilde{F}_3/4M^2 \\
\mathcal{G}_M &= \tilde{F}_1 + \tilde{F}_2 + \varepsilon\nu\tilde{F}_3/4M^2 \\
\mathcal{G}_3 &= \nu\tilde{F}_3/4M^2
\end{aligned}$$

$$\mathcal{G}_E = G_E + \mathcal{O}(\alpha) \qquad \mathcal{G}_M = G_M + \mathcal{O}(\alpha) \qquad \mathcal{G}_3 \sim \alpha$$

$$P = \tfrac{1}{2}(p + p') \qquad K = \tfrac{1}{2}(k + k') \qquad t = q^2 \qquad \nu = s - u = 4PK$$

The first step

We need amplitudes, free from kinematical u and s singularities and zeros. The helicity amplitudes of the process $e^- e^+ \to p\tilde{p}$ are

$$T_{++} = 4\pi\alpha \cdot 2i\cos^2\theta/2 \left(\sqrt{\tau(1+\tau)}\tilde{F}_3 + \tilde{F}_m + \nu\tilde{F}_3/4M^2\right),$$

$$T_{--} = 4\pi\alpha \cdot 2i\sin^2\theta/2 \left(\sqrt{\tau(1+\tau)}\tilde{F}_3 - \tilde{F}_m - \nu\tilde{F}_3/4M^2\right),$$

$$T_{+-} = T_{-+} = 4\pi\alpha \cdot \frac{2M}{\sqrt{t}}\sin\theta\left(\tilde{F}_e + \nu\tilde{F}_3/4M^2\right),$$

where $\cos\theta = -\nu/\sqrt{-t(4M^2-t)}$. **Each of the $T_{\lambda\tilde{\lambda}}$ contains a kinematical factor of** $\sin^{|\lambda+\tilde{\lambda}-1|}\frac{\theta}{2}\cos^{|\lambda+\tilde{\lambda}+1|}\frac{\theta}{2}$**.**

The amplitudes free from kinematical singularities are obtained after removing these factors.

$$G_1 = \Delta\tilde{F}_e + \nu\tilde{F}_3/4M^2, \qquad G_2 = \Delta\tilde{F}_m + \nu\tilde{F}_3/4M^2, \qquad G_3 \equiv \tilde{F}_3.$$

TPE contributions to the amplitudes G_n satisfy fixed-t dispersion relations

$$\pi G_n(\nu) = \int\limits_{\nu_{th}}^{\infty} \frac{\Im\mathrm{m} G_n(\nu' + i0)}{\nu' - \nu} d\nu' - \int\limits_{-\infty}^{-\nu_{th}} \frac{\Im\mathrm{m} G_n(\nu' - i0)}{\nu' - \nu} d\nu'$$

and consequently, vanish at $\nu \to \infty$.

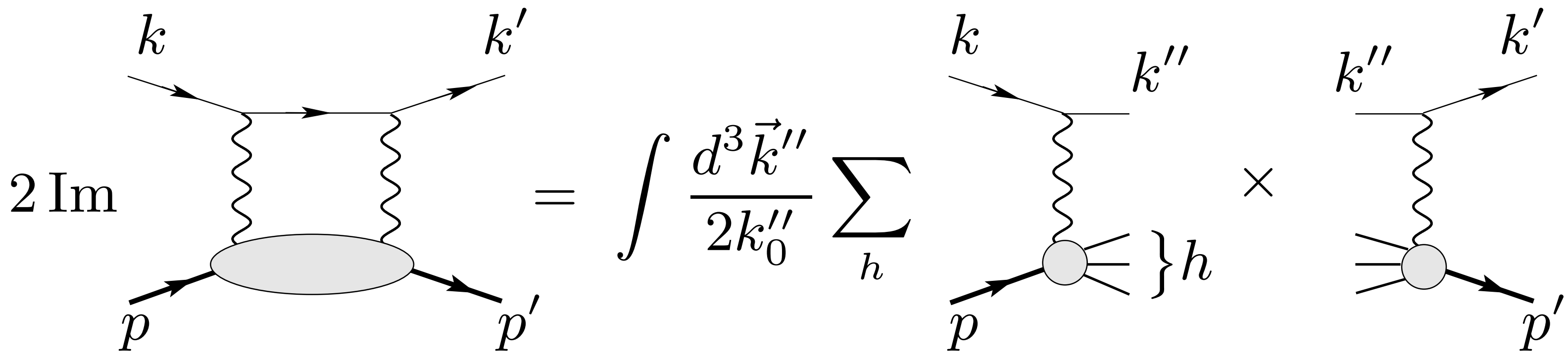
Under crossing $\nu \to -\nu$:

$$G_{1,2}(-\nu) = -G_{1,2}(\nu), \qquad G_3(-\nu) = G_3(\nu).$$

(*Rekalo and Tomasi-Gustafsson, 2004*)

The second step

Equation for the imaginary part of amplitude



The intermediate electron and hadron states are on the mass shell !!!

h =**proton — elastic contribution**
$h = \Delta(1232)$ — **Δ contribution**
ect.

$$\Im\mathrm{m} G_n^{(\mathrm{el})} = -\frac{\alpha}{2\pi} \sum_{i,j=1}^{2} \int \bar{F}_i(t_1)\bar{F}_j(t_2) A_{n,ij}(\nu, t_1, t_2)\theta(k_0'')\delta(k''^2 - m^2)\theta(p_0'')\delta(p''^2 - M^2) d^4k''$$

$$\bar{F}_i(t) = F_i(t)/(t - \lambda^2)$$

The third step

Reconstruction of the real part

$$G_n(\nu) = G_{n,\mathrm{box}}(\nu) + G_{n,\mathrm{xbox}}(\nu)$$

$$G_{n,\mathrm{box}}(\nu) = \pm G_{n,\mathrm{xbox}}(-\nu)$$

Thus to reconstruct G_n it is sufficient to find $G_{n,\mathrm{box}}$.
The analytical structure of FFs is such that

$$\bar{F}_i(t) = \frac{1}{\pi}\int\limits_{\lambda^2}^{\infty} \frac{\Im\mathrm{m}\bar{F}_i(t')}{t'-t}dt'$$

It is important that the final result is reduced to the integral

$$\int_{t_i,t_2<0} dt_1 dt_2 \bar{F}_i(t_1)\bar{F}_j(t_2)A_{n,ij}(\nu,t_1,t_2)...$$

over the time-like region only !

Reply to Prof.V.Karmanov comment to the talk of Prof.C.Perdrisat

Of course the off-mass shell effects change the "naive" results obtained with the standard parametrization of the $NN\gamma$ vertex.

Nevertheless, the expressions for G_1 and G_2 remain the same as in the naive approach

$$G_1 = G_1^{(\text{naive})}, \qquad G_2 = G_2^{(\text{naive})}$$

But

$$G_3 = G_3^{(\text{naive})} + \Delta G_3(t)$$

$$\mathcal{M} = \mathcal{M}^{(\text{naive})} + \frac{4\pi\alpha}{q^2 M^2}\,\bar{u}'\gamma^\mu u\,\bar{U}'(P_\mu \hat{K} - PK\gamma_\mu)U \cdot \Delta G_3(t)$$

$$\boxed{\mathcal{G}_E = G_E^{(\text{naive})}, \qquad \mathcal{G}_M = G_M^{(\text{naive})} - \sqrt{\tau(1+\tau)}\sqrt{1-\varepsilon^2}\Delta G_3(t)}$$

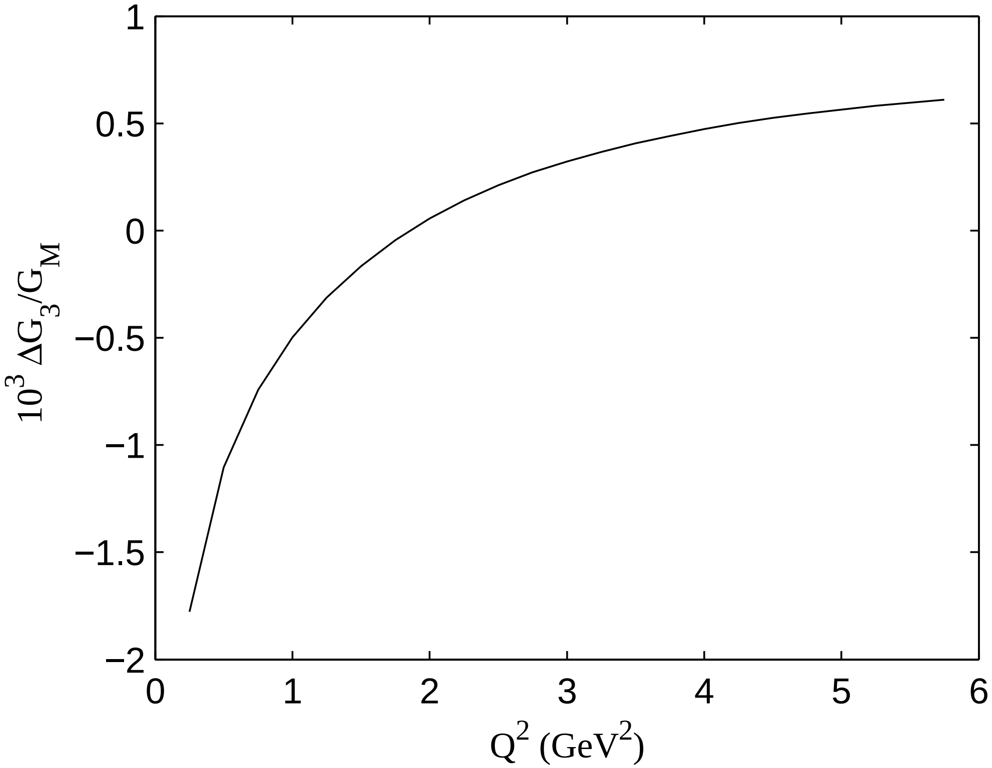


Figure 1: The amplitude change ΔG_3.

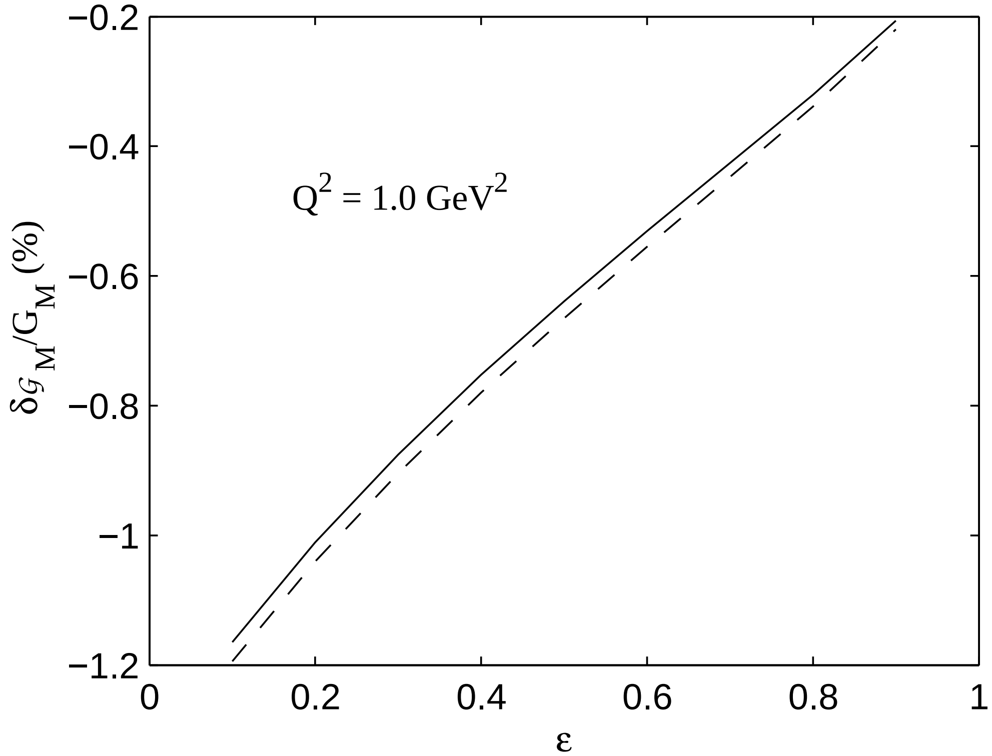
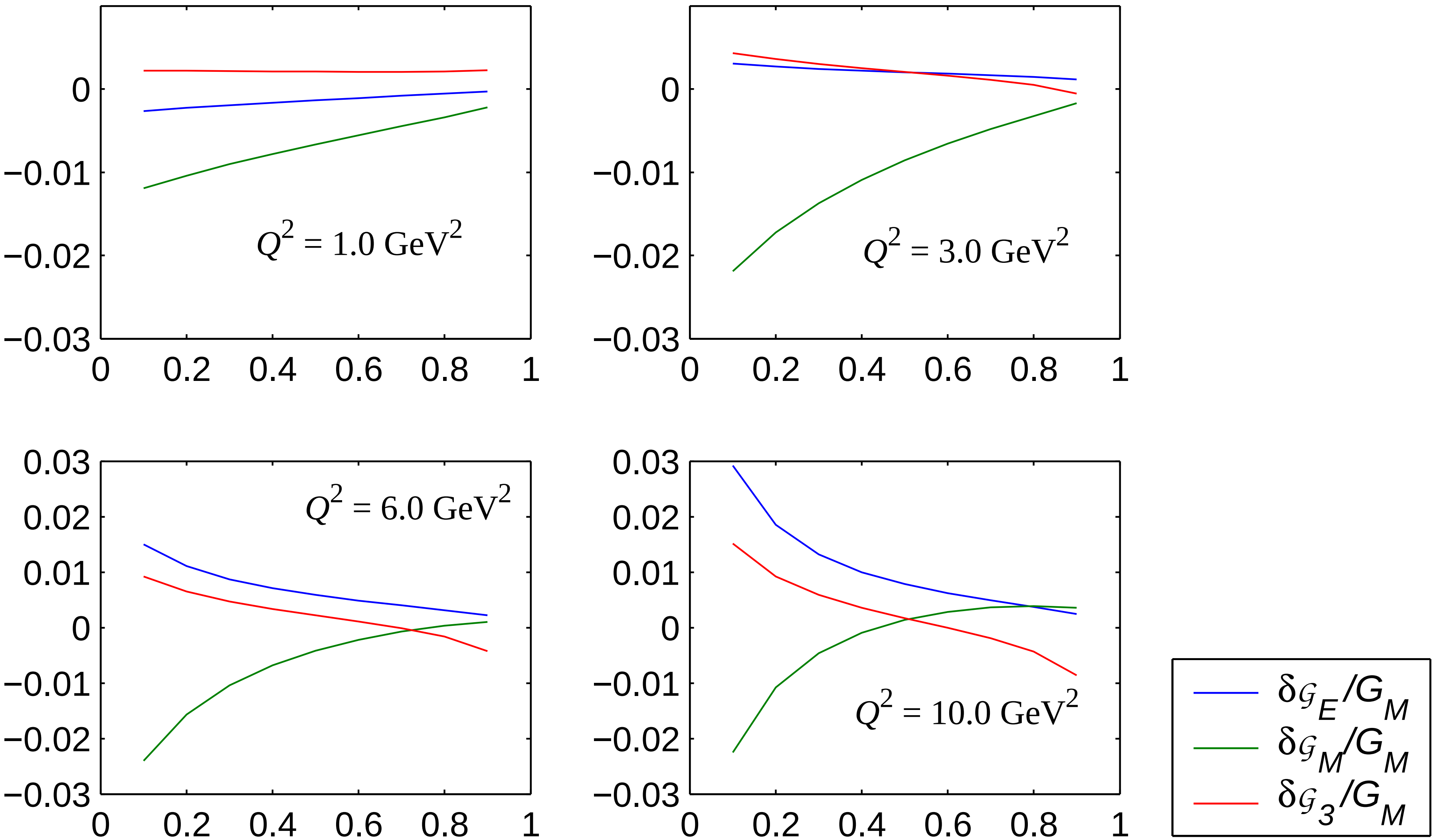


Figure 2: The TPE amplitude $\delta\mathcal{G}_M$ obtained in old (dashed line) and new (solid line) approach.

RESULTS

$$\frac{\delta\mathcal{G}}{G_M} = \frac{\mathcal{G} - \mathcal{G}^{(\mathrm{Born})} - \mathcal{G}^{(\mathrm{Tsai})}}{G_M}$$



PHENOMENOLOGICAL ANALYSIS

D.Borisyuk and A.K., Phys. Rev. C **76** *022201(R) (2007)*

The cross-section is "diagonalizes"

$$d\sigma = \frac{2\pi\alpha^2 dt}{E^2 t}\frac{1}{1-\varepsilon}\left(\varepsilon|\mathcal{G}_E|^2 + \tau|\mathcal{G}_M|^2 + \tau\varepsilon^2\frac{1-\varepsilon}{1+\varepsilon}|\mathcal{G}_3|^2\right) =$$

$$= \frac{2\pi\alpha^2 dt}{E^2 t}\frac{1}{1-\varepsilon}\left(\varepsilon|\mathcal{G}_E|^2 + \tau|\mathcal{G}_M|^2 + \mathcal{O}(\alpha^2)\right)$$

$$\sigma_R = \varepsilon|\mathcal{G}_E|^2 + \tau|\mathcal{G}_M|^2 + \mathcal{O}(\alpha^2)$$

BUT THE ROSENBLUTH SEPARATION IS NOT POSSIBLE !

The amplitudes can be decomposed as

$$\mathcal{G}_E(Q^2,\varepsilon) = G_E(Q^2) + \delta G_E^{(T)}(Q^2,\varepsilon) + \delta\mathcal{G}_E(Q^2,\varepsilon) + O(\alpha^2)$$

$$\mathcal{G}_M(Q^2,\varepsilon) = G_M(Q^2) + \delta G_M^{(T)}(Q^2,\varepsilon) + \delta\mathcal{G}_M(Q^2,\varepsilon) + O(\alpha^2)$$

$\delta G_{E,M}^{(T)} + \delta\mathcal{G}_{E,M}$ **are TPE corrections of order** α**.**

$\delta G_{E,M}^{(T)}$ **denotes the part of the correction, calculated by Tsai. Infrared divergence is contained in it.**

$$\sigma_R = \varepsilon G_E^2 + \tau G_M^2 + 2\varepsilon G_E \delta\mathcal{G}_E + 2\tau G_M \delta\mathcal{G}_M + 2\varepsilon G_E \delta G_E^{(T)} + 2\tau G_M \delta G_M^{(T)} + O(\alpha^2)$$

The terms containing $\delta G_{E,M}^{(T)}$ **are always subtracted from the cross-section by experimenters as a part of radiative corrections, so published cross-sections are, dropping terms of order** α^2

$$\sigma_R = \varepsilon G_E^2 + \tau G_M^2 + 2\varepsilon G_E \delta\mathcal{G}_E + 2\tau G_M \delta\mathcal{G}_M$$

The key point is that G_M is enhanced with respect to G_E by about a factor of $\mu \approx 3$ (proton magnetic moment)

$$\tau G_M^2 \gg \varepsilon G_E^2 \gg 2\varepsilon G_E \delta\mathcal{G}_E, \quad \tau G_M^2 \gg 2\tau G_M \delta\mathcal{G}_M \gg 2\varepsilon G_E \delta\mathcal{G}_E$$

Therefore the term $2\varepsilon G_E \delta\mathcal{G}_E$ is much smaller than three other terms and can be safely neglected. Instead, the term $2\tau G_M \delta\mathcal{G}_M$ can be comparable with εG_E^2 and thus strongly affect the results of Rosenbluth separation.

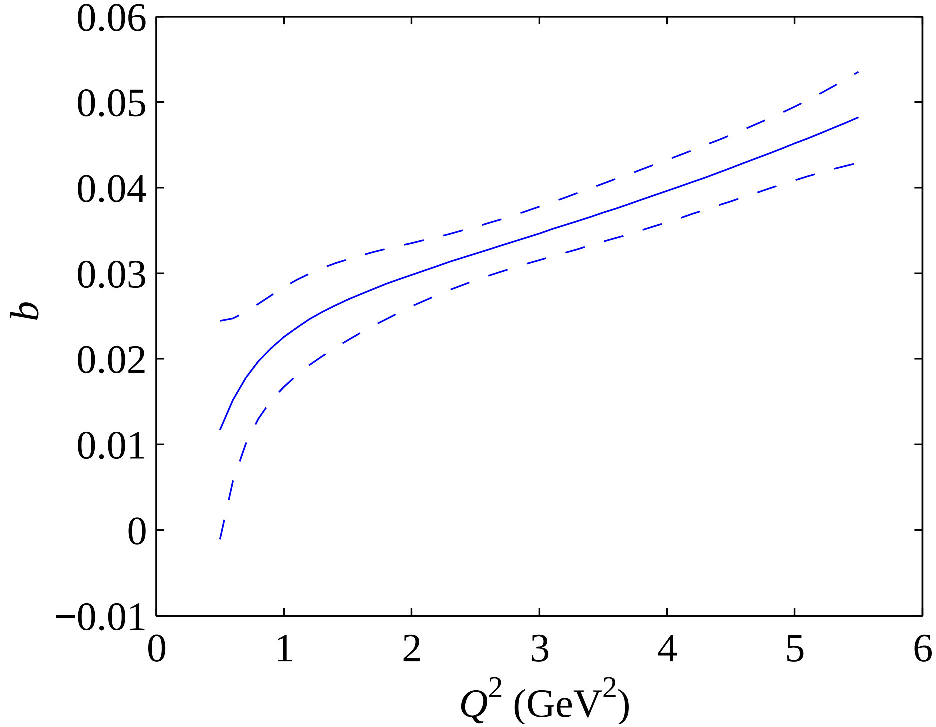
$$\delta\mathcal{G}_M(Q^2, \varepsilon) = [a(Q^2) + \varepsilon b(Q^2)] G_M(Q^2)$$
$$\sigma_R = \tau G_M^2 + \varepsilon(G_E^2 + 2\tau b\, G_M^2)$$

$$\left(\frac{G_E}{G_M}\right)^2\Bigg|_{\rm Ros.S.} \equiv R_{\rm LT}^2 = \frac{G_E^2}{G_M^2} + 2\tau b$$

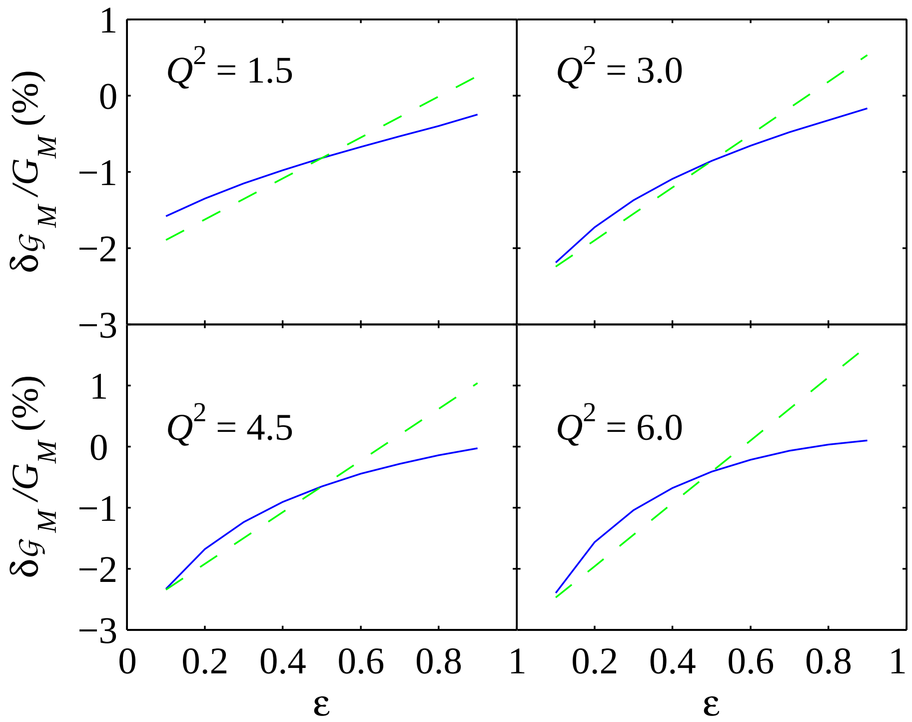
$$\frac{G_E}{G_M}\Bigg|_{\rm Pol.T.} \equiv R_{LT} = \frac{\mathcal{G}_E}{\mathcal{G}_M}\left(1 - \frac{\varepsilon(1-\varepsilon)}{1+\varepsilon} Y_{2\gamma}\right) = \frac{G_E}{G_M} \pm 1\%, \qquad Y_{2\gamma} = \frac{\nu}{4M^2}\frac{\tilde{F}_3}{G_M}$$

$$\boxed{b = \frac{1}{2\tau}(R_{LT}^2 - R_{PT}^2)}$$

Extracted TPE correction slope $b(Q^2)$.
The dashed curves indicate estimated errors.

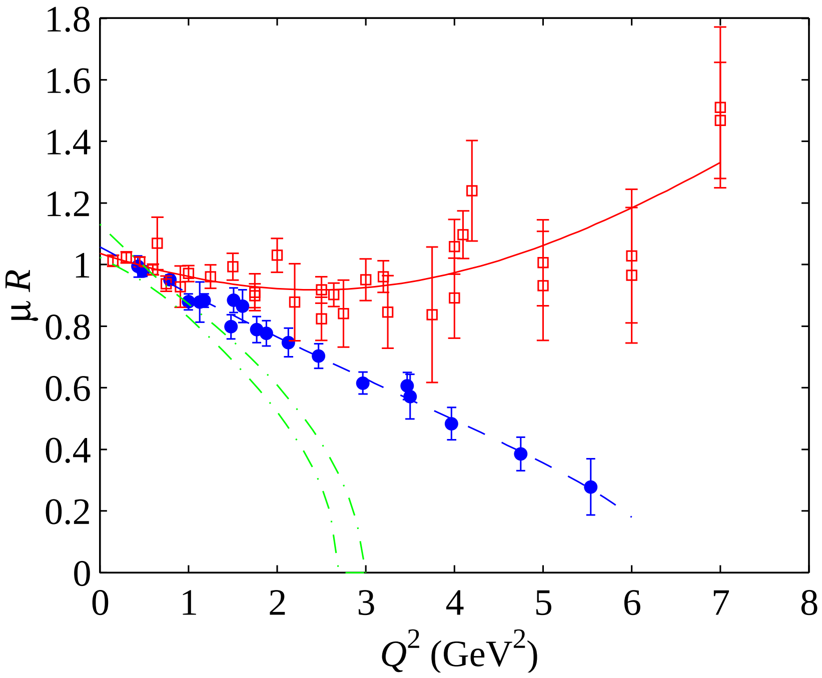


Comparison of extracted (dashed lines) and calculated (solid curves) values of TPE amplitude $\delta\mathcal{G}_M/G_M$.



If the positrons are used instead of electrons, the TPE corrections change their sign. Thus we have for the Rosenbluth FF ratio squared, measured in positron-proton scattering

$$\tilde{R}_{LT}^2 = \frac{G_E^2}{G_M^2} - 2\tau b$$



dash-dotted lines indicate estimated 1σ bounds for $\mu\tilde{R}_{LT}$.

CONCLUSIONS

- Effects beyond the Born approximation strongly affect the results of the e.m. structure of the proton
- In the resonance region the normal beam asymmetry agrees with the calculations
- There are no data for the target normal spin asymmetry
- The TPE correction to the amplitude $\mathcal{G}_M$ is exactly the quantity which is responsible for the discrepancy between Rosenbluth and polarization transfer methods in the measurements of proton FFs.
- Positron-proton scattering is expected to be strongly affected by TPE contribution.