Axial anomaly and $\eta - \eta'$ mesons mixing

O.V.Teryaev[1], Y.N.Klopot[1] and A.G.Oganesian[2]

[1]Joint Institute for Nuclear Research,
Dubna, Russia

[2]Institute of Theoretical and Experimental Physics
Moscow, Russia

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Outline

$\eta \to 2\gamma$ Decay Width Discrepancy

One Angle $\eta - \eta'$ Mixing Scheme

Two Angle $\eta - \eta'$ Mixing Scheme

Summary

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Consider the matrix element for the transition of the 8th component of octet axial current into two photons with momenta p, p' and polarizations ϵ_α , ϵ'_β:

$$T_{\mu\alpha\beta}(p,p') = \langle p, \epsilon_\alpha; p', \epsilon'_\beta | J^{(8)}_{\mu 5} | 0 \rangle ,$$

$$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}} (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s) \ .$$

The general form of $T_{\mu\alpha\beta}(p,p')$ in case of decay into two real photons can be expressed as:

$$T_{\mu\alpha\beta}(p,p') = F_1(q^2) q_\mu \epsilon_{\alpha\beta\rho\sigma} p_\rho p'_\sigma + \frac{1}{2} F_2(q^2)(p_\alpha \epsilon_{\mu\beta\rho\sigma} - p'_\beta \epsilon_{\mu\alpha\rho\sigma}) p_\rho p'_\sigma ,$$

where $q = p + p'$.

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where $q = p + p'$.

The function $F_1(q^2)$ enters the "anomaly sum rule", which can be derived using the dispersive approach:[J.Horejsi,1985; O.Veretin, O.Teryaev, 1995; J.Horejsi, O.Teryaev,1995]

$$\int_0^\infty Im\ F_1(q^2)dq^2 = \sqrt{2}\alpha(e_u^2 + e_d^2 - 2e_s^2)N_c = \sqrt{\frac{2}{3}}\alpha\ .$$

- Notice, that in QCD this equation does't have any perturbative corrections, and it is expected that it does not have any nonperturbative corrections too.
- It is important also that at $q^2 \to \infty$ the function $ImF_1(q^2)$ decreases as $1/q^4$.

Use the definition of η decay constant

$$\langle 0|J_{\mu 5}^{(8)}|\eta\rangle = if_\eta q_\mu$$

and let's try to saturate the above relation by η contribution only. The general form of the η-contribution to $T_{\mu\alpha\beta}(p,p')$ is

$$T_{\mu\alpha\beta}(p,p') = -f_\eta \frac{1}{q^2 - m_\eta^2} \widetilde{A}_\eta q_\mu \epsilon_{\alpha\beta\lambda\sigma} p_\lambda p'_\sigma \ ,$$

where $\widetilde{A}_\eta$ is a constant. In the approximation, when only η contribution is accounted in the l.h.s. of the sum rule relation one can find $\widetilde{A}_\eta$

$$\widetilde{A}_\eta = \sqrt{\frac{2}{3}} \frac{\alpha}{\pi} \frac{1}{f_\eta}.$$

Then one can easily calculate the decay width $\eta \to 2\gamma$:

$$\widetilde{\Gamma}_{\eta\to 2\gamma} = \frac{1}{3}\frac{\alpha^2}{32\pi^3}\,\frac{m_\eta^3}{f_\eta^2}\,.$$

- If we put experimental numbers of α, m_η and $f_\eta = 1.2 f_\pi \approx 150 MeV$ we get the numerical value

$$\widetilde{\Gamma}_{\eta\to 2\gamma} = 0.12 keV,$$

  which is in a serious disagreement with an experimental value

$$\Gamma_{\eta\to 2\gamma} = 0.510 \pm 0.026\, keV\,.$$

- As we mentioned before, the anomaly dispersion relation is exact. That's why this discrepancy motivates us to consider corrections arising from the other states contributions to the sum rule. The resolution of this discrepancy provides us with strict bounds on the mixing parameters. Consider the effects of the mixing of η and η' mesons.

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One Angle $\eta - \eta'$ Mixing Scheme

Let's introduce nonorthogonal states $|P_8\rangle$ and $|P_0\rangle$ and the corresponding fields φ_8, φ_0, coupled to $J^{(8)}_{\mu 5}$ and $J^{(0)}_{\mu 5}$:

$$\langle 0|J^{(k)}_{\mu 5}|P_l\rangle = i\delta_{kl} f_k q_\mu, \quad k = 8, 0.$$

Nonorthogonality of the fields φ_0, φ_8 corresponds to the non-diagonal term $\Delta H = m^2_{\eta\pi}\varphi_8\varphi_0$ in the effective interaction Hamiltonian. In the presence of such term the standard PCAC relation is modified in the following way [B.Ioffe, 1979; B.Ioffe,M.Shifman, 1980] :

$$\partial_\mu J^{(8)}_{\mu 5} = f_\eta (m^2_\eta \varphi_8 + m^2_{\eta\eta'}\varphi_0),$$

The fields φ_8,φ_0 are expressed through the physical fields φ_η, $\varphi_{\eta'}$ as

$$\varphi_8 = \varphi_\eta \cos\theta + \varphi_{\eta'} \sin\theta \ ,$$

$$\varphi_0 = -\varphi_\eta \sin\theta + \varphi_{\eta'} \cos\theta \ .$$

Mixing angle θ can be expressed in terms of masses as:

$$\tan 2\theta = \frac{2m^2_{\eta\eta'}}{m^2_{\eta'} - m^2_\eta} \ .$$

Now $ImF_1(q^2)$ is given by the sum of contributions of η and η' mesons. In order to separate the formfactor $F_1(q^2)$, multiply $T_{\mu\alpha\beta}(p,p')$ by q_μ/q^2. Than, taking the imagenary part we get:

$$Im\; q_\mu \frac{1}{q^2}\langle 2\gamma \mid J^{(8)}_{\mu 5} \mid 0\rangle =$$

$$-\frac{f_\eta}{q^2} Im\langle 2\gamma \mid m^2_\eta(\cos\theta\varphi_\eta + \sin\theta\varphi_{\eta'}) + m^2_{\eta\eta'}(-\sin\theta\varphi_\eta + \cos\theta\varphi_{\eta'}) \mid 0\rangle =$$

$$\pi f_\eta[\delta(q^2 - m^2_\eta)A_\eta \cos\theta + \frac{m^2_\eta}{m^2_{\eta'}}\sin\theta\delta(q^2 - m^2_{\eta'})A_{\eta'} -$$

$$\frac{m^2_{\eta\eta'}}{m^2_\eta}\sin\theta\delta(q^2 - m^2_\eta)A_\eta + \frac{m^2_{\eta\eta'}}{m^2_{\eta'}}\cos\theta\delta(q^2 - m^2_{\eta'})A_{\eta'}]\,,$$

where A_η is the amplitude of the decay $\eta \to 2\gamma$

If we employ the sum rule (...) we'll get:

$$\pi f_\eta [A_\eta \cos\theta + A_{\eta'} \frac{m_\eta^2}{m_{\eta'}^2} \sin\theta - A_\eta \frac{m_{\eta\eta'}^2}{m_\eta^2} \sin\theta + A_{\eta'} \frac{m_{\eta\eta'}^2}{m_{\eta'}^2} \cos\theta] = \sqrt{\frac{2}{3}}\alpha \, .$$

Now let's express the amplitudes in terms of decay widths, employ the relation for a mixing parameter $m_{\eta\eta'}^2$ and finally get the equation for the mixing angle:

$$\boxed{\cos\theta + \beta \frac{m_\eta^2}{m_{\eta'}^2} \sin\theta - \frac{1}{2}(\frac{m_{\eta'}^2}{m_\eta^2} - 1) \tan 2\theta \sin\theta + \frac{\beta}{2}(1 - \frac{m_\eta^2}{m_{\eta'}^2}) \tan 2\theta \cos\theta = \xi \, ,}$$

where the dimensionless parameters β and ξ were introduced:

$$\beta = \frac{A_\eta}{A_{\eta'}} = \sqrt{\frac{\Gamma_{\eta' \to 2\gamma}}{\Gamma_{\eta \to 2\gamma}} \frac{m_\eta^3}{m_{\eta'}^3}},$$

$$\xi = \sqrt{\frac{\alpha^2 m_\eta^3}{96\pi^3 \Gamma_{\eta \to 2\gamma}} \frac{1}{f_\eta^2}}$$

As an input we'll use experimental data (PDG Review 2008):

$$m_\eta = 547.853 \pm 0.024\ MeV$$
$$m_{\eta'} = 957.78 \pm 0.24\ MeV$$
$$\Gamma_{\eta\to 2\gamma} = 0.510 \pm 0.026\ keV$$
$$\Gamma_{\eta'\to 2\gamma} = 4.30 \pm 0.15\ keV$$
$$f_\pi = 130.4 \pm 0.4\ MeV$$

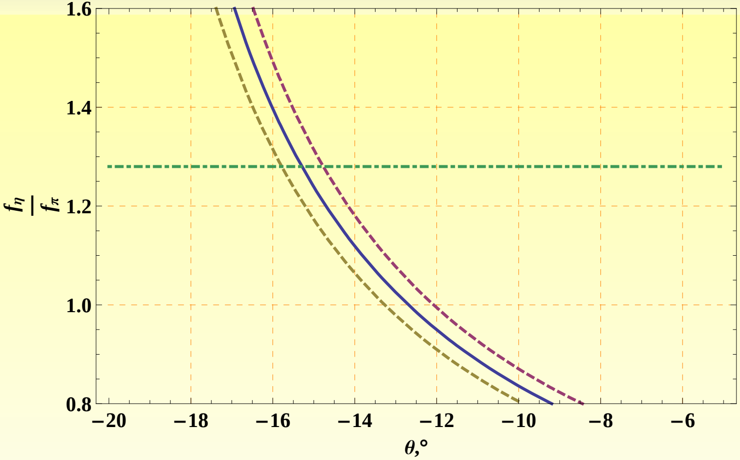

Figure: Mixing angle θ as a function of decay constant f_η in the one angle mixing scheme

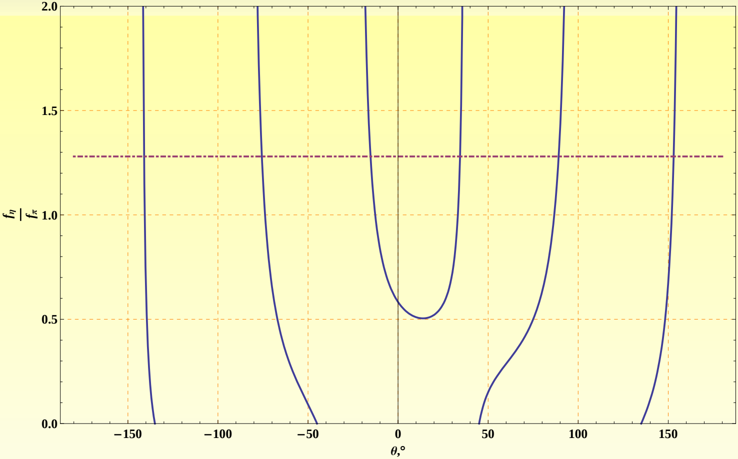

Figure: Mixing angle θ as a function of decay constant f_η in the one angle mixing scheme - the full range of parameters

For $f_\eta = 1.28 f_\pi$ we get the mixing angle:

$$\theta = -15.3^\circ \pm 0.5^\circ$$

Two Angle $\eta - \eta'$ Mixing Scheme

Let's introduce the fields φ_8,φ_0 which are expressed through the physical fields φ_η, $\varphi_{\eta'}$ as

$$\varphi_8 = \varphi_\eta \cos\theta_2 + \varphi_{\eta'} \sin\theta_1 \ ,$$
$$\varphi_0 = -\varphi_\eta \sin\theta_2 + \varphi_{\eta'} \cos\theta_1 \ .$$

And the equation for the two angle mixing scheme will be than:

$$\cos\theta_2 + \beta \frac{m_\eta^2}{m_{\eta'}^2} \sin\theta_1 - \frac{m_{\eta\eta'}^2}{m_\eta^2} \sin\theta_2 + \beta \frac{m_{\eta\eta'}^2}{m_{\eta'}^2} \cos\theta_1 = \xi,$$

where

$$m_{\eta\eta'}^2 = \frac{1}{2} \frac{m_\eta^{2\,\prime} \sin 2\theta_1 - m_\eta^2 \sin 2\theta_2}{\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2},$$

$$\beta = \frac{A_\eta}{A_{\eta'}} = \sqrt{\frac{\Gamma_{\eta' \to 2\gamma}}{\Gamma_{\eta \to 2\gamma}} \frac{m_\eta^3}{m_{\eta'}^3}},$$

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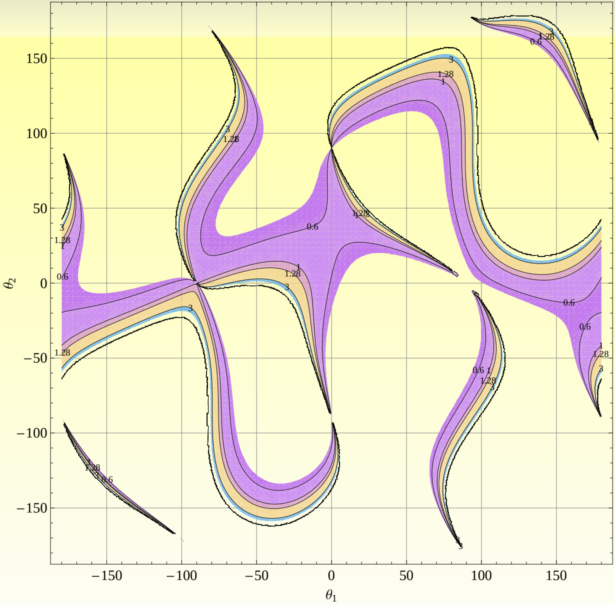
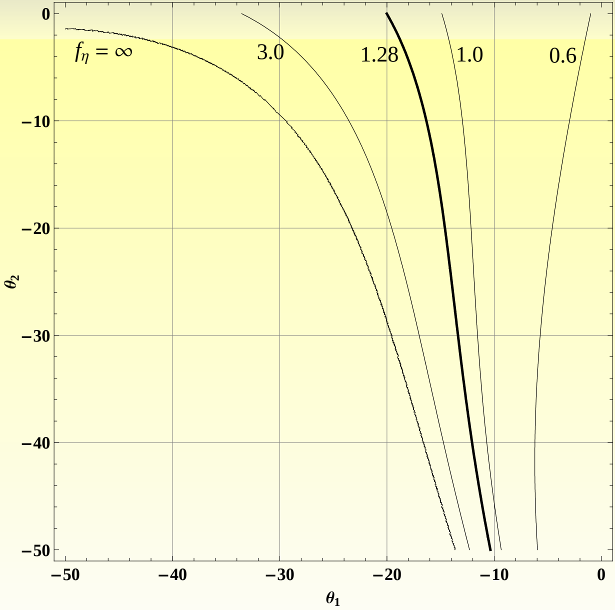

θ_1
θ_2
−150
−100
−50
0
50
100
150


$f_\eta = \infty$
3.0
1.28
1.0
0.6
0
−10
−20
−30
−40
−50
θ_2
−50
−40
−30
−20
−10
0
θ_1

Summary

- Anomaly condition in it's dispersive approach allowed us to get a precise value for a mixing angle in a one-angle mixing scheme. The value of the mixing angle is consistent with most of previous calculations done in other approaches, but is more precise.
- For a two-angle mixing scheme the relationship between two mixing angles was gotten. The dramatic behaviour of $\theta_1 - \theta_2$ relationship allows us get the limits for the one of the mixing angles.

Thank you for your attention!

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