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Manifestation of three-body forces in Bethe-Salpeter and light-front equations

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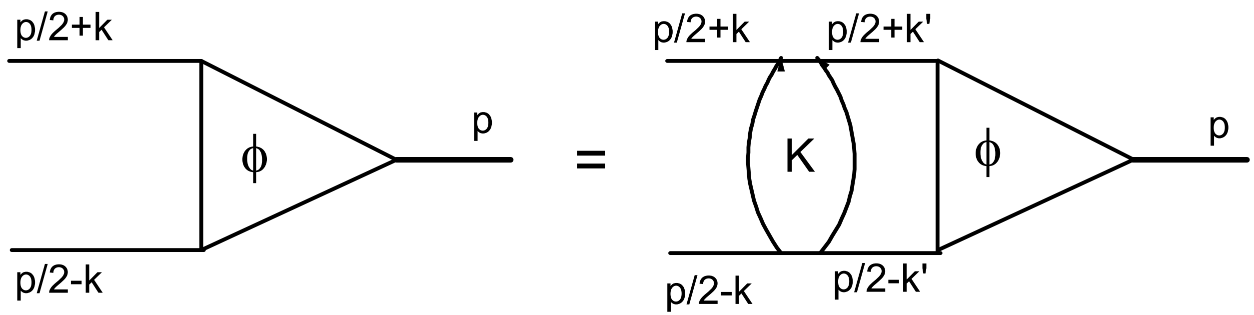
• Aim

- To present the results of first solution, for one-boson exchange interaction, of
  - Three-body Bethe-Salpeter equation
  - Three-body light-front equation
- To incorporate two exchanged bosons in flight.
- To obtain important physical conclusions (three-body forces).

- **First – two-body problem**

• Two-body Bethe-Salpeter equation



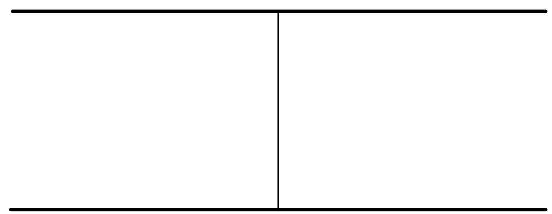
$$\left((\frac{p}{2}+k)^2-m^2\right)\left((\frac{p}{2}-k)^2-m^2\right)\Phi(k,p)$$
$$= \quad = -i\int \frac{d^4k'}{(2\pi)^4} V_{Feyn}(k,k',p)\Phi(k',p)$$

One can find $\Phi(k,p)$ and mass $M^2=p^2$.

• OBE kernel

All particles are spinless

$$V_{Feyn}(k,k',p) = -\frac{g^2}{(k-k')^2-\mu^2+i\epsilon}$$



In more realistic case (two fermions) it was intensively studied in Dubna:
V. Burov, S. Bondarenko
S. Dorkin, L. Kaptari, S. Semykh

• Two-body light-front equation

$$\left(\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} - M^2\right)\psi(\vec{k}_\perp, x)$$

$$= \; -\frac{m^2}{2\pi^3}\int \psi(\vec{k}'_\perp, x')V(\vec{k}'_\perp, x'; \vec{k}_\perp, x, M^2)\frac{d^2k'_\perp\, dx'}{2x'(1-x')}$$

OBE kernel:

$$V(\vec{k'}_\perp, x'; \vec{k}_\perp, x; M^2) =$$

$$\begin{cases} -\dfrac{4\pi\alpha}{\mu^2+\frac{x'}{x}\left(1-\frac{x}{x'}\right)^2 m^2+\frac{x'}{x}\left(\vec{k}_\perp-\frac{x}{x'}\vec{k'}_\perp\right)^2+(x'-x)\left(\frac{m^2+\vec{k'}_\perp^2}{x'(1-x')}-M^2\right)}, & \text{if } x \le x' \\ -\dfrac{4\pi\alpha}{\mu^2+\frac{x}{x'}\left(1-\frac{x'}{x}\right)^2 m^2+\frac{x}{x'}\left(\vec{k'}_\perp-\frac{x'}{x}\vec{k}_\perp\right)^2+(x-x')\left(\frac{m^2+\vec{k}_\perp^2}{x(1-x)}-M^2\right)}, & \text{if } x \ge x' \end{cases}$$

• Techniques of calculation

Graph techniques developed by V.G. Kadyshevsky (1964), modified (V. Karmanov) for the case of light-front dynamics.

Review:
J. Carbonell, B. Desplanques, V.A. Karmanov and J.-F. Mathiot, Phys. Reports **300**, 215 (1998).

• Two-body light-front wave function

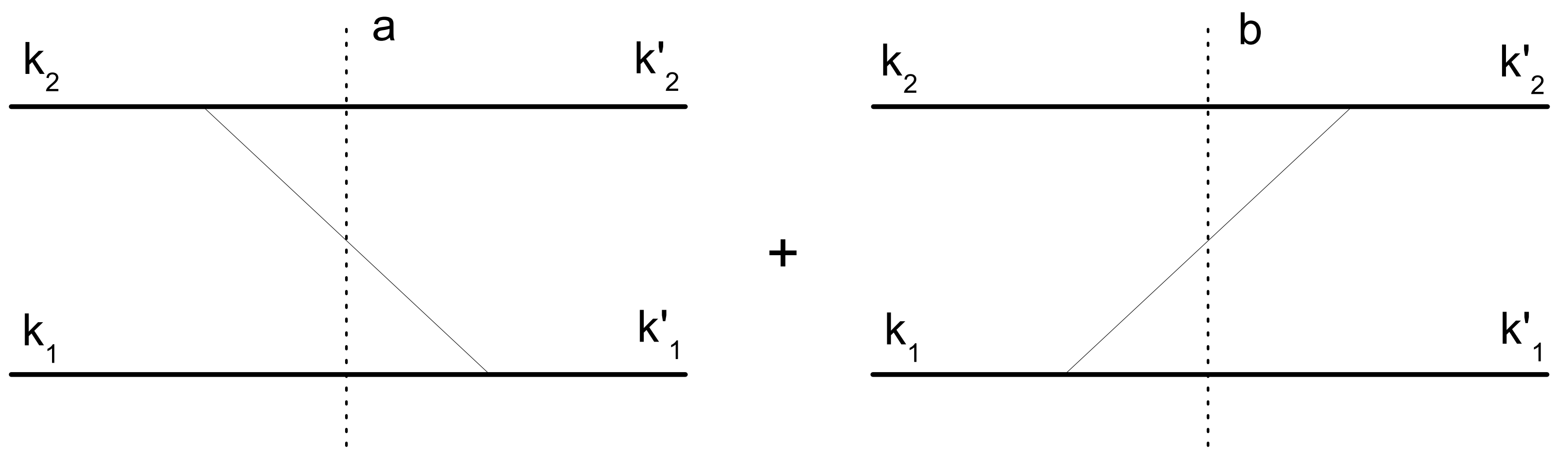
$$\psi \;=\; \psi(\vec{k},\vec{n})$$

Wave function depends on the orientation $\vec{n}$ of the light-front plane!
Another pair of variables

$$\vec{k}_\perp = \vec{k} - \vec{n}(\vec{n}\cdot\vec{k}), \quad x = \frac{1}{2}\left(1 - \frac{\vec{n}\cdot\vec{k}}{\varepsilon_k}\right),$$

$$\psi = \psi(\vec{k}_\perp, x)$$

• OBE light-front kernel



Ladder kernel

Nonrelativistic limit

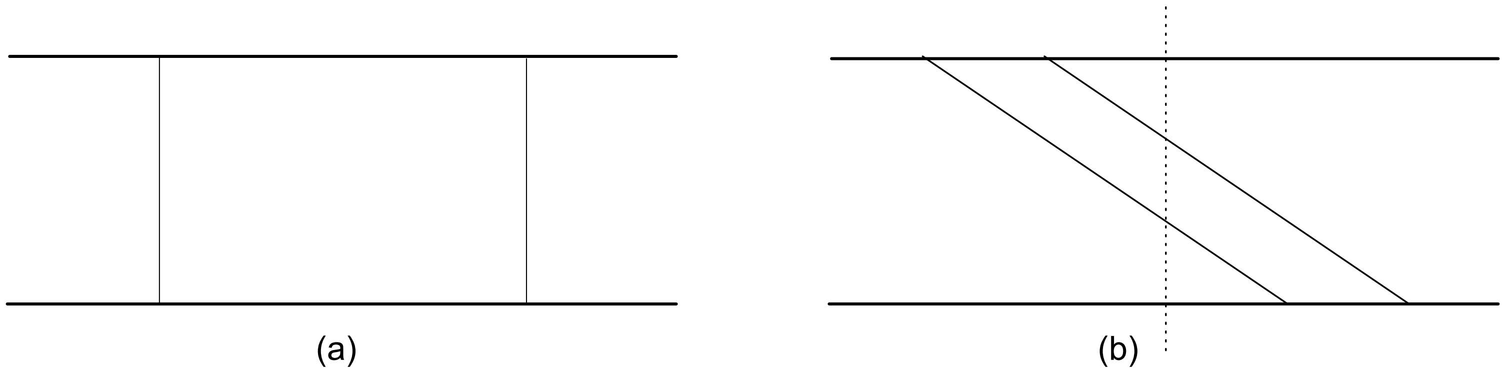
$$V(\vec{k}, \vec{k'}) = \frac{-4\pi\alpha}{(\vec{k} - \vec{k'})^2 + \mu^2},$$

Coordinate space:
well-known Yukawa potential.

$$V(r) = -\frac{\alpha}{r} \exp(-\mu r)$$

Difference between BS and LF approches

Stretched boxes



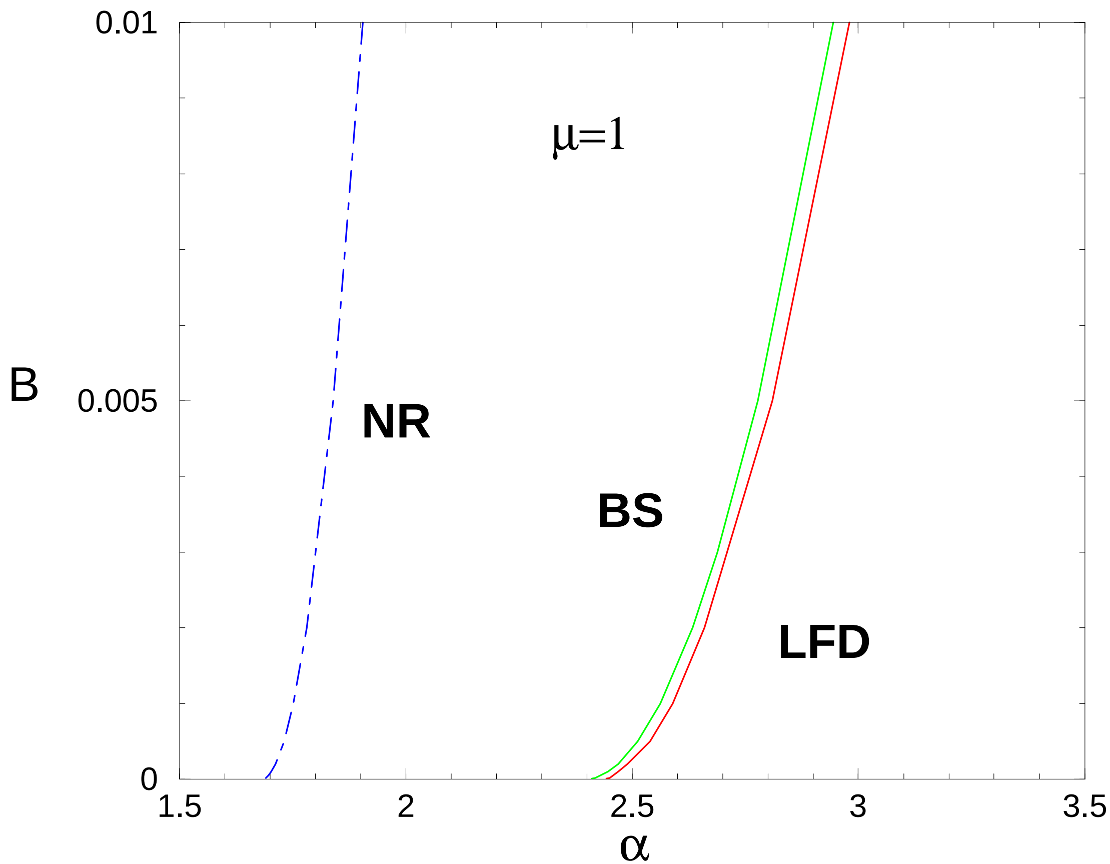
(a) Ladder graph with two exchanges (taken into account).
(b) "Stretched box" graph.

- Automatically included in iterations of BS equation.
- Not included in the LF equation.

However, inspection shows that stretched boxes are small.

• Comparison of BS and LFD results

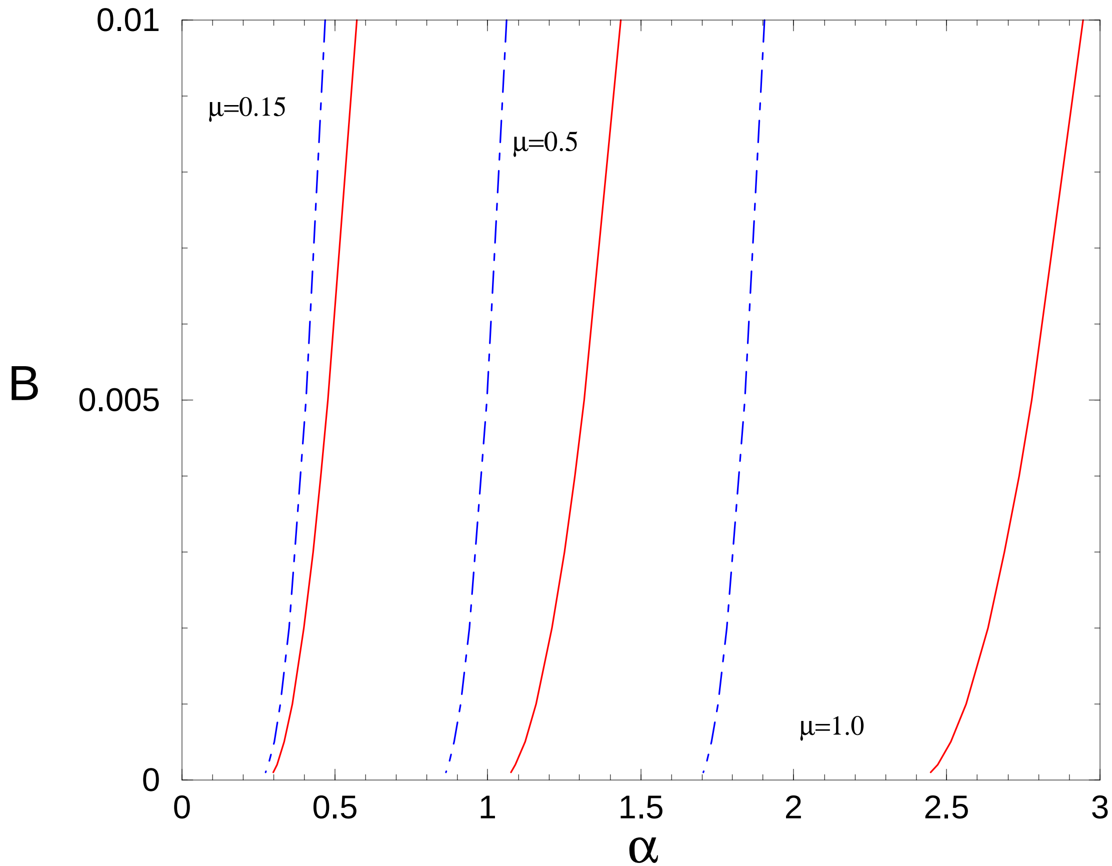
Mangin-Brinet and J. Carbonell,
Phys. Lett., **B474** (2000) 237.



Coincidence of BS and LFD results!

Huge difference between NR and REL results.

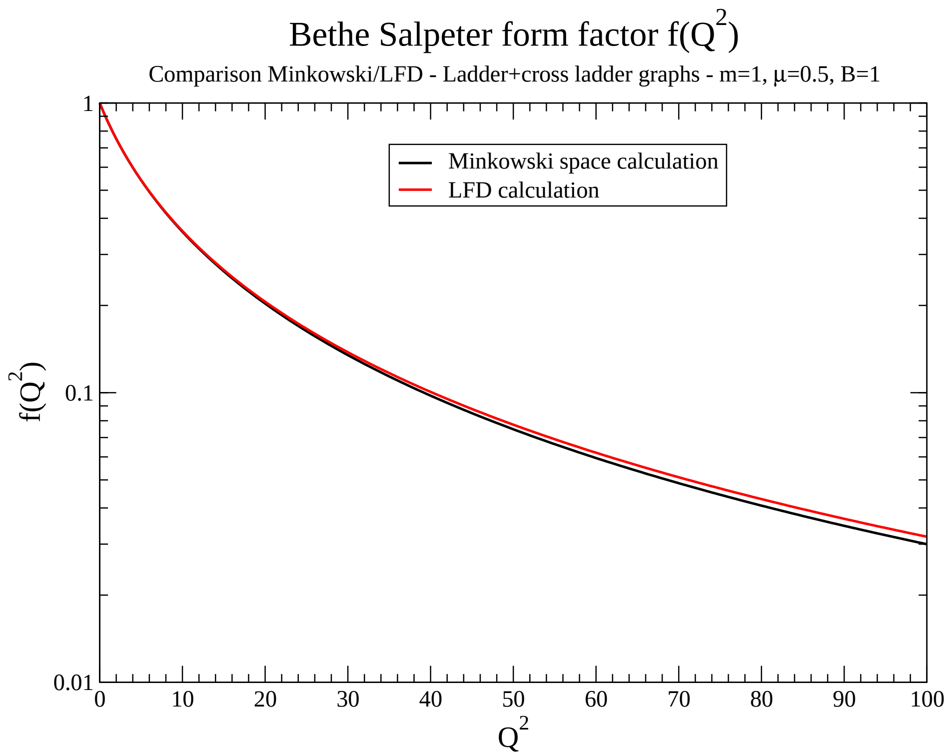
• Dependence on the exchange mass μ



Two-body binding energy B versus coupling constant α

• EM form factor: BS vs. LFD

V.A. Karmanov, J. Carbonell, M. Mangin-Brinet,
Nucl. Phys. A **790**, 598c, (2007).



EM form factor

Black curve: Bethe-Salpeter calculation.

Red curve: LFD calculation, via $\psi(\vec{k}_\perp, x)$.

• Two-body conclusion

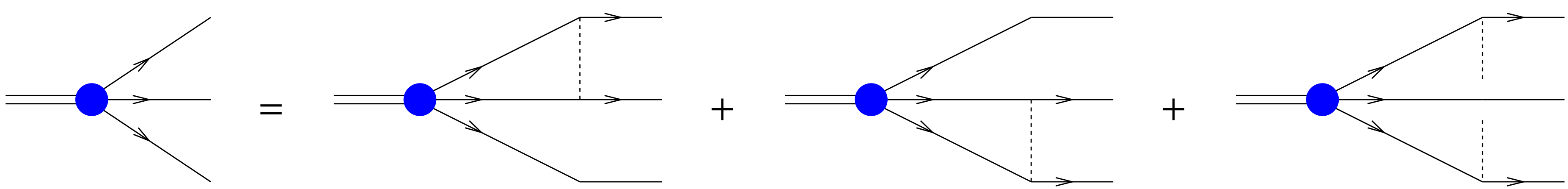
- Binding energies calculated by BS and LFD are very close to each other.

- **Relativistic three-body problem**

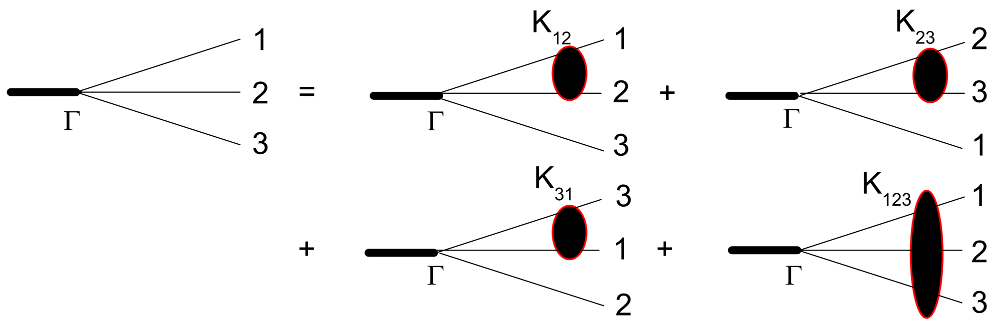
• Three-body Bethe-Salpeter equation

$$(k_1^2 - m^2)(k_2^2 - m^2)(k_3^2 - m^2)\Phi(k_1, k_2, k_3; p)$$

$$= -i \int \Phi(k_1', k_2', k_3'; p)(V_{12} + V_{23} + V_{31})$$
$$\times (2\pi)^4 \delta^{(4)}(k_1' + k_2' + k_3' - p) \frac{d^4 k_1'}{(2\pi)^4} \frac{d^4 k_2'}{(2\pi)^4} \frac{d^4 k_3'}{(2\pi)^4}$$



• Three-body relativistic equation in LFD



Equation with two-body and three-body forces

$$
\begin{aligned}
(s_{123} - M_3^2)\,\psi &= -\frac{m^2}{2\pi^3}\int \psi\, V_{12}\, \frac{d^2k'_{\perp}dx'}{(2\pi)^3 2x'(1-x')(1-x_3)} \\
&+ (231) + (312) \\
&+ \int \psi\, K_{123}\, \frac{d^2k'_{\perp}dx'}{(2\pi)^3 2x'(1-x')}\, \frac{d^2k'_{3\perp}dx'_3}{(2\pi)^3 2x'_3(1-x'_3)},
\end{aligned}
$$

• LFD equation for Faddeev component

Faddeev decomposition:

$$\begin{aligned}\psi(1,2,3) &= \psi_{12}(1,2,3)+\psi_{23}(1,2,3)+\psi_{31}(1,2,3)\\ &= \psi_{12}(1,2,3)+\psi_{12}(2,3,1)+\psi_{12}(3,1,2)\end{aligned}$$

Equation:

$$\begin{aligned}&(s_{123}-M_3^2)\,\psi_{12}(\vec{k}_\perp,x;\vec{k}_{3\perp},x_3)\\ &= -\frac{m^2}{2\pi^3}\int\frac{d^2k'_\perp\,dx'}{2x'(1-x')}\;\frac{1}{(1-x_3)}\;V(\vec{k}_\perp,x;\vec{k'}_\perp,x';M_{12}^2)\\ &\times\Big[\psi_{12}\left(\vec{k'}_\perp,x';\vec{k}_{3\perp},x_3\right)+\psi_{12}\left(\vec{k'}_{23\perp},x'_{23};\vec{k'}_{1\perp},x'_1\right)\\ &\qquad+\psi_{12}\left(\vec{k'}_{31\perp},x'_{31};\vec{k'}_{2\perp},x'_2\right)\Big]\end{aligned}$$

$\vec{k}_\perp,x;\vec{k}_{3\perp},x_3$ – light-front Jacobi variables.

• Five scalar variables

$$\psi_{12} = \psi_{12}(\vec{k}_\perp, x; \vec{k}_{3\perp}, x_3)$$

depends on:

$$k_\perp, \quad k_{3\perp}, \quad u = \frac{\vec{k}_\perp \cdot \vec{k}_{3\perp}}{k_\perp k_{3\perp}}, \quad x, \quad x_3$$

After discreteization, five variables are converted in one huge index.

Computer facilities – in USA, the same as used for lattice calculations, but small part of them.

• Nonrelativistic Faddeev equation

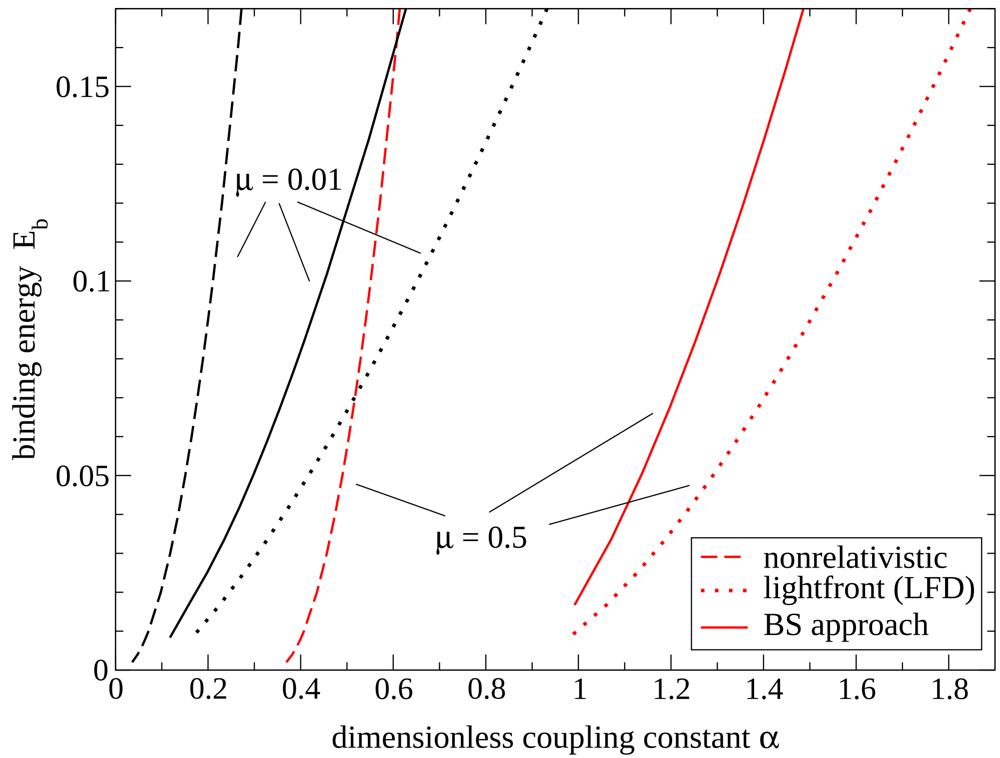
$$\left(E_3 - \frac{\vec{p}^{\,2}}{m} - \frac{3\vec{q}^{\,2}}{4m}\right)\psi_{12}(\vec{p},\vec{q})$$
$$= \int V(\vec{p}-\vec{p'})\left[\psi_{12}(\vec{p'},\vec{q}) + \psi_{12}\left(-\frac{1}{2}\vec{p'} + \frac{3}{4}\vec{q},\ -\vec{p'} - \frac{1}{2}\vec{q}\right)\right.$$
$$\left. + \psi_{12}\left(-\frac{1}{2}\vec{p'} - \frac{3}{4}\vec{q},\ \vec{p'} - \frac{1}{2}\vec{q}\right)\right]\frac{d^3p'}{(2\pi)^3}.$$

Kernel: $V(\vec{p}-\vec{p'}) = -\dfrac{4\pi\alpha}{(\vec{p}-\vec{p'})^2 + \mu^2}$

Kernel in the coordinate space: $V(r) = -\dfrac{\alpha e^{-\mu r}}{r}$

3 variables, solved immediately.

• Three-body results: BS, LFD, non.-rel.

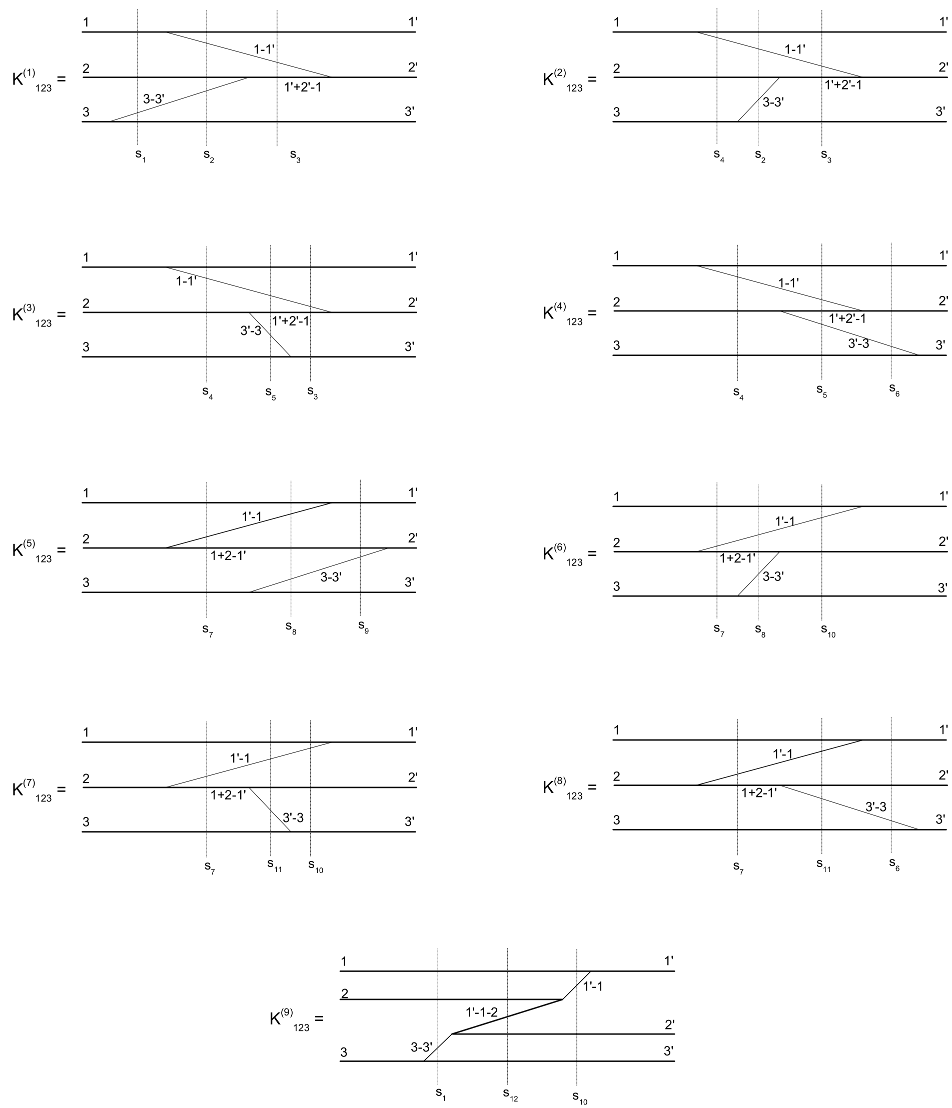


Three-body binding energy in the limit when it tends to zero, for $\mu = 0.01$ and $\mu = 0.5$

LFD and BS results are **very different**, in contrast to two-body case!

• Three-body forces of relativistic origin

(Two mesons in flight)



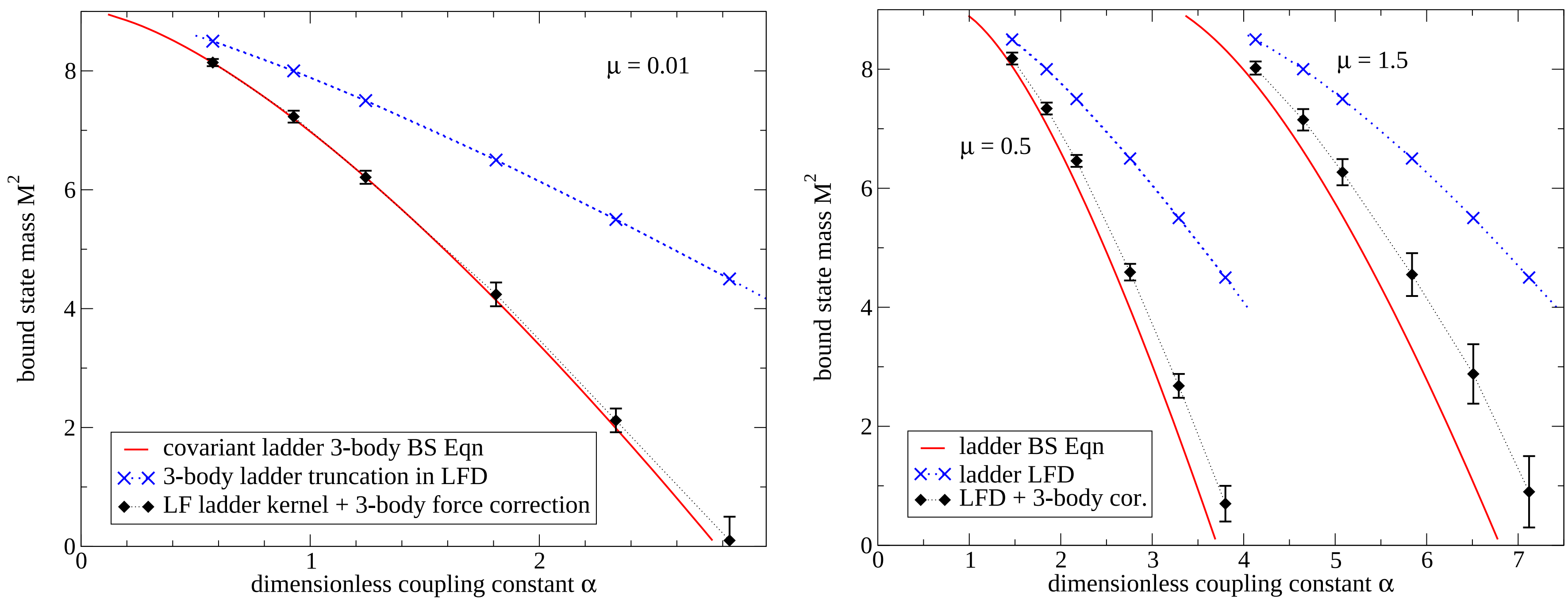
• Perturbative correction

$$\Delta M_3^2 = -\int \frac{d^2k_{3\perp}}{(2\pi)^3} \frac{d^2k_\perp}{(2\pi)^3} \frac{d^2k'_{3\perp}}{(2\pi)^3} \frac{d^2k'_\perp}{(2\pi)^3}$$
$$\times \int_0^1 \frac{dx_3}{2x_3(1-x_3)} \frac{dx}{2x(1-x)} \frac{dx'_3}{2x'_3(1-x'_3)} \frac{dx'}{2x'(1-x')}$$
$$\times \psi(\vec{k}_\perp, x; \vec{k}_{3\perp}, x_3)$$
$$\times K_{123}(\vec{k}_\perp, x; \vec{k}_{3\perp}, x_3; \vec{k'}_\perp, x'; \vec{k'}_{3\perp}, x'_3)$$
$$\times \psi(\vec{k'}_\perp, x'; \vec{k'}_{3\perp}, x'_3)$$

11-dimensional integral

$$K_{123} = 6 \times (9 \quad \text{terms}) = 54 \quad \text{graphs}$$

• Including correction



Three-body bound state mass squared M_3^2 vs. coupling constant α for exchange masses $\mu = 0.01, 0.5$ and 1.5.

• Three-body forces

$$V(\vec{r}_1, \vec{r}_2, \vec{r}_3) \neq V_{12}(\vec{r}_1, \vec{r}_2) + V_{23}(\vec{r}_2, \vec{r}_3) + V_{31}(\vec{r}_3, \vec{r}_1)$$

An indication: underbinding of tritium:

Paris:	$E = -7.38\ MeV$
RSC:	$E = -7.23\ MeV$
experiment:	$E = -8.48\ MeV$

Two mesons in flight as a first order relativistic correction in the framework of the three-body Schrödinger equation:
Shin-Nan Yang, Phys. Rev. **C10** (1974) 2067.
Shin-Nan Yang, W, Glöckle, Phys. Rev. **C33** (1986) 1774.

• Conclusions

- Three-body relativistic LFD and BS equations with OBE kernel are solved.
- Binding energies considerably differ from each other (in contrast to two-body case)
- Three-body forces of relativistic origin appear in three-body system.
- Three-body forces remove the difference between between LFD and BS.
- Undoubted manifestation of three-body forces is found.

- **For the present precision,**

we always need three-body forces.