ASME method
and particle reconstruction

A.Jerusalimov *JINR-LHEP, Dubna*

Outlook

1. **Charged particle reconstruction**
   - **.1. What is ASME ?**
   - **.2. ASME for CBM**
   - **.3. ASME for HADES**
   - **.4. ASME for HADES → FullFit**

2. **V^0 reconstruction**
   - **.1. Features of V^0 – decay**
   - **.2. V^0 – Finder**
   - **.3. V^0 – Fitter**

3. **Ξ^-/Ω^- reconstruction**

4. **Conclusion**

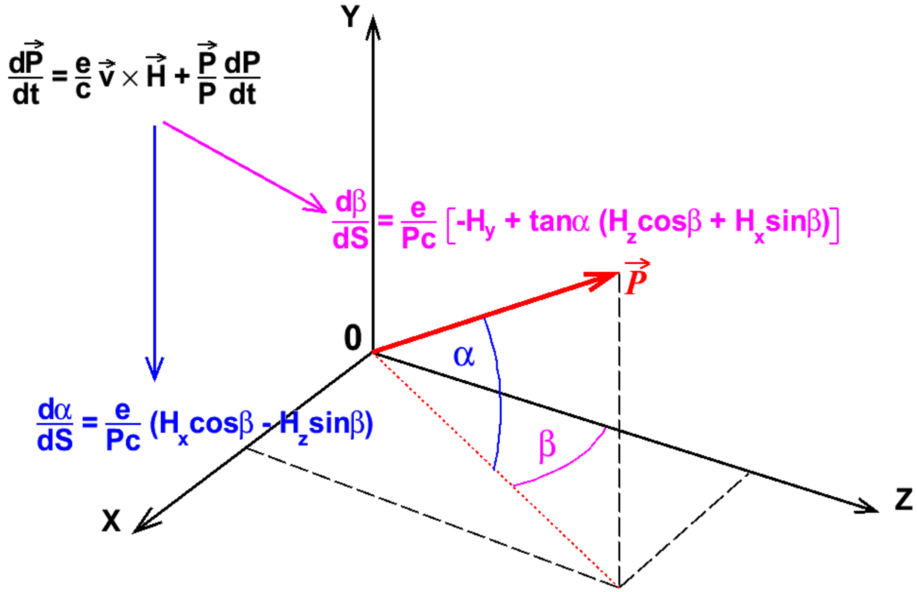
ISHEPP XIX ***JINR Dubna*** Sep 29 – Oct 4, 2008

1. Charged particle reconstruction

1.1 What is ASME ? → Approximate Solution of Motion Equation

Equation of motion of charged particle in magnetic field

$$\frac{d\vec{P}}{dt} = \frac{e}{c}\,\vec{v}\times\vec{H} + \frac{\vec{P}}{P}\cdot\frac{dP}{dt}$$



P – momentum,
β - azimuthal angle,
α - deep angle,
S - length of track
H – Magnetic field

1[st] integration →

$$\beta(S) = \beta_0 + \frac{e}{c}\int_0^S \frac{1}{p}\left[-H_y + \tan\alpha\cdot(H_z\cos\beta + H_x\sin\beta)\right]dS$$

$$\sin\alpha(s) = \sin\alpha_0 + \frac{e}{c}\int_0^s \frac{1}{p}\left(H_x\cos\beta - H_z\sin\beta\right)dS$$

2^{nd} integration: →

$$x(s) = x_0 + \int_0^s \sin\beta(s)ds$$

$$y(s) = y_0 + \int_0^s \tan\alpha(s)ds$$

But

$\boldsymbol{x}=\boldsymbol{x}(x_0, y_0, \beta_0, \alpha_0, P_0)$
$\boldsymbol{y}=\boldsymbol{y}(x_0, y_0, \beta_0, \alpha_0, P_0)$

Therefore to find $(x_0, y_0, \beta_0, \alpha_0, P_0)$ *it is necessary*

$$\chi^2 = \sum_{i,j}^{n} (x_i^{exp} - x_i)(G + D_x E)_{ij}^{-1}(x_j^{exp} - x_j) \Rightarrow$$ ***min* in X0Z plane**

$$w^2 = \sum_{i,j}^{n} (y_i^{exp} - y_i)(G + D_y E)_{ij}^{-1}(y_j^{exp} - y_j) \Rightarrow$$ ***min* in Y0Z plane**

where *G* – matrix of multiple scattering,
D_x, D_y – matrix of errors,
***E* – unity matrix**

To minimize:

needs initial values

$x_0^{init}, y_0^{init}, \beta_0^{init}, \alpha_0^{init}, P_0^{init}$

How to minimize ?

1. Gradient downhill method
2. Variation procedure:

$$x_i^{k+1} = x_i^k + \delta x_0 + \frac{\partial x}{\partial \beta}\Big|_i \delta\beta_0 + \frac{\partial x}{\partial p}\Big|_i \delta p_0$$

$$y_i^{k+1} = y_i^k + \delta y_0 + \frac{\partial y}{\partial \tan\alpha}\Big|_i \delta \tan\alpha_0$$

min χ^2 $\Downarrow$

$$\begin{cases} a_{11}\,\delta x_0 + a_{12}\,\delta\beta_0 + a_{13}\,\delta p_0 &= b_1 \\ a_{21}\,\delta x_0 + a_{22}\,\delta\beta_0 + a_{23}\,\delta p_0 &= b_2 \\ a_{31}\,\delta x_0 + a_{32}\,\delta\beta_0 + a_{33}\,\delta p_0 &= b_3 \end{cases}$$

min w^2 $\Downarrow$

$$\begin{cases} a_{44}\,\delta y_0 + a_{45}\,\delta\tan\alpha_0 &= b_4 \\ a_{54}\,\delta y_0 + a_{55}\,\delta\tan\alpha_0 &= b_5 \end{cases}$$

$par_0^{iter} = par_0^{iter-1} + \delta par_0$

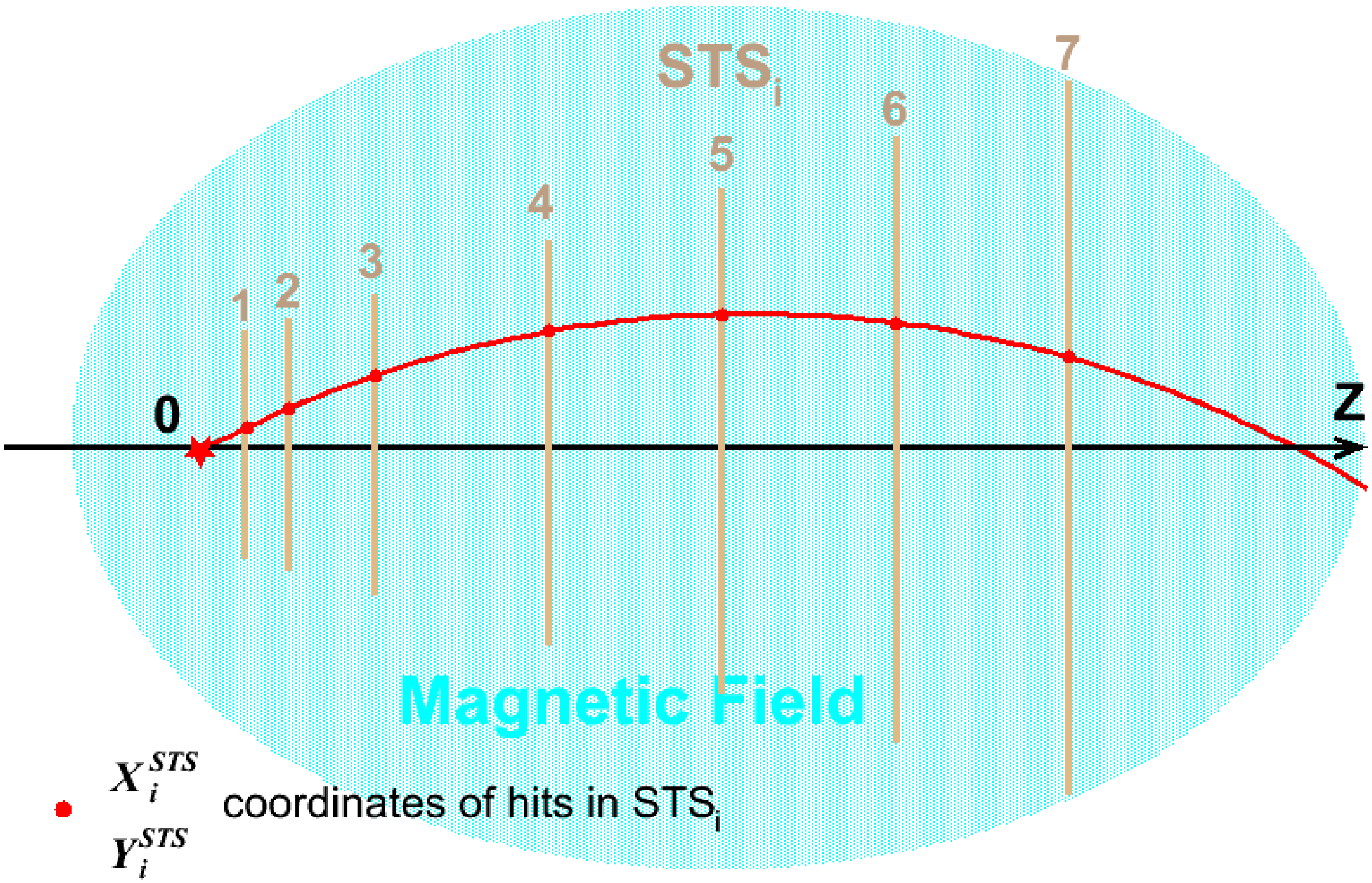
Needs 2 – 3 iterations to get minimum

Results: x_0^{fit}, y_0^{fit}, β_0^{fit}, α_0^{fit}, p_0^{fit} - parameters

σ^2_x, σ^2_y, σ^2_β, σ^2_α, σ^2_P, $\delta_{P\beta}$ – errors and correlations

Procedure is very robust: accuracy of P_0^{init} is ≈ 50 %

1.2 ASME for CBM



Input data:

x^i, y^i - coordinates of hits,

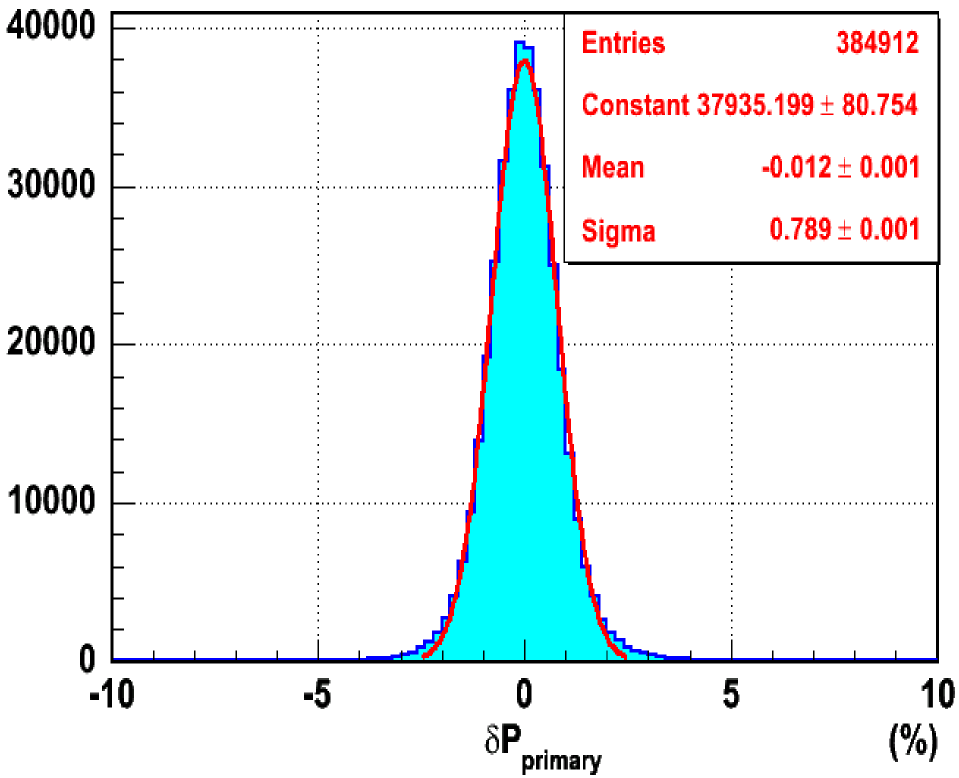
initial values: α_0, β_0, P_0 – from parabola approximation.

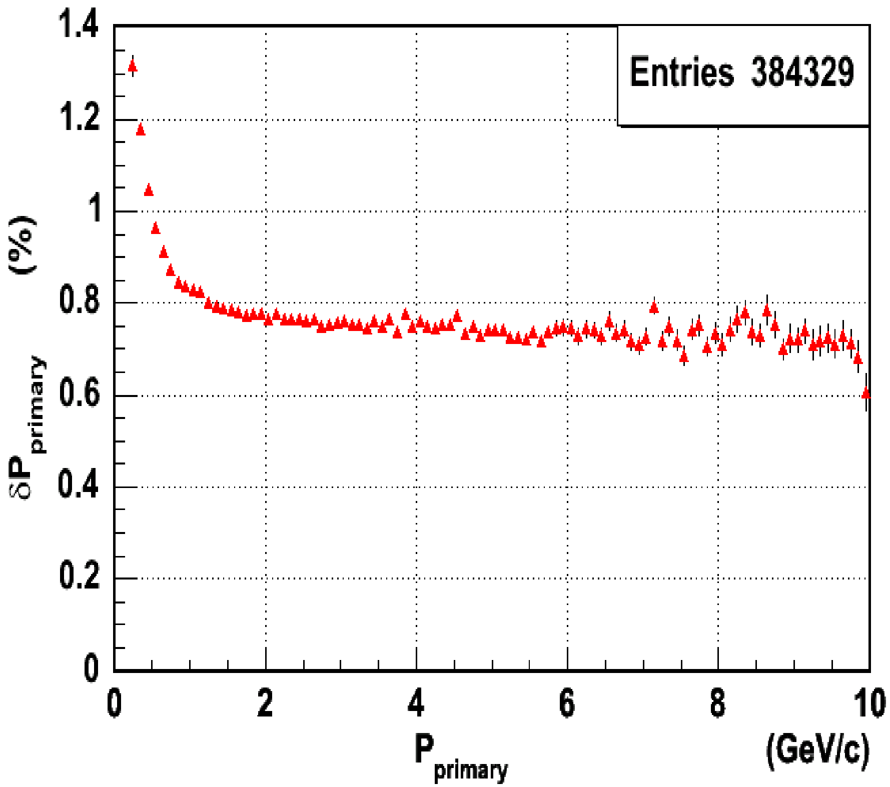
Results: Central Au+Au collisions at 25A GeV

GEANT 1k events

7STS → dtot (Si)=1200μm

ASME TrackFitter (into 1st hit):



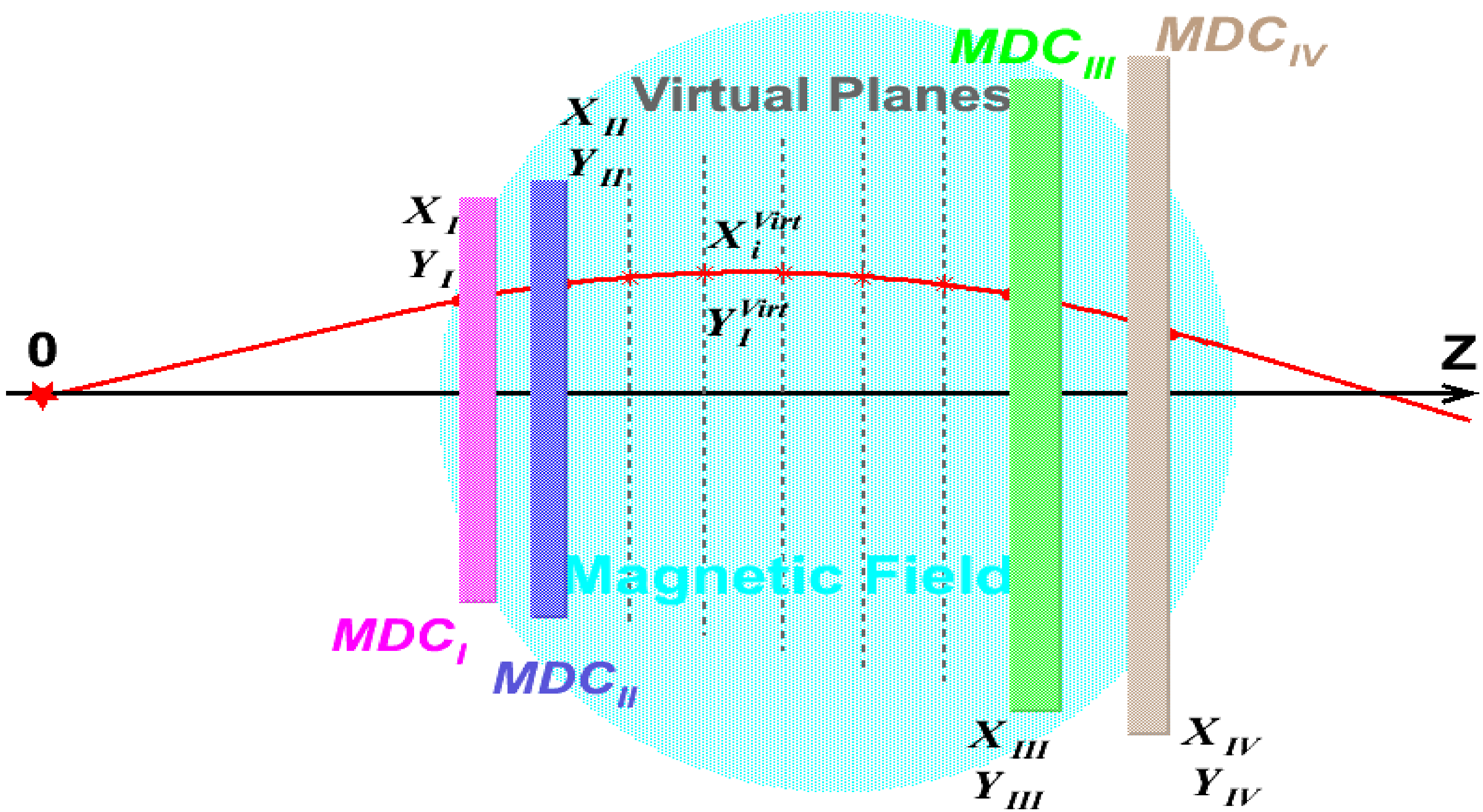


δP_0 = 0.79 %

δP_0 vs P_0

8STS → dtot (Si)=3500μm → δP_0 = 1.40 %

1.3 ASME for HADES



Now track reconstruction is double step procedure:

1st - determination of track segments (MDC1-MDC2 & MDC3-MDC4) parameters ← **fit by straight line**

2nd - determination of momentum ← **Spline** or **RK**

Input data for ASME:

coordinates of hits x_{MDCi}, y_{MDCi} i=1…4

P_0, x^{Virt}_{MDCi}, y^{Virt}_{MDCi} – from **Spline** or **4-parameters parabola**

Results: Momentum resolution

for electron (C+C at 2.0 AGeV)

Spline:

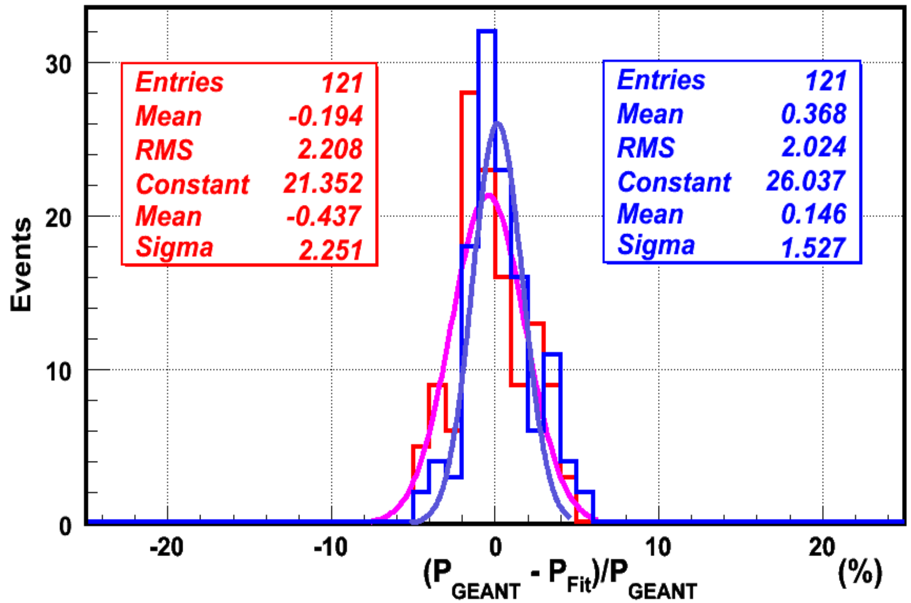
RMS_P = 2.21 %

σ_P = 2.25 %

ASME:

RMS_P = 2.02 %

σ_P = 1.53 %



for protons (pp elastic at 2.2 GeV)

Spline:

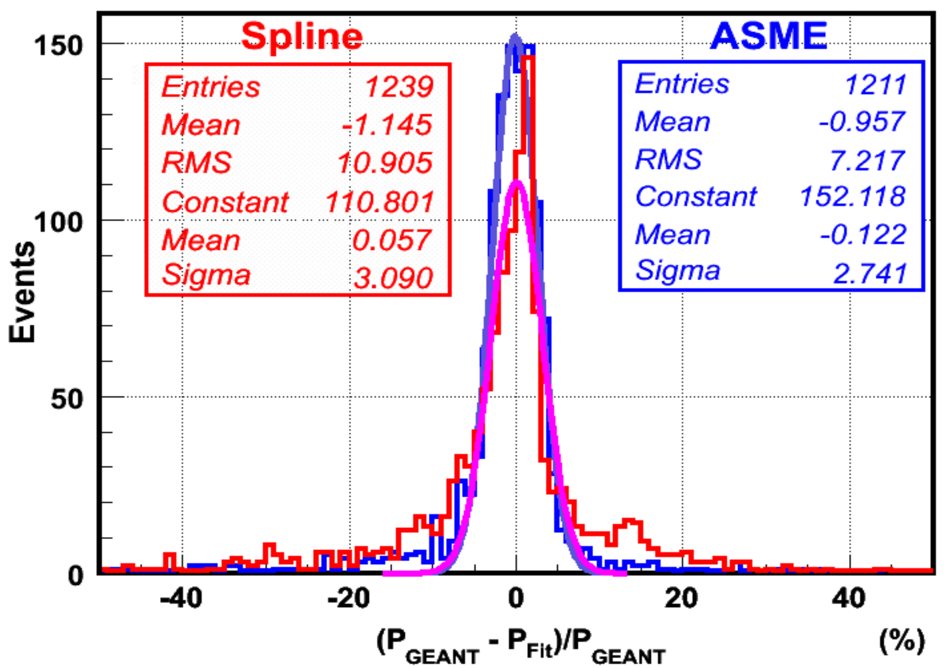
RMS_P = 10.9 %

σ_P = 3.1 %

ASME:

RMS_P = 7.2 %

σ_P = 2.7 %



1.4 ASME for HADES – FullFit

Details of MDC tracking:

- **Data of track: "hits" → t_i – drift time (track trajectory ↔ i^th sense wire)**
- ***Trackfinder* select hits produced by concrete track: $\{t_i^{inner}\}$ & $\{t_i^{outer}\}$**
- **Track model for inner and outer segments of track – straight line**

1st step: Functional to be minimized $F = \sum_i \frac{(t_i + t_{off} - T_i)^2}{(\Delta T_i)^2} w_i$

T_i – measured drift times
t_i – drift times (calc.)
w_i – Tukey weights
ΔT_i – drift time errors

Results: $\{x_1, y_1, z_1\}, \{x_2, y_2, z_2\}, \{x_3, y_3, z_3\}, \{x_4, y_4, z_4\}$ – **coordinates of track segments**

2nd step: SPLINE + RK(*optionally*) to determine **momentum**

Weakness:
- 2 step procedure
- do not take into account energy loss and multiple scattering
- RK is more precise but some times slow than SPLINE
- hard to propagate errors (momentum and angles) for track
- global min sometimes was not found
- not quite sufficient fakes rejection (hits filtering)
- problem of close tracks

Solution: (**segments trackfitter**) + (**Spline / RK**) → **FullFit** (single step procedure)

Input data: **hits** (t_i) from **all MDC's**,

P_0^{init} and x_i^{Virt}, y_i^{Virt} from **Spline** or **Parabola4**

Functional to be minimized:

$$w^2 = \sum_{i,j}^{n} (t_i^{exp} + t_{off} - T_i) (G_t + D_t E)_{ij}^{-1} (t_j^{exp} + t_{off} - T_j) w_{ij}$$ "time like"

or

$$w^2 = \sum_{i,j}^{n} \left(d(t_i^{exp} + t_{off}) - D_i\right) (G_d + D_t E)_{ij}^{-1} \left(d(t_j^{exp} + t_{off}) - D_j\right) w_{ij}$$ "space like"

where $T_i = T(d_i) = T(x_0, y_0, \beta_0, \alpha_0, P_0)_{i\text{-layer}}$ - drift time (calc.)

$D_i = D(x_0, y_0, \beta_0, \alpha_0, P_0)_{i\text{-layer}}$ - track ↔ wire distance

w_{ij} – Tukey weights

G_t (G_d) – **matrix of multiple scattering**

D_t – matrix of errors

Expected results:

- sufficient **accuracy** of determination of **track parameters** (better than for SPLINE and at least not worse than for RK),
- calculation of **errors** of parameters (especially important for Kine Fit)
- better **"hits filtering"** during track reconstruction:

***Necessary** to test.*

2. V^0 reconstruction

$\Lambda^0(K^0) \rightarrow p(\pi^+) + \pi^-$

2.1 Features of V^0 – decay →

Primary Vertex position ?
Track : Primary or Secondary ?
Secondary Vertex (V^0) position ?

Primary VertexFinder

used
track propagation procedure
and virtual planes

X_V (μm)	**0.7 ± 2.2**
Y_V (μm)	**- 0.3 ± 1.4**
Z_V (μm)	**- 1.9 ± 4.1**

Practically the same
both for 7STS and 8STS

For further analysis were used
$X_V = Y_V = Z_V = 0.0$

Tracks separation

used impact parameter: distance between track and Primary Vertex position

Primary Tracks	**94 %**
Secondary Tracks	**6 %**

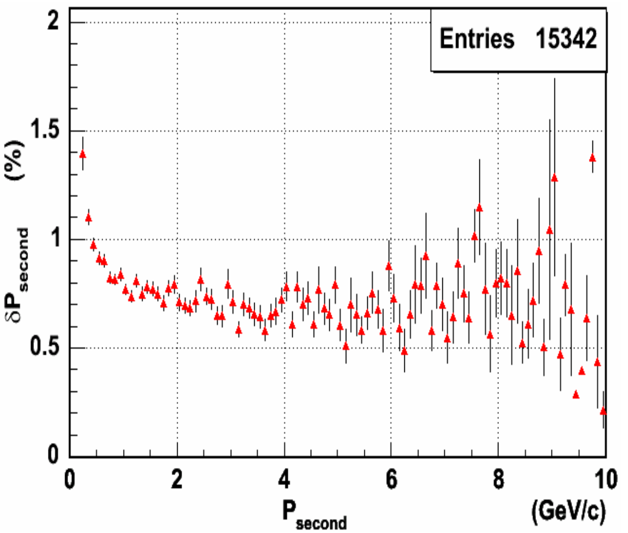
↓

7STS
5 % lost (→ to primary)
24 % false (→ from primary)

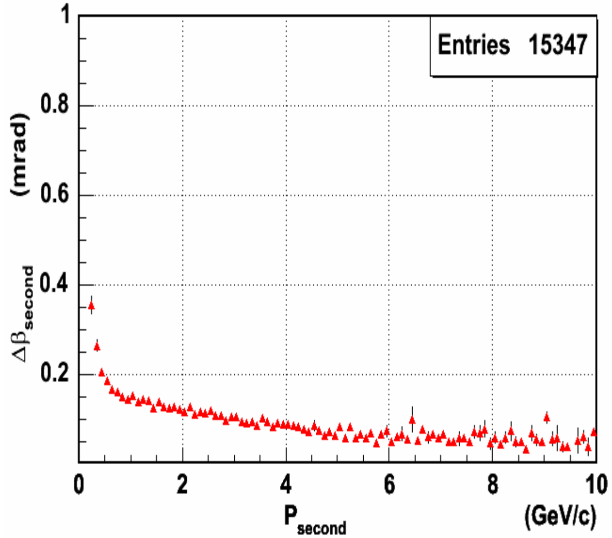
8STS
8 % lost (→ to primary)
36 % false (→ from primary)

2.2 V^0-Finder (for 7STS)

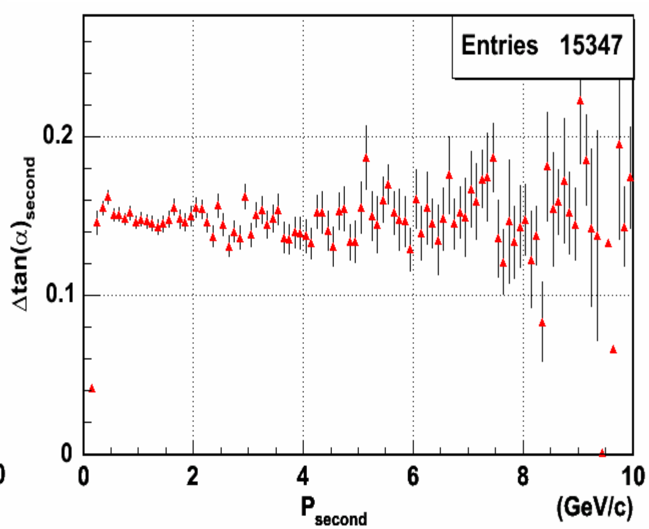
Accuracy for secondary tracks at 1st hit



σP = **0.77** (%)



$\sigma\beta$ = **0.14** (mrad)

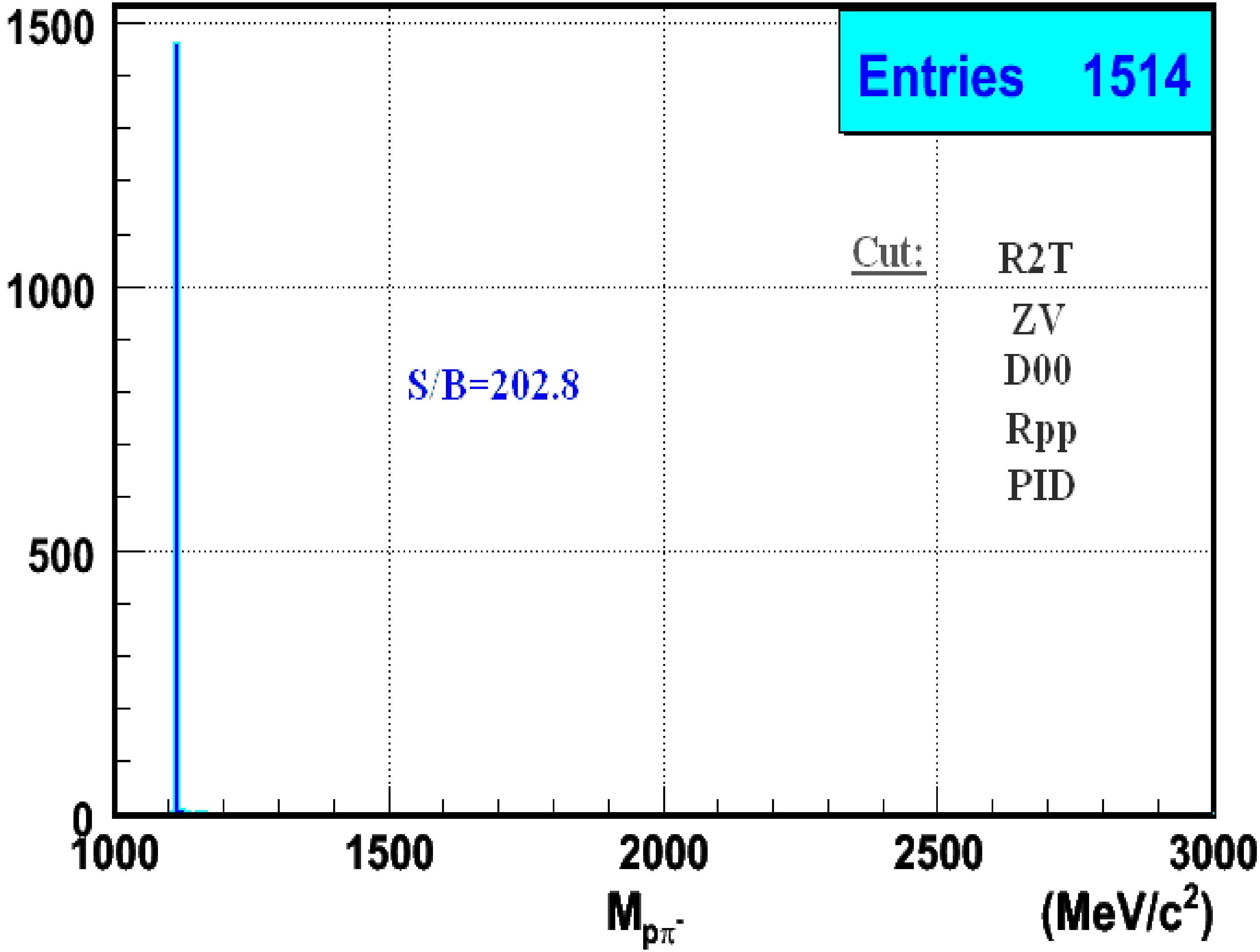


$\sigma\tan(\alpha)$ = **0.15**

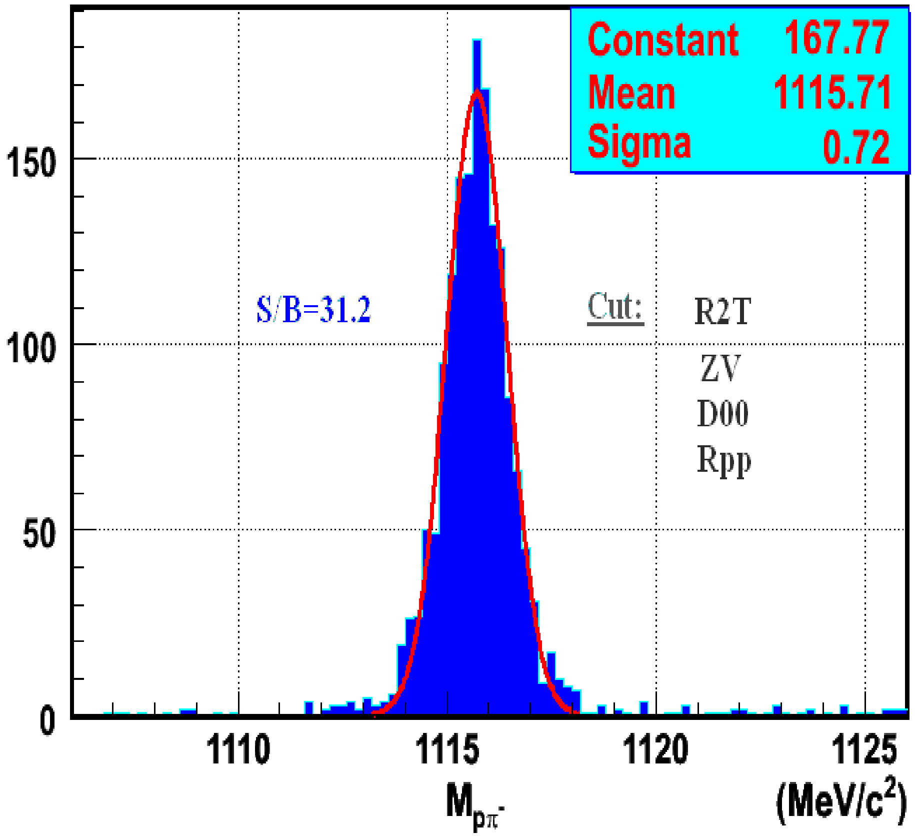
All "+/-" pairs of secondary tracks are tested !

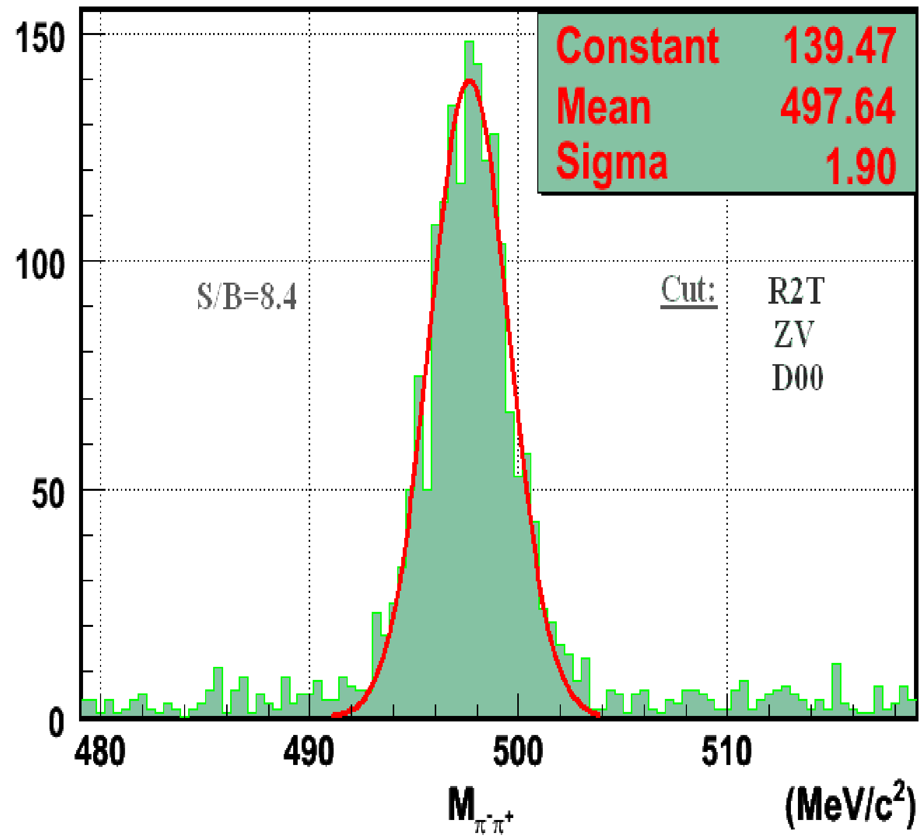
Cuts

- **R2T**: $R_{2Tr} < R_{2Tr}^{lim}$ - min distance between 2 tracks
- **ZV** : $Z_V > Z_V^{lim}$ - Z position of pair
- **D00**: $D_{00} < D_{00}^{lim}$ - impact parameter for pair (for primary V^0)
- **Rpp**: $R_{pp} > R_{pp}^{lim}$, where $R_{pp} = P^+/P^-$ (only for Λ^0 !)
- **PID:** Taken from GEANT

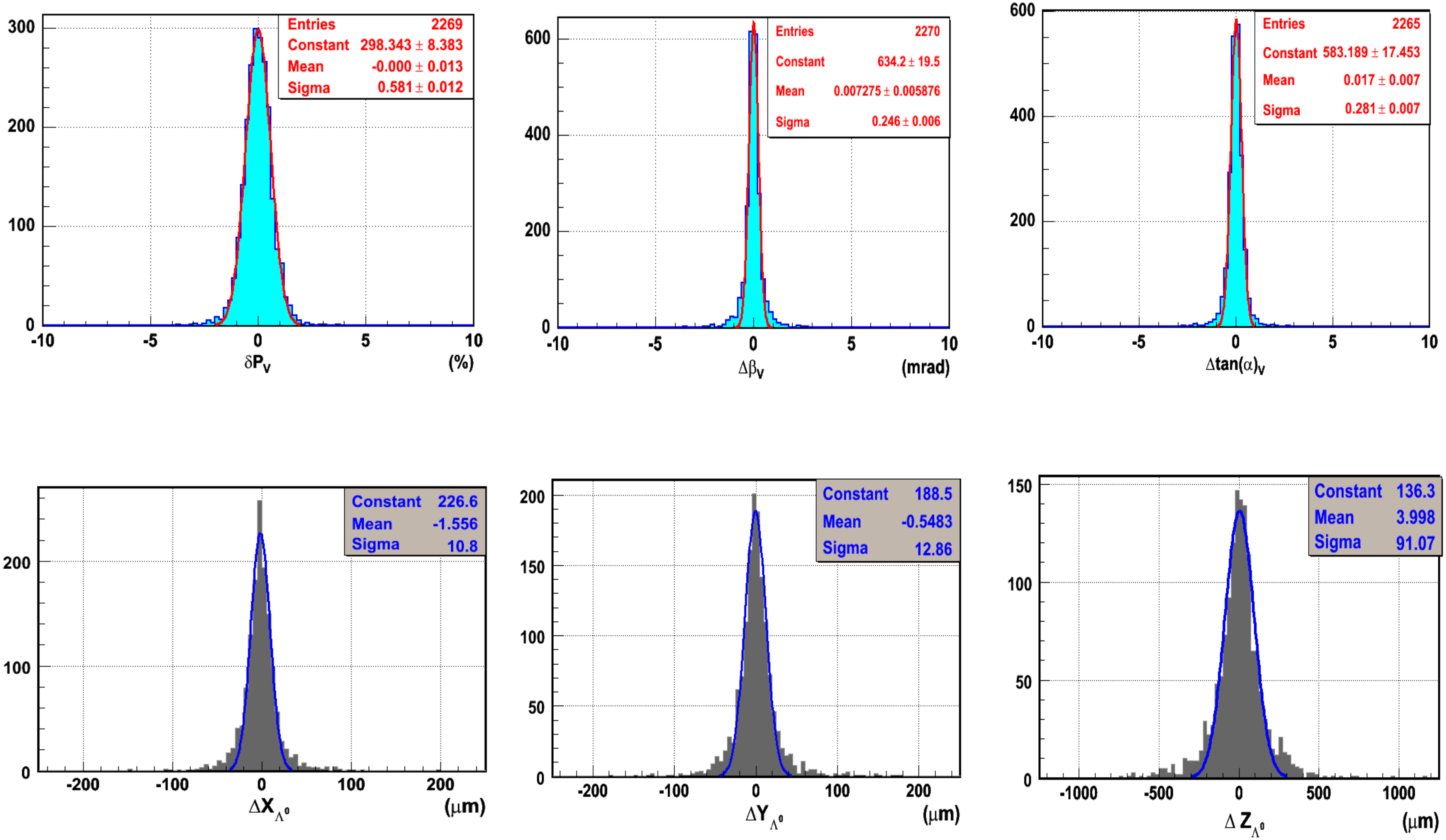


Entries 1514
Cut: R2T
ZV
D00
Rpp
PID
S/B=202.8
1500
1000
500
0
1000
1500
2000
2500
3000
$M_{p\pi^-}$ (MeV/c^2)


Constant 167.77
Mean 1115.71
Sigma 0.72
S/B=31.2
Cut:
R2T
ZV
D00
Rpp
150
100
50
0
1110
1115
1120
1125
$M_{p\pi^-}$
(MeV/c^2)


Constant 139.47
Mean 497.64
Sigma 1.90
S/B=8.4
Cut:
R2T
ZV
D00
150
100
50
0
480
490
500
510
$M_{\pi^-\pi^+}$
(MeV/c^2)

Accuracy of Λ^0 parameters



V0-Finder: results and 8STS vs 7STS

	Λ^0 7STS	Λ^0 8STS	K^0 7STS	K^0 8STS
S/B*	**31.2**	**28.1**	**8.4**	**7.1**
σP_V (%)	**0.58**	**1.10**		
$\sigma \beta_V$ (mrad)	**0.24**	**0.36**		
$\sigma \tan(\alpha_V)$	**0.28**	**0.33**		
σX_V^0 (μm)	**10.8**	**16.0**	**9.4**	**15.1**
σY_V^0 (μm)	**12.8**	**17.1**	**9.5**	**14.6**
σZ_V^0 (μm)	**91.1**	**154.3**	**52.8**	**98.8**
σM_V^0 MeV/c²)	**0.72**	**1.16**	**1.90**	**3.13**

* **Cuts:** R2T, ZV, D00 & Rpp for **Λ0**; R2T, ZV, D00 for **K0**

d_{tot} (Si)$_{7STS}$=1200μm **d_{tot}(Si)$_{8STS}$=3500μm**

Resolution ~ $\sqrt{}$ d_{tot} (Si)

2.3 V^0- Fitter (for 8STS)

2 versions of **V^0-Fitter** were tuned and tested

1. Simplified V^0-Fit :

   input: parameters of 2 tracks at secondary vertex

   +

   $(M_{pair} - M_V)^2 \rightarrow \min$

2. Full V^0-Fit

   input: hits of 2 tracks &

   secondary vertex as additional hit

   ↓

   simultaneous ASME fit for hits

   +

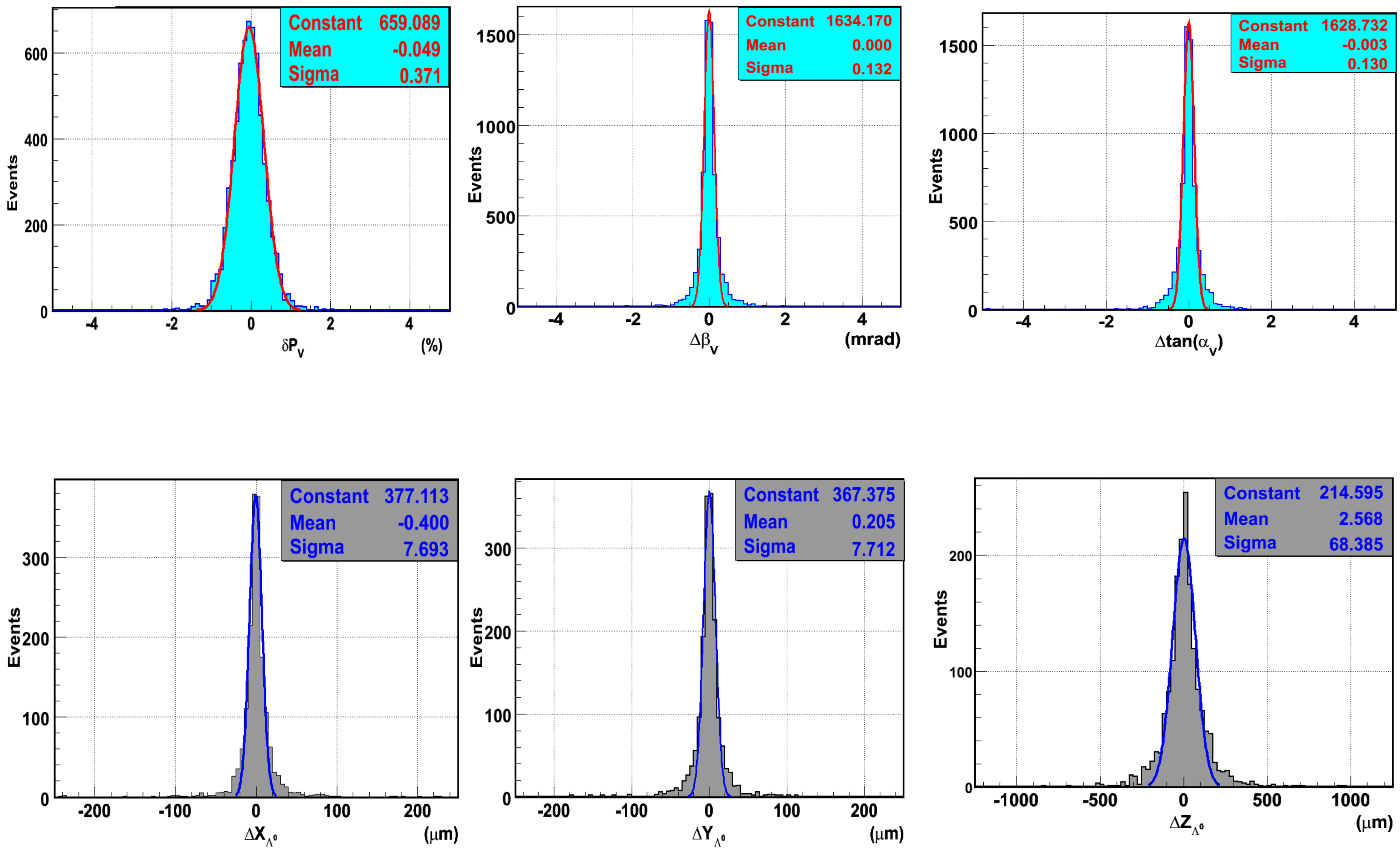
   $(M_{pair} - M_V)^2 \rightarrow 0$ (1^{st} constraint)

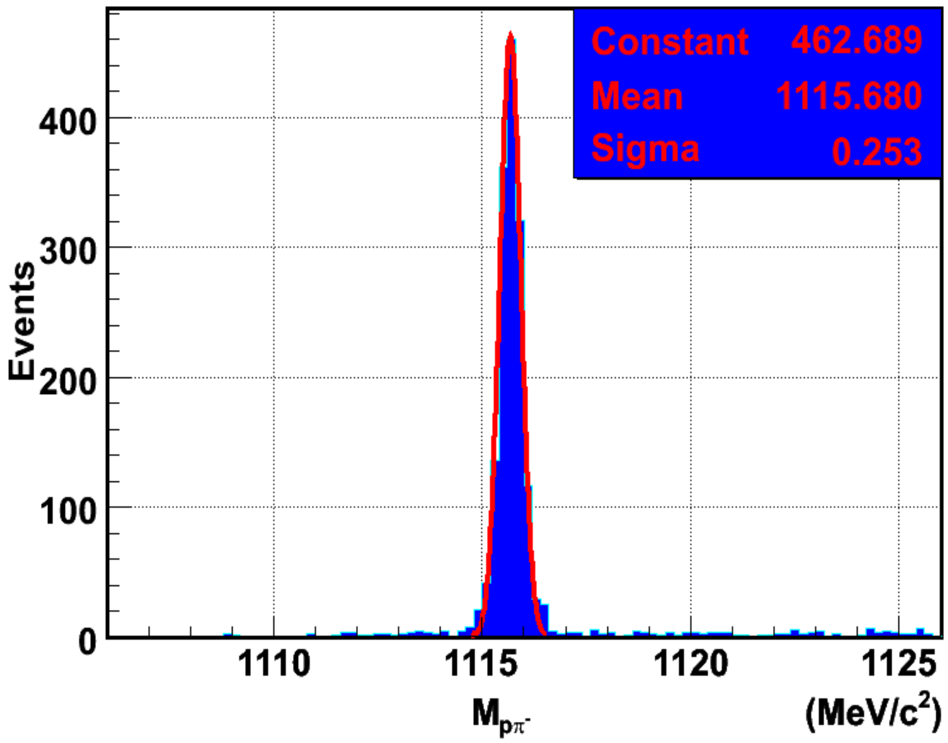
   +

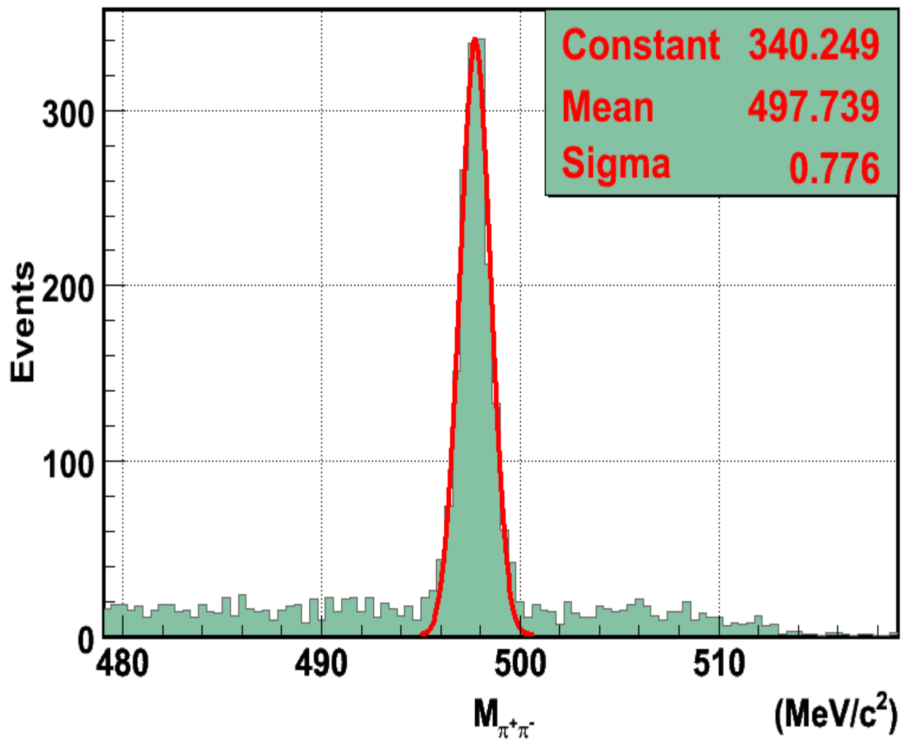
   $D_{00}^V \rightarrow 0$ (impact parameter for V^0 - 2^{nd} constraint)

   Needs 3 iteration to get minimum

Accuracy of Λ^0 parameters (Full V^0-Fit)







M_{+-} are not δ-function because of the simple method of minimization.

Necessary use Lagrange method (or some another) for the last iteration.

↓

Space resolution → 1.5 – 2 times better (estimation)

V^0-Finder vs V^0-Fitter (8STS)

	Λ^0 - Finder	Λ^0-Fitter	K^0 - Finder	K^0 - Fitter
S/B*	**28.1**	**32.8**	**7.1**	**8.8**
σP_V (%)	**1.10**	**0.37**		
$\sigma \beta_V$ mrad)	**0.36**	**0.13**		
$\sigma \tan(\alpha_V)$	**0.33**	**0.13**		
σX_V^0 (μm)	**16.0**	**7.7**	**15.1**	**6.3**
σY_V^0 (μm)	**17.1**	**7.7**	**14.6**	**6.7**
σZ_V^0 (μm)	**154.3**	**68.4**	**98.8**	**53.5**
σM_V^0 (MeV/c²)	**1.16**	**0.25**	**3.13**	**0.78**

* **Cuts:** R2T, ZV, D00 & Rpp for Λ^0; R2T, ZV, D00 for K^0

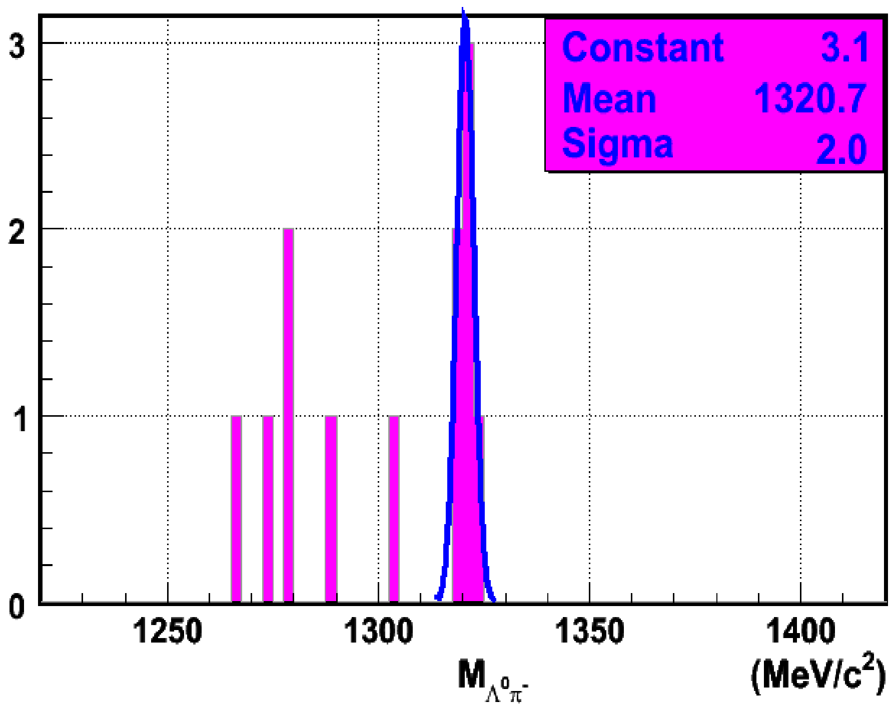
3. Ξ^-/Ω^- reconstruction

$$\Xi^-(\Omega^-) \rightarrow \pi^- (K^-) + \Lambda^0 \rightarrow \pi^- (K^-) + p + \pi^-$$

"+/-" pairs → **Cuts for Λ^0 :** R2T, ZV, anti-D00 (**to select secondary Λ^0**), Rpp

&

Λ^0 "–" pairs → **Cuts for $\Xi^-(\Omega^-)$:** R2Th, D00h – impact parameter



6 reference Ξ^-, all 6 are found (in the pick).
no Ω^- were found
(due to a small statistics?)

Next step → **$\Xi^-(\Omega^-)$ – Fit:** the same strategy as for Λ^0 Fit

4. Conclusion

ASME method

- takes into account energy loss and multiple scattering
- provides a good momentum resolution
- permits to calculate both track parameters and errors
- sufficient count rate

Full Fit (single step procedure, further development of ASME for HADES) seems to be more effective for track reconstruction, especially at large multiplicities.

The presented algorithm of V^0-Finder gives good accuracies both for kinematical parameters and vertex position of V^0's and provides effective V^0 reconstruction.

The algorithm of V^0-Fitter permits to get an essentially better resolutions both for V^0 kinematical parameters and vertex position.

V^0-Finder/Fitter algorithm can be implemented for Ξ^-/Ω^- reconstruction.

ASME method was successfully used for particle reconstruction on HADES and CBM setup.

ASME method can be used for particle reconstruction in another coordinate detectors such as MPD (project NICA)