Wavelet Analysis of the Measurements $R_{e^+e^-}$ to Determine Parameters of the Higher Vector States of Charmonium

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Data from several collaborations (Crystal Ball, BES and Mark I) on the total cross section $e^+e^- \to$ hadrons in the resonance region 3.5-5 GeV are examined. All the knowledge of charmonium resonances $\psi(1^{--})$ above the $D\bar{D}$ threshold at 3.73 GeV comes from the measurement of $R_{e^+e^-}$. The first part of our analysis uses wavelet methodology to provide more reliable parameters of these relatively wide states. In the second part we use the wavelet method to compare the QCD prediction of with experimental data over the energy range 0-8.0 GeV.

We use WA to reduce statistical noise in experimental data in order to clear out resonance contribution.

Wavelet analysis of function $f(t)$ - in our case it's *R(t)*

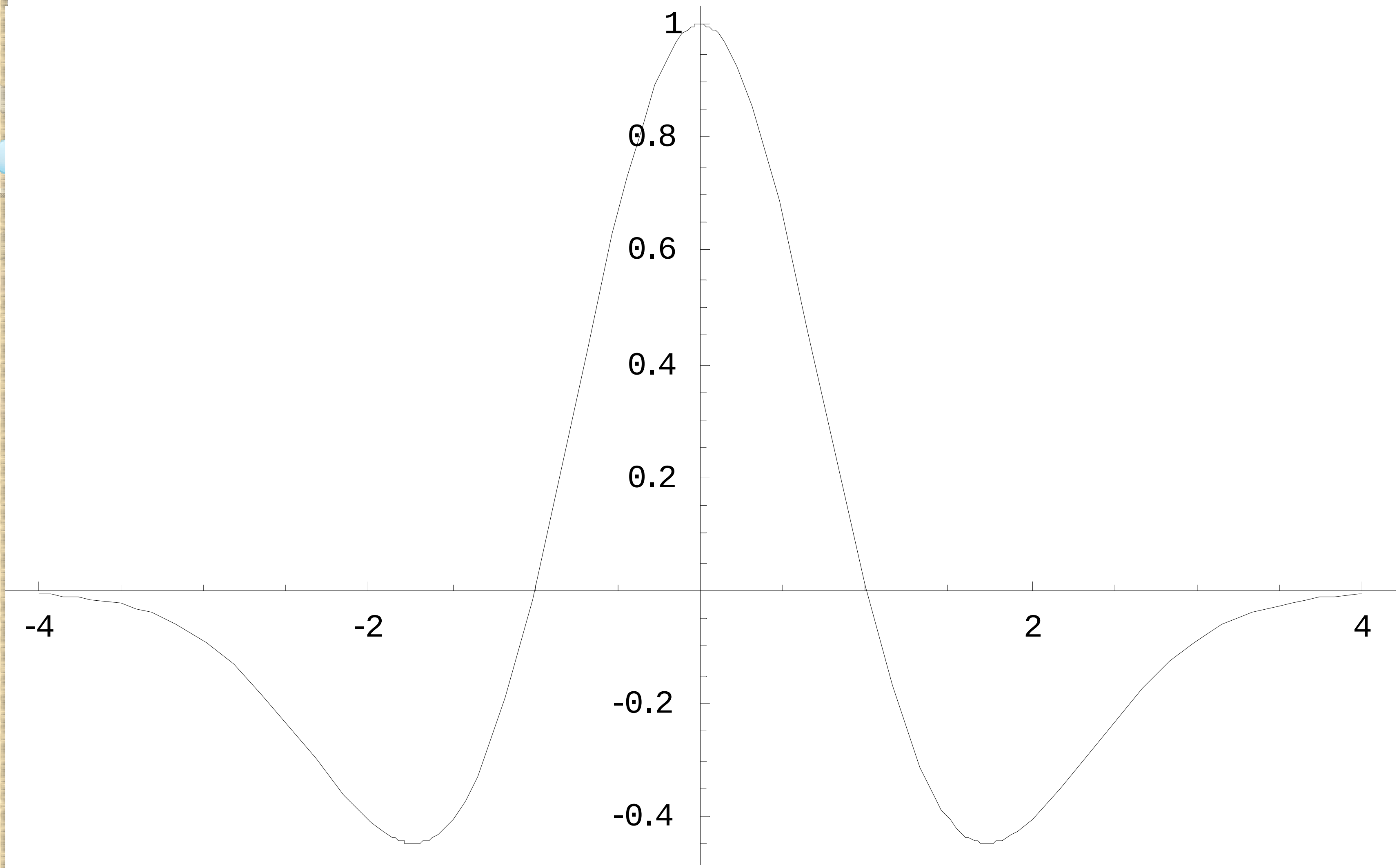
$$w(a,t) = C_{\psi}^{-1/2} a^{-1/2} \int_{-\infty}^{+\infty} \psi^{*}\left(\frac{t'-t}{a}\right) f(t')\,dt'$$

$a^{-1/2}$ **is frequency scale parameter (dilatations). High frequency wavelets are narrow, low frequency are broader. Contrary to Fourier analysis $w(a,t)$ depends both on *t* and frequency a *providing an optimal compromise with uncertainty principal.***

$C_{\psi}^{-1/2}$ **Normalization coefficient depends on the wavelet $\psi(t)$**

Popular wavelet: "Mexican hat"

$$\psi(t) = (1 - t^2) e^{-t^2/2}$$


1
0.8
0.6
0.4
0.2
-4
-2
2
4
-0.2
-0.4

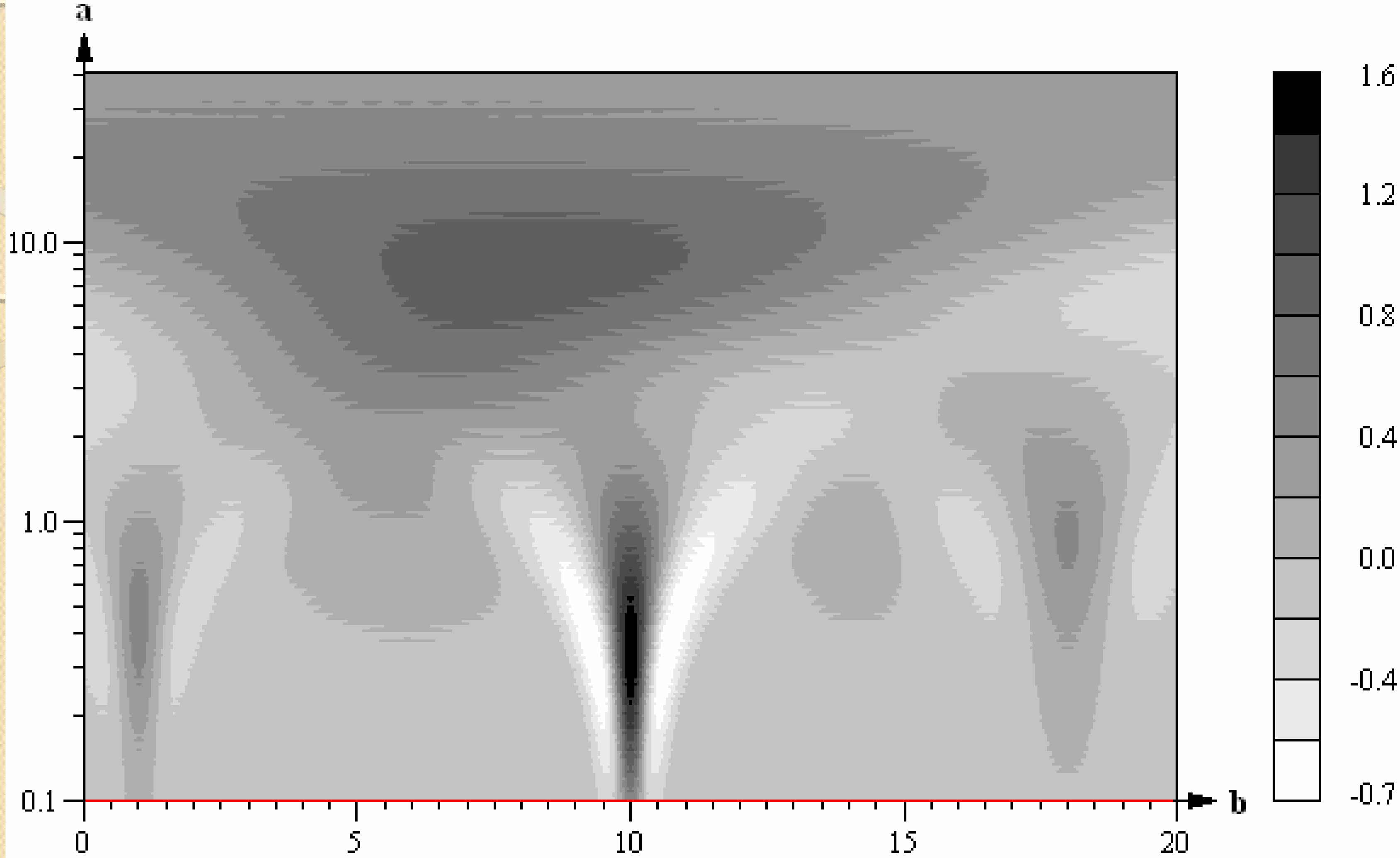
The example similar to data in physics of resonances:

$$f(t) = e^{-5(t-1)^2} + 2e^{-(t-6)^2/20} + 3e^{-10(t-10)^2} + e^{-(t-14)^2/10} + e^{-(t-18)^2}$$

Then $f(t)$ was made discrete and high-frequency noise randomly added at each point.

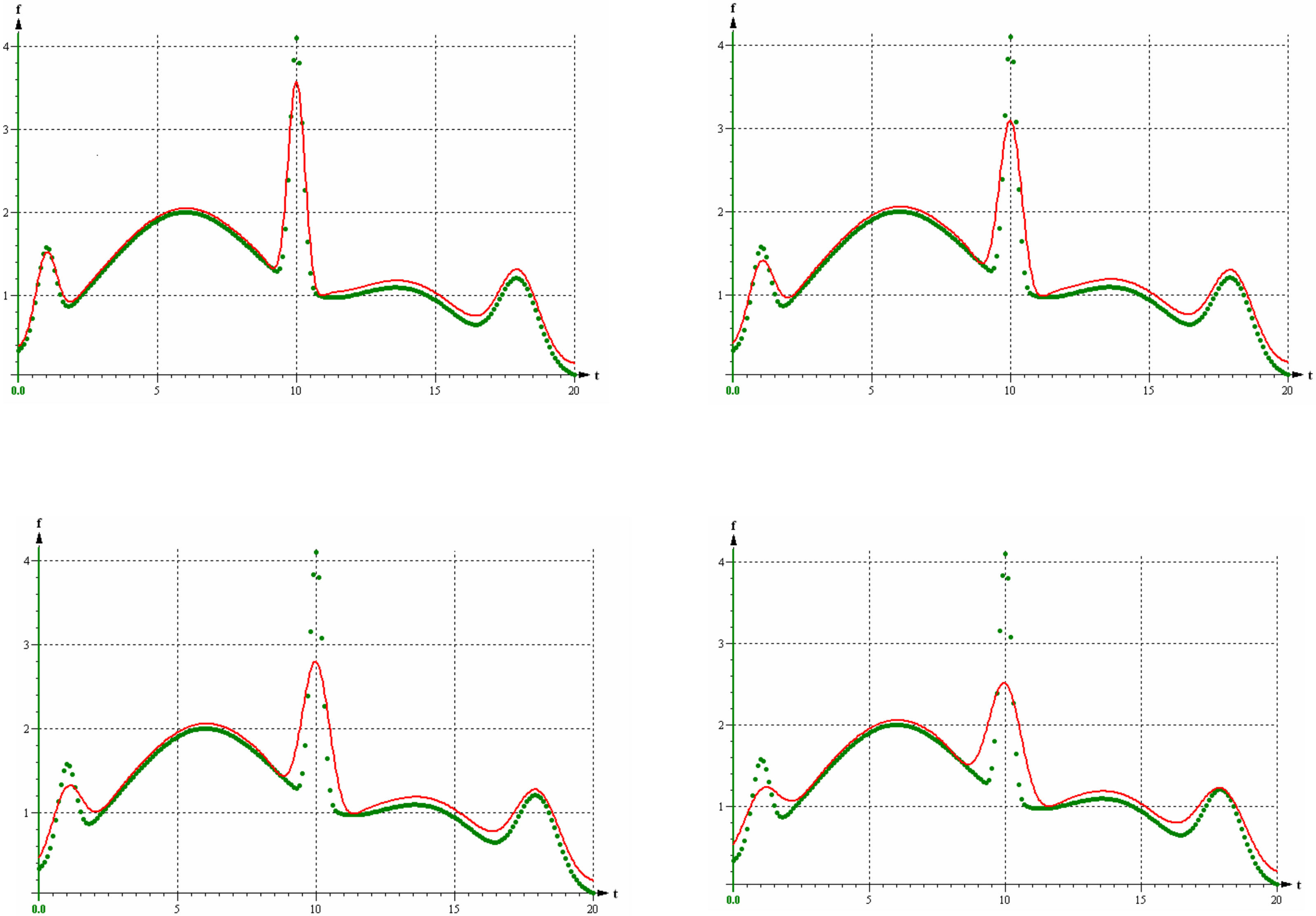
Fourier reconstruction of such a “data” gives a smooth curve but with false locations of the peaks, their widths, heights and, also false maxima.

The situation is quite different with the WA: smooth restored signal has correct structures parameters.



$$f(t) = C_{\psi}^{-1/2} \int_{-\infty}^{+\infty} \int_{0}^{+\infty} \psi\left(\frac{t-t'}{a}\right) w(a,t') \frac{dt'da}{a^{5/2}}$$

inversed transformation - restored signal (data)



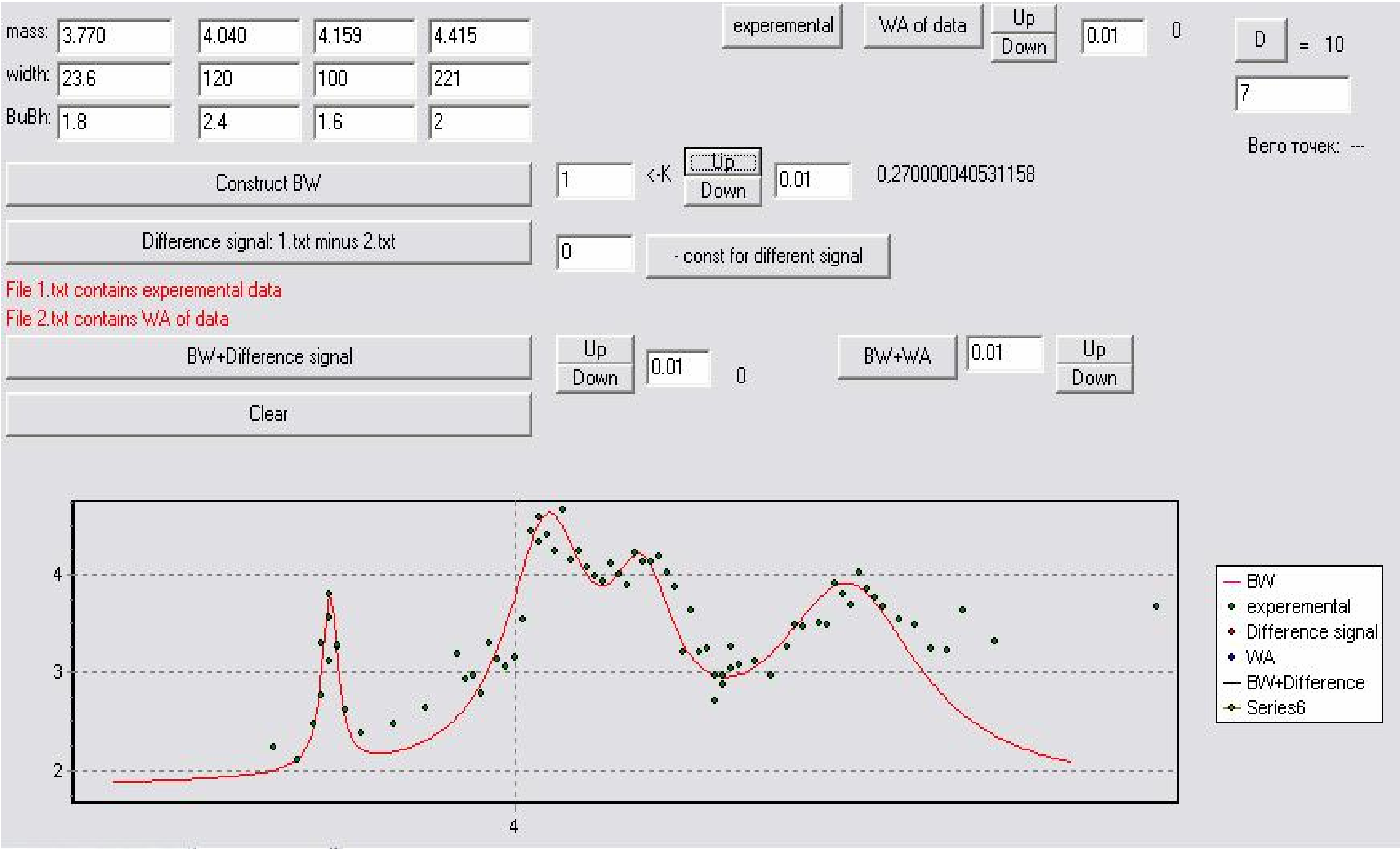
Restored $f(t)$ for different values of a

Another approach: The smeared ratio is defined (Weinberg et al) by

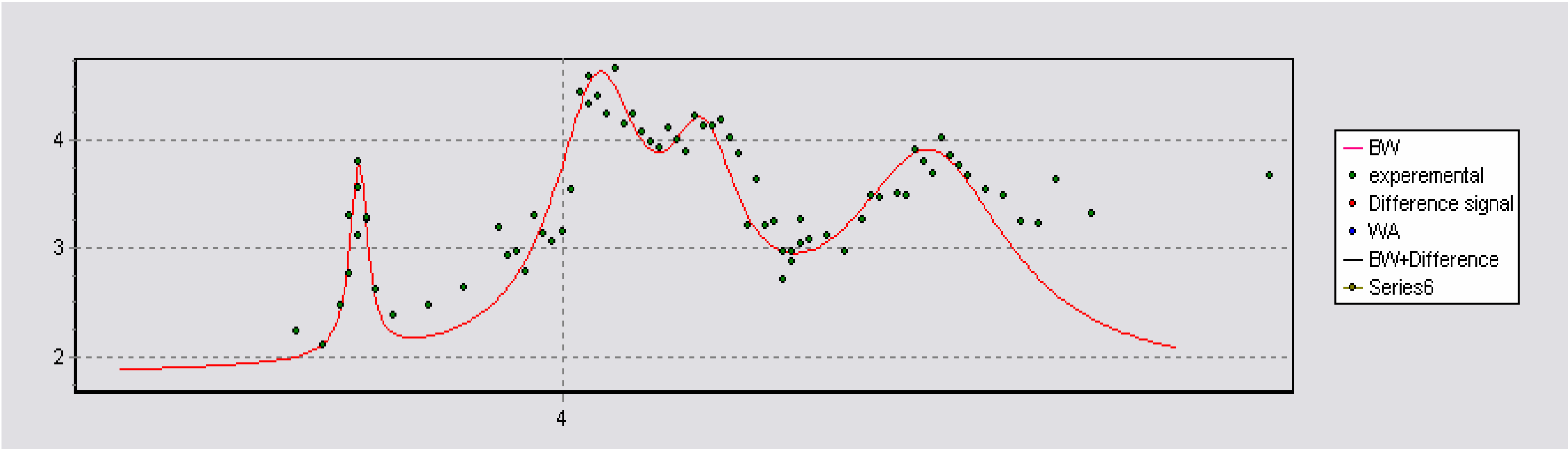
$$R(s,\Delta)=\frac{\Delta}{\pi}\int_0^\infty \frac{R(s')ds'}{(s-s')^2+\Delta^2}$$

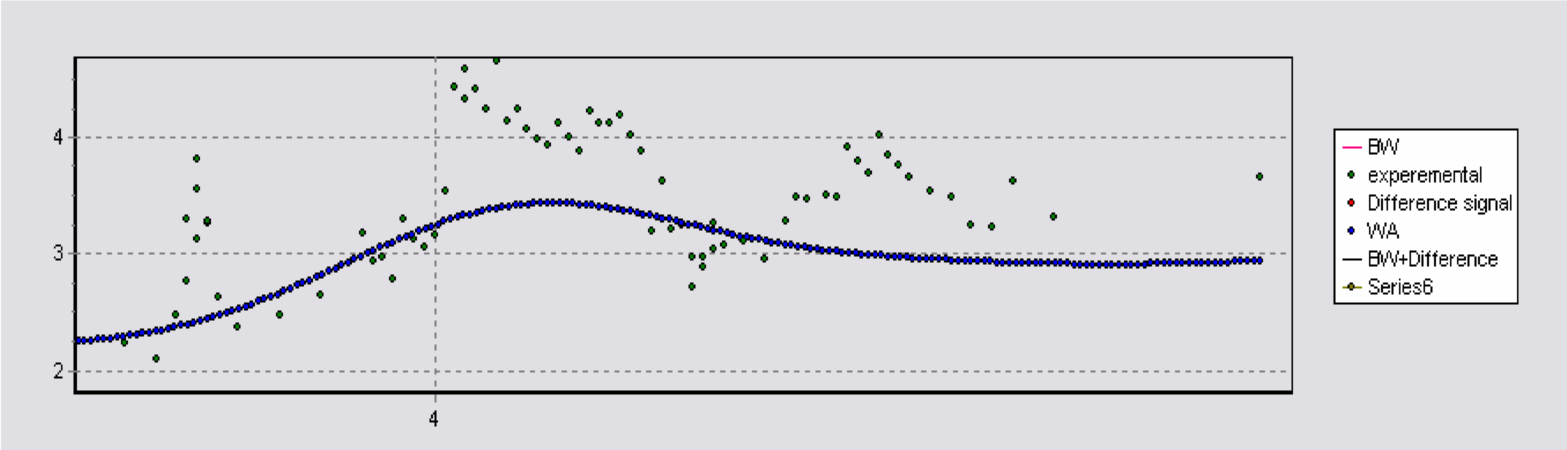
The integral averages out both the quark-gluon thresholds in the theoretical (perturbation QCD) cross-section and resonances.
For different values of Δ they got smooth $R(s,\Delta)$ to compare to QCD

Screenshot from program (BW)

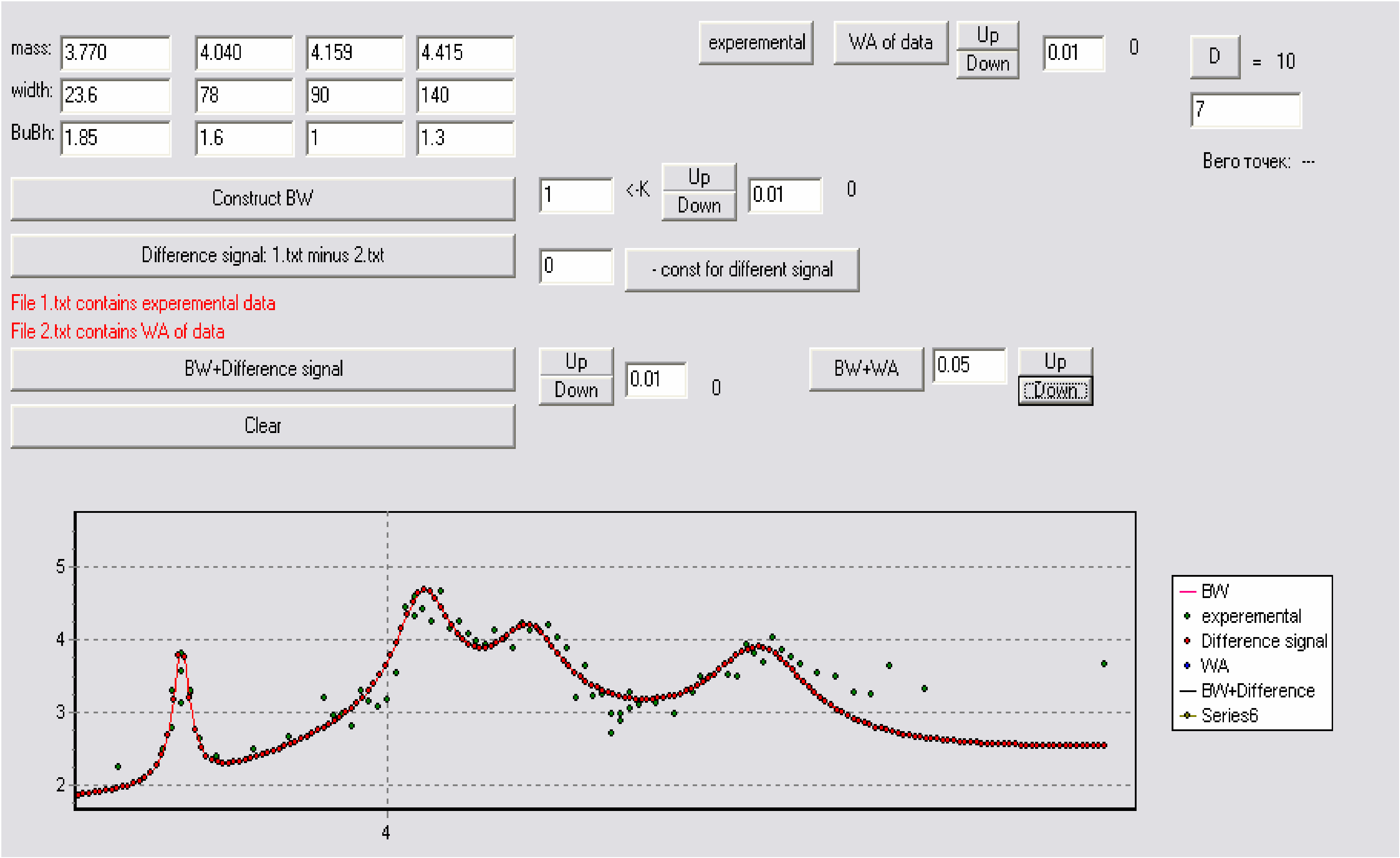


a) BW;   b) data smearing (WA)

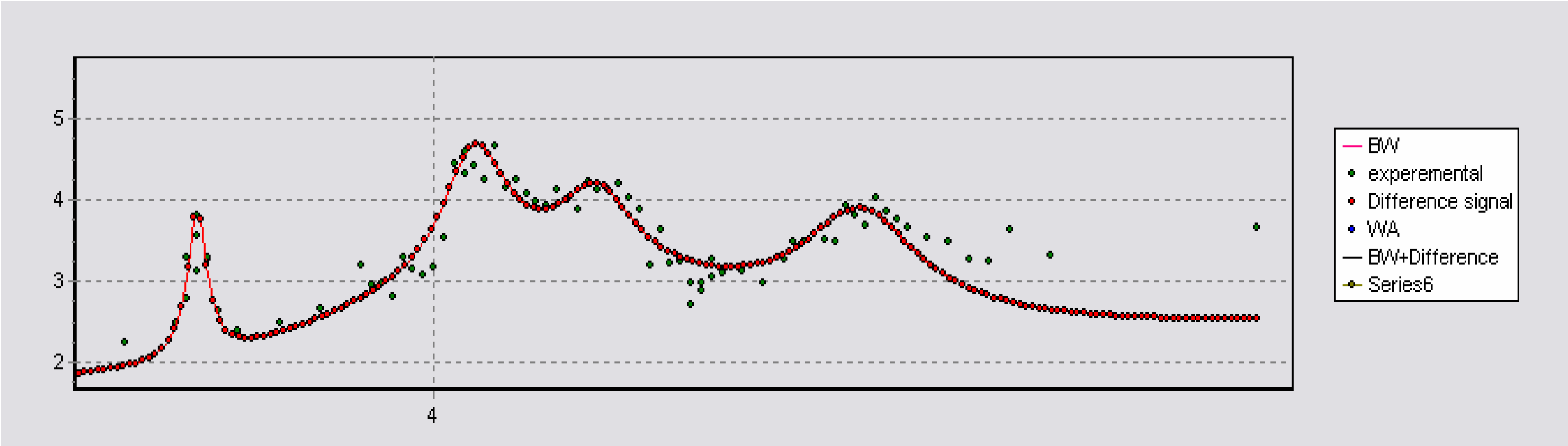




Screenshot from program (BW+WA)



BW and wavelet background



The e^+e^- annihilation experimental data exhibit several vector states above 4 GeV. The properties of these states and even their number are not well

established. The main difficulties encountered are large statistical errors

in the data and disagreement between data sets obtained by different

collaborations. Due to the excellent scaling property of wavelet

methodology one can consider the experimental data with varying resolution, allowing the separation of resonances from noise and background, and from each other.

Wavelet analysis (WA) of experimental data yields useful starting conditions for a description of the ψ states with the Breit-Wigner (BW) method.

The procedure based on both WA and BW approaches enables us to obtain a more reliable measure of the parameters of the ψ states. $R_{e^+e^-}$ data over the range 0.0-8.0 GeV have a wide range of structures and large statistical errors, making a direct comparison with QCD very difficult. However, a meaningful comparison can be made provided that some kind of "smearing" procedure is used, similar to the work described in reference [1], to smooth out any rapid variations in $R_{e^+e^-}$. A wavelet analysis can be used to achieve this smearing effect. We compare the WA of theoretical (perturbation) QCD and experimental data. The wavelet reconstruction of the R experimental data preserves its main features, but with damped statistical errors and threshold singularities [2], that makes such a comparison possible. The WA of QCD perturbation theory is nicely compatible with the WA of experimental data sets.