Where is the $0^{\pm+}$ glueball?

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Content

- Preliminaries.
- Underlying reasons and realisation
- The meson mass-relations - from octets via nonets to a kind of decuplets.
- Discussion, prospects and conclusion

Preliminaries

We concentrate on the mass region $1.3 \div 1.7$ GeV occupied by the spin-zero $0^{\pm+}$ mesons. In each group of the positive (or negative) parity mesons there are three isoscalar mesons with similar masses which, in the presence of the nearly lying isotriplet and isodublet ones, suggest on the overpopulated nonet where a possible glueball is hidden within structures of the three isoscalar states.

Whether this idea is right or wrong one should to deduce from data on the reactions creating them as well as from the relations between branching ratios of their decays. With this in mind, we present results of a simple approach enabling to discuss an acute problem of the existence and properties of glueballs with quantum numbers $I^G J^{PC} = 0^{\pm}0^{\pm+}$.

Both the scalar and pseudoscalar sector is problematic up to now in spite of very large number of works devoted to problems connected with reliable experimental identification of glueballs and the theoretical description of their properties.In this respect we would especially mention very recent note contained in the review section of the Particle Data Properties and clarifying the situation with possible low-lying ($\sim 1.5 GeV$) pseudoscalar state extra to the near pseudoscalar nonet of meson states which could be the pseudoscalar glueball and several reviews devoted to long-lasting searches of scalar glueball.

Underlying relations

We define the 3×3 mass-matrix $\widehat{V(i)}$ as acting on the basis vectors N, S, G to transform them into one of three vectors of the physical meson states, either $\eta(i)$, or $f_0(i)$

$$\left(\begin{array}{c} \eta(i) \end{array} \right) = \hat{V}(i) \cdot \left(\begin{array}{c} N \\ S \\ G \end{array} \right) \tag{1}$$

where

$$N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad S = s\bar{s},$$

and G is the glueball. The matrix for the scalar mesons is obtained for the $f_0(i)$ replacing $\eta(i)$. As an example illustrating the relations between the amplitudes of the physical processes and amplitudes of the mixed configuration state vector of a meson we choose to consider relations between the two-photon widths of the η_i-resonances. We parameterize each $\Gamma_{\gamma\gamma}(\eta_i)$ in a simple and transparent manner as

$$\Gamma_{\gamma\gamma}(\eta_i) = const \cdot | \sum_{\alpha=N,S,\Gamma} \langle 2\gamma|\alpha\rangle \cdot a_i(\alpha)|^2 \cdot m(\eta_i)^3,$$
$$\langle 2\gamma|N\rangle : \langle 2\gamma|S\rangle : \langle 2\gamma|G\rangle = \frac{5}{9\sqrt{2}} : \frac{1}{9} : 0, \quad (2)$$

where $a_i(\alpha)$ is the amplitude of the α-configuration in the mixing-matrix $V(i,\alpha)$. The explicit form of $\widehat{V}$ is given later together with the numerical estimates. We present also some useful relations for the amplitudes of the radiative decays of the vector charmonium $J/\Psi \rightarrow \gamma + \eta_i$ by applying the mixing-matrix $\widehat{V}_D$ and following from it the flavor structure of the η_i final state resonances. We use the parametrization of needed amplitudes by taking for granted the two-step mechanism of the reaction $J/\Psi \rightarrow \gamma + 2g^* \rightarrow \gamma + \eta_i$, where g^* denote the virtual gluon, the flavor-independent quark-gluon interaction and afterwards, by analogy with the amplitude of $2\gamma\eta_i$-transition, we write down the relation of the width of the $J/\Psi \to \gamma + 2g^* \to \gamma + \eta_i$-transition in the form

$$\epsilon(f_i)(\frac{\Gamma(J/\Psi \to \gamma\eta_i)}{k_i^3})^{1/2} = \qquad (3)$$

$$f_i = g_Q([a_i(N) + a_i(S)]) + g_G a_i(G), \qquad (4)$$

where $\epsilon(f_i) = \pm 1$ for $f_i \gtrless 0$, $k_i = (M^2(J/\Psi) - m^2(\eta_i))/(2M(J/\Psi))$ is the photon momentum in the decay $J/\Psi \to \gamma + \eta_i$, the effective constants $g_Q = g_N = g_S$ and g_G are assumed to absorb all the dynamics ingredients of the respective field-theoretical transitions

$$J/\Psi \to c\bar{c} \to \gamma + g^* g^* \to \gamma + N(S,G) \to \gamma + \eta_i,$$

the $a_i(N,S,G)$ being the elements of the $\widehat{V}_D$ mixing-matrix defining the flavor structure of the $\eta(i)$-resonances. The necessity of a proper definition of the signs results in the appearance of several relations including $C_i \equiv \sqrt{(\Gamma(J/\Psi \to \gamma\eta_i)/(k_i^3)}$, $i = 1,2,3$, each depending on the specific relation between g_Q and g_G

We consider the mass-matrices $\widehat{V(i)}$ taking into account explicitly the different appearance of the two types of gluon effects in mixing states of the differing flavor. In a certain sense, we follow the way proposed in old works by Isgur to connect the strong "non-ideality" of the $SU(3)$-singlet-octet mixing angle in the lowest pseudoscalar and scalar meson nonet with the overwhelmingly strong, as compared with the respective term in the vector or tensor meson nonets, annihilation term in the mass-matrix, inducing the non-diagonal $q\bar{q} \leftrightarrow s\bar{s}$ transitions.

Underlying relations.Mass formulas.

We remind, that . the celebrated Gell-Mann–Okubo (GMO) formula

$$3m_{f_8}^2 = 4m_{K_0^*}^2 - m_{a_0}^2$$

is following as the mass sum rule after exclusion of parameters introduced into the general mass term of phenomenological meson lagrangian

$$M^2 \cdot 2Tr(V_8V_8) - \mu^2 \cdot Tr(V_8V_8\lambda_8)$$

.

Okubo proposed to replace $V_8 \rightarrow V_9$ in the GMO mass operator and drop the term proportional to $Tr(V_9)$. The well-known ”ideal mixing ” mass relations

$$m^2(\rho) = m^2(\omega)$$

,

$$2m^2(K^*) - m^2(\rho) = m^2(\phi)$$

fulfilled for the vector and reasonably well for tensor nonet, but poor for the pseudoscalar one. We indicate also the hierarchy of meson masses, following from effective lagrangian GMO

$$m^2(s\bar{s}) \geq m^2(q\bar{s}) \geq m^2(q\bar{q})$$

. The idea to relate the apparently specific situation for the pseudoscalar meson sector with additional strong annihilation mechanism transforming the quark field combinations into each other was put forward phenomenologically by Isgur, and interpreted now as mediated by short-range fluctuations in the quark-gluon vacuum.

We follow this idea in the further generalized form via the introducing the ”bare” scalar or pseudoscalar glueball mass and non-diagonal glueball-quarkonium transition-mass into the spin-zero meson mass-matrices. Hence, in the $N = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), S = s\bar{s}$ basis our symmetric mass-matrix acquires the following form

$$\widehat{M}^2 = \left(\begin{array}{ccc} M_N^2 + A_Q & \sqrt{2}A_G & \sqrt{2}rA_Q \\ \sqrt{2}A_G & M_G^2 & rA_G \\ \sqrt{2}rA_Q & rA_G & M_S^2 + r^2A_Q \end{array} \right) \quad (4)$$

After reducing it to the diagonal form we should get the matrix of the eigenvalues $\widehat{M}^2{}_{ph}$:

$$\widehat{M}^2{}_{ph} = \left(\begin{array}{ccc} M_{f_0}^2(1) & 0 & 0 \\ 0 & M_{f_0}^2(2) & 0 \\ 0 & 0 & M_{f_0}^2(3) \end{array} \right)$$

and matrix $\widehat{V}(i)$ of eigenvectors in chosen basis

$$\widehat{V} = \left(\begin{array}{ccc} a_{11}(1) & a_{12}(1) & a_{13}(1) \\ a_{21}(2) & a_{22}(2) & a_{23}(2) \\ a_{31}(3) & a_{32}(3) & a_{33}(3) \end{array} \right)$$

The isoscalar ”sub-multiplets” in pseudoscalar and scalar- sector

We start the treating of mass relations with the higher-mass, scalar 0^{++}-sector:

$$M_{a_0} = 1474 \pm 19, M_{K^*{}_0} = 1425 \pm 50$$

$$M_{f_0}(1) = 1370 \pm 50, M_{f_0}(2) = 1505 \pm 6,$$

$$M_{f_0}(3) = 1724 \pm 7$$

where all values are in MeV.

We define the ”bare” mass values M_N and M_S devoid of the strong annihilation contributions via

$$M_N = M_{a_0}, {M_S}^2 = 2{M_{K^*{}_0}}^2 - {M_{a_0}}^2 \qquad (2)$$

(A short digression: the second relation is alike of the S-wave vector quarkonia, but we would like to note the opposite mass hierarchy sequence

$$M^2(s\bar{s}) \leq M^2(q\bar{s}) \leq M^2(q\bar{q})$$

which follows if one changes the sign of the parameter μ^2 in the GMO mass operator and serves as demonstration of the significant flavor dependence in the spin-dependent (the spin-orbit, etc.) mass terms of the P-wave scalar mesons).

The (presumably) intermediate gluon-matter originated terms A_Q and A_G and the "bare" gluon mass M_G are unknown variables which have to be found by solution of the system of three equations representing the equalities of three invariants of the diagonalizing process: the trace, the determinant and the sum of main minors of the matrices under consideration. The small correction factor $r = \sqrt{M_S/M_N}$ is introduced as reflecting the relativistic normalization of the meson state vectors. Successively excluding unknown variables A_Q and A_G in favor of M_G, we solve numerically the last equation by varying remaining unknown M_G under constraint ${A_G}^2 \geq 0$. There are 2 solutions for M_G and $A_G = 0$ and none for $A_G > 0$. We have chosen as physically acceptable the value of the decoupled physical glueball mass

$M_G(ph) \simeq 1730\ MeV$ vis-a-vis $M_{f_0}(3) = 1724 \pm 7\ MeV$

The state vectors of the $f_0(1506)$ and $f_0(1370)$ are obtained by the diagonalizing the rest 2×2 matrix:

$$|f_0(1506)> = 0.868|N> +(-)0.496|S>$$

$$|f_0(1370)> = -(+)0.496|N> +0.868|S>$$

The choice of signs remains to be done on the physics ground.

The case for pseudoscalar glueball

The same procedure is applied to the mass-matrix of the pseudoscalar mesons $\eta(i)$,i=1,2,3:

$$M_{\eta(1)} = 1294 \pm 4, M_{\eta(2)} = 1409.8 \pm 2.5$$

$$M_{\eta(3)} = 1476 \pm 4$$

all values are in MeV.
The masses of the isovector and isospinor radial excited states are either unknown or known very poorly. For illustrative purposes we take for the lowest mass of the radial excited state of the pion $\pi(2S) = 1345 \pm 8 \pm 10$ obtained by StPt Group(Schegelsky,2006) from analysis of L3(CERN) data on $\gamma\gamma \to 3\pi$.It is in accord also with E-852 Collab. data on $\pi N \to 3\pi N$ (BNL,Chung,2002), while the PDG average gives 1300 ± 100 MeV.

On the mass value of the $K(2S)$ very old experiments give $M(K(2S)) \sim 1460$ (CERN,1981) and ~ 1400 (SLAC,1976). In our estimates we take $M(K(2S)) \simeq 1420 \ MeV$ that corresponds to maximal value of the $K(0^-)$-mass at which our system of equations have physically acceptable solution for the sought parameters:

$$M_G(0^{-+}) \simeq 1.37 \, GeV, A_Q(0^{-+}) \simeq .003 \, GeV^2,$$

$$A_G(0^{-+}) \simeq 0.122\ GeV^2$$

The state vectors of the η_i are represented as follows

$$|\eta_1(1295)> = 0.739|N> -0.653|G> +0.164|S>$$

$$|\eta_2(1405)> = 0.634|N> +0.591|G> -0.498|S>$$

$$|\eta_3(1474)> = 0.228|N> +0.472|G> +0.851|S>$$

Comparing the mixing-mass parameters of the scalar and pseudoscalar isoscalar mesons, one can noted the surprising low value of $A_Q(0^{-+})$ as compared with $A_Q(0^{++}) \simeq 0.021\ GeV^2$ and with a large value of the pseudoscalar glueball-quarkonium non-diagonal coupling A_G. This suggests the interesting noticeable effects of the glueball mixing with the ground state pseudoscalar mesons $\eta(547)$ and $\eta'(958)$.

On viability of the presented scheme: A few examples.

1. A large amount of the strange quarks in the $f_0(1370)$: $J/\Psi \to \phi\pi\pi$-decay cannot be fitted without excitation of the $f_0(1370)$-resonance unlike the reaction with ϕ replaced by ω (BEPS+BES-II)

2. Dominance of the pion(s) decay mode of $f_0(1506)$ - large amount of the non-strange $q\bar{q}$-quarks.

3.The hierarchy of the radiative decay modes:

$$BR(J/\Psi \to \gamma f_0(1370)) \leq BR(J/\Psi \to \gamma f_0(1506))$$

$$< BR(J/\Psi \rightarrow \gamma f_0(1710))$$

(BEPS+BES-II) $\leftrightarrow$ large amount of ”gluon-matter” in the $f_0(1710)$.

4. The evidence of inter-multiplet mixing via the pseudoscalar glueball between the η’s of the radial-excited and ground 0^{-+}-multiplets.

Much more exciting information from the upgraded $BEPS-II + BES-III, CLEO, Belle$, $COMPASS$&$CERN$,...