Confinement
Bethe-Salpeter equation
Meson spectrum

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Mesons = quark−antiquark bound states

- Hadronization scale $\approx$ confinement scale.
- Relativistic description.
- Quarks and gluons can not be described by plane waves in the confinement region.
- Behavior of quarks and gluons in the confinement region.
- Equation :
  - Ladder Bethe − Salpeter equation.
  - Small coupling constant $\alpha_{QED} \ll \alpha_{QCD} < 1$.
  - Calculation method.

Analytical confinement

$$L_x \phi(x) = 0 \qquad \Longrightarrow \qquad \phi(x) = 0$$

$$L_x \phi(x) = J(x) \qquad \Longrightarrow \qquad \phi(x) = \frac{1}{L} J(x) \neq 0$$

Example

$$e^{l^2 \partial^2} \phi(x) = 0 \qquad \Longrightarrow \qquad \phi(x) = 0$$

$$e^{l^2 \partial^2} \phi(x) = J(x) \qquad \Longrightarrow \qquad \phi(x) = e^{-l^2 \partial^2} J(x)$$

Confinement $\Longrightarrow$ vacuum fluctuations in space and time

QCD – vacuum gluon field

$$\mathcal{L} = -\frac{1}{8}\mathrm{Tr}\ \breve{G}^2_{\mu\nu} + \left(\bar{q}\left[\hat{p} + g\hat{A} - m\right] q\right)$$

$$\breve{G}_{\mu\nu}(x) = \partial_\nu \breve{A}_\mu - \partial_\mu \breve{A}_\nu + g[\breve{A}_\mu(x), \breve{A}_\nu(x)],$$

$$\breve{A}_\mu(x) \ \Rightarrow \ \breve{A}_\mu(x) + \breve{B}_\mu(x)$$

$$\breve{B}_\mu(x) = \Lambda^2 \breve{n} b_{\mu\nu} x_\nu, \qquad \breve{n} = n^a t^a, \qquad n^a n^a = 1$$

$$b_{\mu\nu} = -b_{\nu\mu}, \quad b_{\mu\rho} b_{\rho\nu} = -\delta_{\mu\nu}, \quad \epsilon_{\mu\nu\alpha\beta} b_{\alpha\beta} = \pm b_{\mu\nu}$$

$\breve{B}_\mu(x)$ - vacuum field

$\breve{B}_\mu(x)$ - vacuum gluon field

$$(\gamma_\mu(\partial_\mu - i\breve{B}_{\mu\nu}x_\nu) - m)q(x) = 0 \Longrightarrow q(x) = 0,$$

$$(\gamma_\mu(\partial_\mu - i\breve{B}_{\mu\nu}x_\nu) - m)S(x) = -\delta(x),$$

$$\tilde{S}_\pm(p) \sim \frac{1}{2\Lambda^2}\int_0^1 du\ e^{-u\frac{p^2}{2\Lambda^2}}\left(\frac{1-u}{1+u}\right)^{\frac{m^2}{4\Lambda^2}}\left\{i\hat{p} + m\frac{1\mp\gamma_5 u^2}{1-u^2}\right\}.$$

Self-dual homogeneous vacuum gluon field $B_\mu(x)$ realizes true QCD vacuum

QED

$$E_{vac}(\Lambda) = \frac{\Lambda^4}{12\pi^2}\left[\sum_F \ln\left(1+\frac{2\Lambda^2}{M_F^2}\right) - \sum_B \ln\left(1+\frac{2\Lambda^2}{m_B^2}\right)\right]$$

$$\Lambda_{min} = 0$$

Fundamental particles are fermions

QCD

$$E_{vac}(\Lambda) = \frac{\Lambda^4}{12\pi^2}\left[\sum_f \ln\left(1+\frac{2\Lambda^2}{M_f^2}\right) - \ln\left(\frac{2\Lambda^2}{\Lambda_{QCD}^2}\right)\right].$$

$$\Lambda_{min} > 0$$

Ladder Bethe – Salpeter equation for fermions

$$\Lambda(p^2)\mathcal{Y}(y) = g^2 \int dy' \; \mathcal{K}(y,y')\mathcal{Y}(y')$$

$$\mathcal{K}(p;y,y') = \sqrt{D(y)}\Pi(p;y-y')\sqrt{D(y')}$$

$$\begin{aligned}
\Pi(p;y-y') &= \int dx \; e^{ipx} \; \mathrm{Tr}\left[\Gamma S\left(x+\frac{y-y'}{2}\right)\Gamma S\left(\frac{y-y'}{2}-x\right)\right] \\
&\sim \int \frac{dk}{(2\pi)^4} \frac{e^{-ik(y-y')}[k^2C_1+C_2]}{\left(k^2+m^2-\frac{M^2}{4}\right)^2+M^2k_4^2}
\end{aligned}$$

$$\mathrm{Tr}\;\mathcal{K}^2 = \iint dydy' \; \mathcal{K}(y,y')\mathcal{K}(y',y) = \infty$$

$$
\begin{aligned}
\mathcal{K}(y,y') &= \mathcal{K}_0(y,y') + \mathcal{K}_I(y,y') \\
&= \sqrt{D(y)}\Pi_0(y-y')\sqrt{D(y')} + \sqrt{D(y)}\Pi_I(y-y')\sqrt{D(y')}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_0(y-y') &= \int \frac{dk}{(2\pi)^4} \frac{e^{-ik(y-y')}k^2 C_1}{\left(k^2+m^2-\frac{M^2}{4}\right)^2 + M^2 k_4^2} \sim \frac{1}{(y-y')^2}, \\
\Pi_I(y-y') &= \int \frac{dk}{(2\pi)^4} \frac{e^{-ik(y-y')}C_2}{\left(k^2+m^2-\frac{M^2}{4}\right)^2 + M^2 k_4^2} \sim \ln(y-y')^2.
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Tr}\ \mathcal{K}_0^2 &= \iint dydy'\ \mathcal{K}_0(y,y')\mathcal{K}_0(y',y) = \infty, \\
\mathrm{Tr}\ \mathcal{K}_I^2 &= \iint dydy'\ \mathcal{K}_I(y,y')\mathcal{K}_I(y',y) < \infty.
\end{aligned}
$$

$$\Lambda(g^2) \cdot U = g^2 \left[\mathcal{K}_0 + \mathcal{K}_I\right] U.$$

$$\mathcal{K}_0(y, y') \sim \sqrt{D(y)} \frac{C_1}{(2\pi)^2 (y - y')^2} \sqrt{D(y')},$$

Let us put $U(y) = \sqrt{D(y)}\Phi(y)$, then one can obtain

$$\left[-\Box_y - \frac{g^2 C_1}{y^2}\right] \Phi(y) = p^2 \Phi(y)$$

$$\Phi(r) \sim \frac{J_\nu(pr)}{r}, \qquad r = \sqrt{y^2}, \quad p = \sqrt{p^2},$$

$$\nu = \sqrt{1 - g^2 C_1} \quad \text{and} \quad g^2 < g_c^2 = \frac{1}{C_1}$$

The kernel $\mathcal{K}_0$ contains a continuous spectrum only and requires a small coupling constant.

$$[I - g^2\mathcal{K}_0]U = g^2\mathcal{K}_I \cdot U \Rightarrow U = \frac{1}{\sqrt{I - g^2\mathcal{K}_0}} A$$

$$A = g^2\mathcal{K}_G \cdot A, \qquad \mathcal{K}_G = \frac{1}{\sqrt{I - g^2\mathcal{K}_0}} \mathcal{K}_I \frac{1}{\sqrt{I - g^2\mathcal{K}_0}}. \qquad (1)$$

This kernel for small $g^2 < g_c^2$ is of the Fredholm type

$$\mathrm{Tr}\ \mathcal{K}_G^2 < 0$$

The solution of equation does exist and can be calculated by the variational method

$$1 = g^2 \max_A \frac{\left(A \frac{1}{\sqrt{I - g^2\mathcal{K}_0}} \mathcal{K}_I \frac{1}{\sqrt{I - g^2\mathcal{K}_0}} A\right)}{(AA)}$$

or

$$1 = g^2 \max_U \frac{(U\mathcal{K}_I U)}{(U\,[I - g^2\mathcal{K}_0]\,U)}$$

Gauge

The ladder Bethe-Salpeter equation is not gauge invariant.

Quantum electrodynamics

$$L = -\frac{1}{4} F_{\mu\nu}^2(x) + (\overline{\psi}(x)(\hat{p} + e\hat{A}(x) - m)\psi(x)),$$

$$\text{Feynman gauge}: \qquad \tilde{D}_{\mu\nu}(k) = \frac{\delta_{\mu\nu}}{k^2}$$

$$\text{Coulomb gauge}: \quad \tilde{D}_{\mu\nu}(k) = \begin{cases} \left[\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right] \frac{1}{k^2} \\ -\frac{1}{\mathbf{k}^2} \end{cases}$$

Binding energy ϵ (eV) of the 1^- state

α	0.0005	0.001	$\frac{1}{137}$ 0.0073	0.01	0.1	0.3	0.5
Feynman	0.032	0.126	6.47	12.0	893	5 700	12 600
Coulomb	0.032	0.127	6.8	12.8	1 270	10 800	27 800
Schrödinger	0.032	0.127	6.8	12.8	1 280	11 500	31 900

$$\begin{array}{lll} \textit{Feynman} & V & \to \mathcal{J}_j(x) = \left(\overline{\Psi}(x) V(\overleftrightarrow{p}_x)\gamma_j \Psi(x)\right) \\ \textit{Coulomb} & V + iT & \to \mathcal{J}_j(x) = \left(\overline{\Psi}(x) V(\overleftrightarrow{p}_x)(1+\gamma_0)\gamma_j \Psi(x)\right) \end{array}$$

Fermion propagator in a self-dual field

$$\tilde{S}_{\pm}(p) \sim \int\limits_0^1 \frac{du}{2\Lambda^2} e^{-u\frac{p^2}{2\Lambda^2}} \left(\frac{1-u}{1+u}\right)^{\frac{m^2}{4\Lambda^2}} \left\{ i\hat{p} + m\frac{1 \mp \gamma_5 u^2}{1-u^2} \right\}.$$

$$1 = \frac{\alpha_s}{\pi} \iint dy_1 dy_2 V_Q(y_1) \Pi_Q(y_1 - y_2|p) V_Q(y_2).$$

$$1 = \frac{\alpha_s}{\pi} \iint\limits_0^1 du_1 du_2 P_Q(\ldots) e^{\frac{(m_1+m_2)^2}{2\Lambda^2} E(\mu,\mu_1,\mu_2,u_1,u_2)}.$$

$$\mu = \frac{M}{m_1+m_2}, \qquad \mu_j = \frac{m_j}{m_1+m_2}, \quad j = 1,2$$

$$\mathrm{E}(\mu, \mu_1, \mu_2, u_1, u_2)$$
$$= \mu^2 \cdot \frac{u_1 u_2 + 2(\mu_1^2 u_1 + \mu_2^2 u_2)}{u_1 + u_2 + 2} - \frac{\mu_1^2}{2} \ln\left(\frac{1+u_1}{1-u_1}\right) - \frac{\mu_2^2}{2} \ln\left(\frac{1+u_2}{1-u_2}\right)$$

$$M > m_1 + m_2$$

$$1 = \iint_0^1 du_1 du_2 P_Q(\dots) e^{\frac{(m_1+m_2)^2}{2\Lambda^2} E(\dots)} \approx C_Q e^{\frac{(m_1+m_2)^2}{2\Lambda^2} \max\limits_{u_1,\ u_2} E(\mu,\mu_1,\mu_2,u_1,u_2)}$$

$$M_Q \approx (m_1 + m_2) \left[1 + \frac{A_Q}{(m_1+m_2)^{1.23}}\right].$$

$$Q = J^P \;\Rightarrow\; \begin{pmatrix} uV_Q\bar{u} & uV_Q\bar{s} & uV_Q\bar{c} & uV_Q\bar{b} \\ & sV_Q\bar{s} & sV_Q\bar{c} & sV_Q\bar{b} \\ & & cV_Q\bar{c} & cV_Q\bar{b} \\ & & & bV_Q\bar{b} \end{pmatrix}.$$

$$Q = \; 1^-,\; 0^+,\; 1^+,\; 2^+(n=0),\; 2^+(n=2)$$

$$m_u = m_d = 260, \quad m_s = 434, \quad m_c = 1500, \quad m_b = 4700 \qquad (MeV)$$

$$M_Q \approx (m_1 + m_2)\left[1 + \frac{A_Q}{(m_1+m_2)^{1.23}}\right].$$

$$V(1^-) = \begin{pmatrix} \omega(782) & K^*(892) & D^*(2007) & B^*(5325) \\ & \phi(1020) & D_s^*(2112) & - \\ & & J/\psi(3100) & - \\ & & & \Upsilon(9460) \end{pmatrix};$$

$$S(0^+) = \begin{pmatrix} f_0(980) & - & - & - \\ & f_0(1370) & - & - \\ & & \chi_{c0}(3415) & - \\ & & & \chi_{b0}(9893) \end{pmatrix};$$

$$A(1^+) = \begin{pmatrix} a_1(1260) & K_1(1330) & - & - \\ & f_1(1420) & - & - \\ & & \chi_{c1}(3510) & - \\ & & & \chi_{b1}(9892) \end{pmatrix};$$

$$D(2^+) = \begin{pmatrix} f_2(1270) & K_2^*(1430) & D_2^*(2460) & - \\ & f_2'(1525) & - & - \\ & & \chi_{c2}(3556) & - \\ & & & \chi_{b2}(9912) \end{pmatrix}.$$

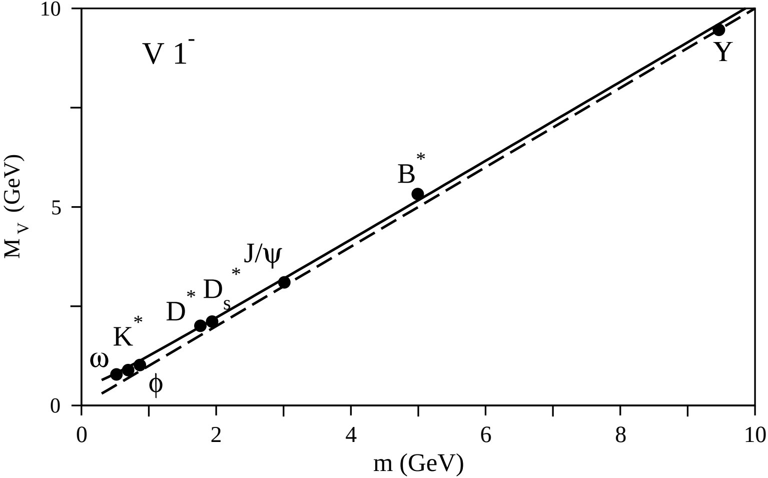
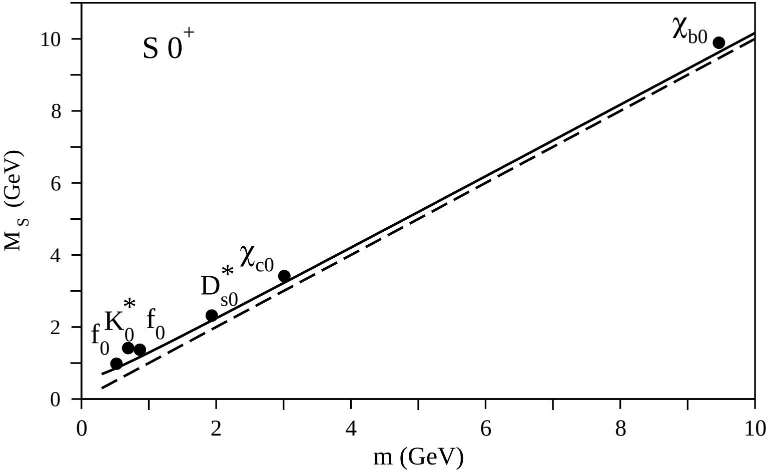


Рис.:



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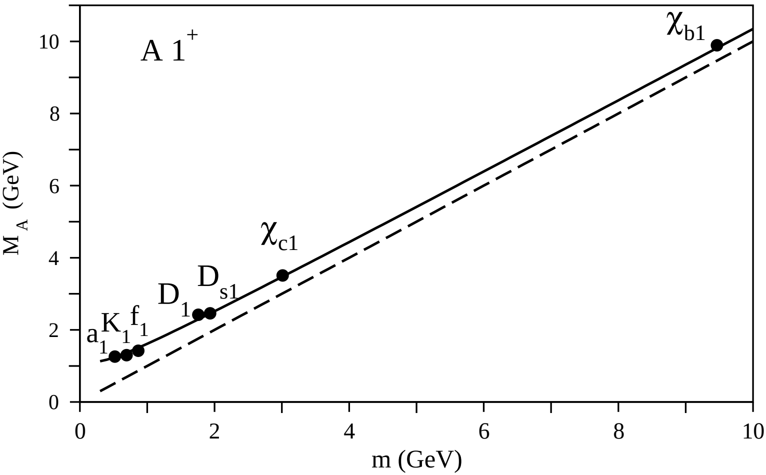

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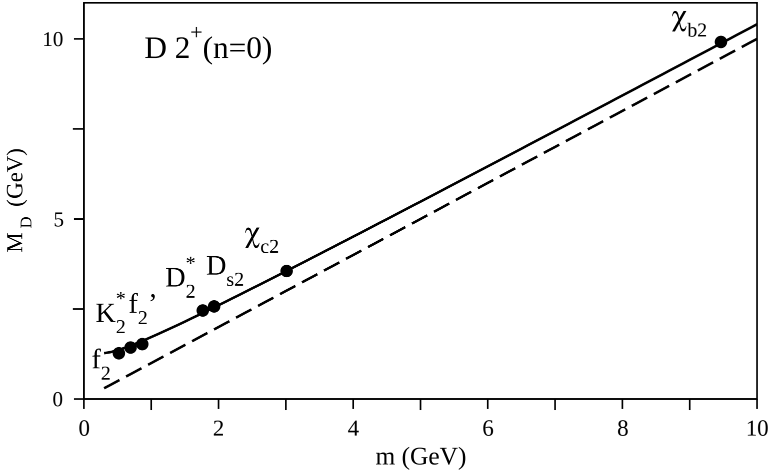

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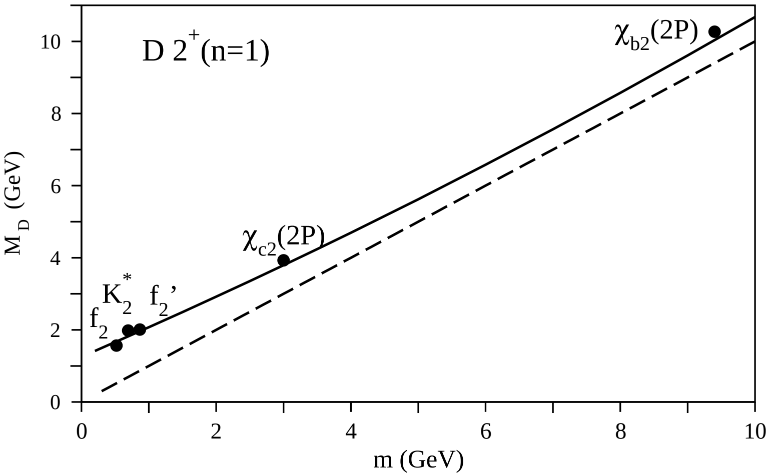

Рис.:

Radial excitations for vector 1^- mesons.

n	$\rho = u\bar{u}$	$\psi = c\bar{c}$	$\Upsilon = b\bar{b}$	O_V
0	$\rho(770)$ - 778	$J/\psi(3100)$ - 3178	$\Upsilon(1S)(9460)$ - 9589	0.0079
1	$\rho(1450)$ - 1323	$\psi(2S)(3655)$ - 3530	$\Upsilon(2S)(10023)$ - 9853	0.0037
2	$\rho(1700)$ - 1699	$\psi(3770)$ - 3772	$\Upsilon(3S)(10365)$ - 10035	0.0011
3	$\rho(1900)$ - 2081	$\psi(4040)$ - 4018	$\Upsilon(4S)(10580)$ - 10220	0.0034
4	$\rho(2150)$ - 2390	$\psi(4160)$ - 4217	$\Upsilon(10860)$ - 10370	0.0049
5	$\rho(-)$ - 2698	$\psi(4415)$ - 4416	$\Upsilon(11020)$ - 10518	$-$

$$A_V(n) \approx 0.1897 + 0.3236\ln(n+1) + 0.1634n.$$

Conclusion

♢ Analytical onfinement ⟶ vacuum fluctuations in space and time.

♢ Self-dual homogeneous vacuum gluon field can realize true QCD vacuum and leads to analytical confinement .

♢ Hadronization of quarks takes place in confinement region.

♢ Bethe-Salpeter equation is an acceptable tool for describing bound states in QCD.

♢ Mass formula

$$M_Q \approx (m_1 + m_2)\left[1 + \frac{A_Q}{(m_1 + m_2)^{1.23}}\right]$$

gives an acceptable description of meson multiplets.