Recent results on a solving Bethe–Salpeter equations for spinor particles

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Bethe-Salpeter equation

$$\mathcal{G}(p) = ig^2 \int \frac{d^4k}{(2\pi)^4} V(p,k)\,\Gamma(1)\,S(k_1)\,\mathcal{G}(k)\,\tilde{S}(k_2)\,\tilde{\Gamma}(2),$$

$$S(k) = \frac{\hat{k} + m}{k^2 - m^2 + i\varepsilon}, \quad V(p,k) = \frac{1}{(p-k)^2 - \mu^2 + i\varepsilon}.$$

$\Gamma(1) = 1$	$\tilde{\Gamma}(2) = -1$	scalar
$\Gamma(1) = \gamma_5$	$\tilde{\Gamma}(2) = -\gamma_5$	pseudoscalar
$\Gamma(1) = \gamma_\mu - \frac{i\kappa}{2m}\sigma_{\mu\rho}q^\rho$	$\tilde{\Gamma}(2) = \gamma_\nu + \frac{i\kappa}{2m}\sigma_{\nu\rho}q^\rho$	vector

Vertex function structure

$$\mathcal{G}(p_0, \mathbf{p}) = \sum_\alpha g_\alpha(p_0, |\mathbf{p}|)\, \Gamma_\alpha(\mathbf{p})$$

$^1S_0$	$^1S_0^{++}$	$^1S_0^{--}$	$^3P_0^e$	$^3P_0^o$
$^3S_1 - {}^3D_1$	$^3S_1^{++}$	$^3D_1^{++}$	$^3S_1^{--}$	$^3D_1^{--}$
	$^3P_1^e$	$^3P_1^o$	$^1P_1^e$	$^1P_1^o$

$$\Gamma_1(\mathbf{p}) = \frac{1}{\sqrt{16\pi}}\gamma_5 \quad \Gamma_2(\mathbf{p}) = \frac{1}{\sqrt{16\pi}}\gamma_0\gamma_5$$

$$\Gamma_3(\mathbf{p}) = \frac{1}{\sqrt{16\pi}}\frac{(\mathbf{p}\gamma)}{|\mathbf{p}|}\gamma_0\gamma_5 \quad \Gamma_4(\mathbf{p}) = \frac{1}{\sqrt{16\pi}}\frac{(\mathbf{p}\gamma)}{|\mathbf{p}|}\gamma_5$$

Expansion over hyperspherical harmonics

$$
\begin{aligned}
Z_{nlm}(\chi,\theta,\phi) &= X_{nl}(\chi)Y_{lm}(\theta,\phi),\\
X_{nl}(\chi) &= \sqrt{\frac{2^{2l+1}}{\pi}\frac{(n+1)(n-l)!l!^2}{(n+l+1)!}}\sin^l\chi\, C^{l+1}_{n-l}(\cos\chi),
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{(p-k)^2+\mu^2} &= 2\pi^2\sum_{nlm}\frac{1}{n+1}V_n(\tilde{p},\tilde{k})Z_{nlm}(\chi_p,\theta_p,\phi_p)Z^*_{nlm}(\chi_k,\theta_k,\phi_k\\
V_n(\tilde{p},\tilde{k}) &= \frac{4}{(\Lambda_+ + \Lambda_-)^2}\left(\frac{\Lambda_+ - \Lambda_-}{\Lambda_+ + \Lambda_-}\right)^n,\\
\Lambda_\pm &= \sqrt{(\tilde{p}\pm\tilde{k})^2+\mu^2}.
\end{aligned}
$$

$$
g_i(p_4,\mathbf{p}) = \sum_{j=1}^{\infty} g_i^j(\tilde{p})\ Z_{jlm}(\chi_p,\theta_p,\phi_p)
$$

Equations

- $$g_i^j(\tilde{p}) = -g^2 \int_0^\infty \frac{d\tilde{k}\,\tilde{k}^3}{8\pi^2(2j-1)}\, V_{ij}(\tilde{p},\tilde{k}) \sum_{n=1}^{4} \sum_{m=1}^{\infty} A_{jm}^{in}(\tilde{k})\, g_n^m(\tilde{k}),$$

- $$X = g^2\, AX,$$

$$X^T = \Big([\{g_1^m(\tilde{k}_i)\}_{i=1}^{N_G}]_{m=1}^{M_{\max}}, [\{g_2^m(\tilde{k}_i)\}_{i=1}^{N_G}]_{m=1}^{M_{\max}}, \ldots, [\{g_n^m(\tilde{k}_i)\}_{i=1}^{N_G}]$$

Equations

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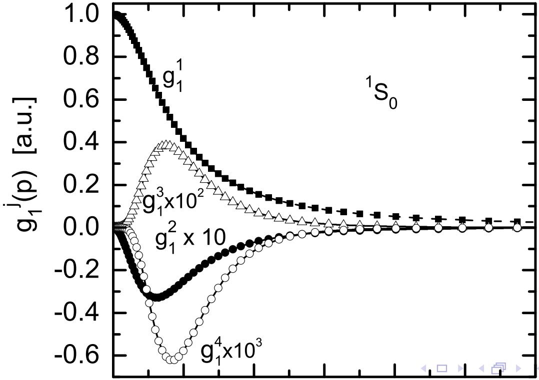
Convergence of the calculated values of mass M

$g^2 = 15$	$\mu = 0.15\ GeV/c^2$			$\mu = 0.5\ GeV/c^2$		
$M_{\max}$	$N_G = 32$	$N_G = 64$	$N_G = 96$	$N_G = 32$	$N_G = 64$	$N_G = 96$
1	1.9399	1.9399	1.9399	1.9984	1.9984	1.9984
2	1.9370	1.9370	1.9370	1.9982	1.9982	1.9982
3	1.9368	1.9368	1.9368	1.9982	1.9982	1.9982
4	1.9368	1.9368	1.9368	1.9982	1.9982	1.9982

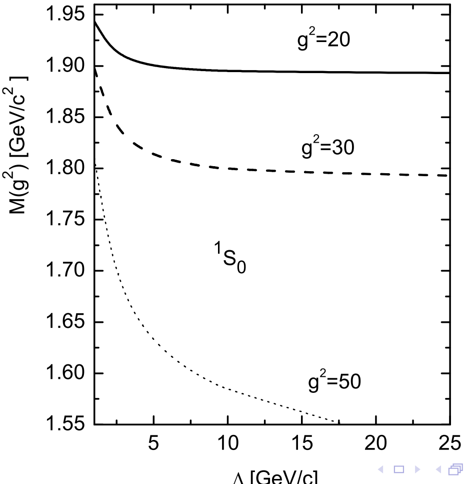
$g^2 = 30$	$\mu = 0.15\ GeV/c^2$			$\mu = 0.5\ GeV/c^2$		
$M_{\max}$	$N_G = 32$	$N_G = 64$	$N_G = 96$	$N_G = 32$	$N_G = 64$	$N_G = 96$
1	1.7932	1.7910	1.7905	1.9167	1.9142	1.9137
2	1.7897	1.7875	1.7871	1.9152	1.9127	1.9122
3	1.7896	1.7874	1.7870	1.9152	1.9127	1.9122
4	1.7896	1.7874	1.7870	1.9152	1.9127	1.9122

Results: components of g_1

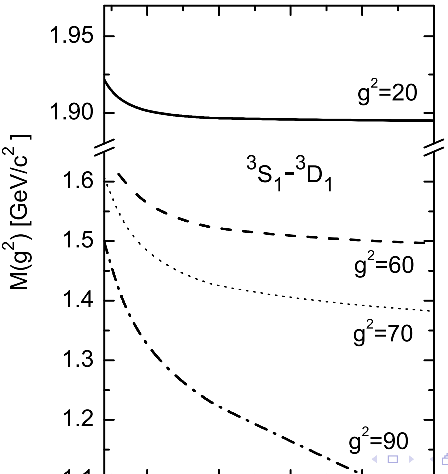
$$g^2 = 15 \qquad M = 1.937\,GeV/c^2$$



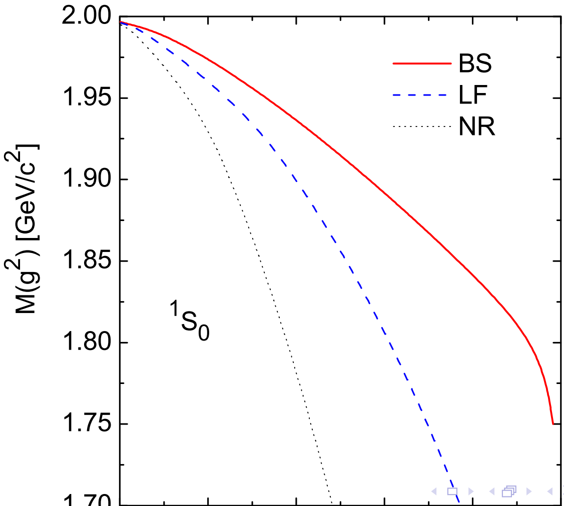
Results: stability of solutions



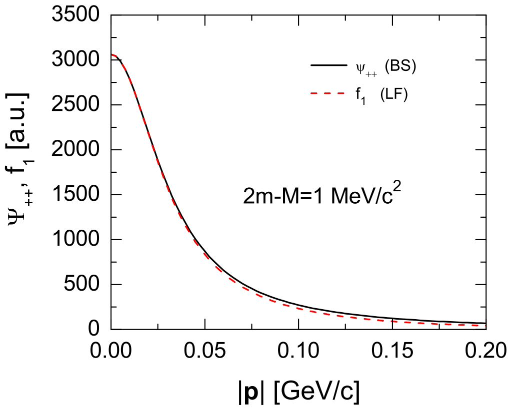
Results: stability of solutions



Results: g^2 vs M



Results: comparison to LF calculations



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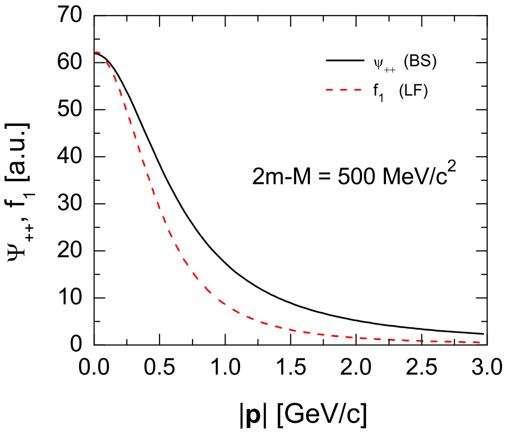

Conclusions

Novel method of solving Bethe-Salpeter equation is proposed.
Advantages:

- fast convergence
- simple parameterization of solutions
- separable form of a kernel