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Determination of the odderon mass spectrum with relativistic corrections

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⋆The Model.
⋆The Two − gluon Mass Spectrum
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⋆Conclusion

Determination the mass and constituent mass bound state system

In this section, we will present one of the alternative methods of the bound state mass determination when the nonperturbative and relativistic character of the interaction is taken into account. Let us consider an interaction between n-charged scalar particles in the external gauge field. We assume that these particles constitute a bound state. Let us determine the mass of a bound state by investigating the asymptotic behavior of the polarization loop function for a charged scalar particle in the external gauge field. The polarization loop function for a scalar particle can be written as

$$\Pi(x-y) = \langle G_{m_1}(x,y|A) G_{m_2}(y,x|A) G_{m_3}(x,y|A) \cdots G_{m_n}(x,y|A) \rangle_A . \quad (1)$$

Here the averaging over the external gauge field $A_\alpha(x)$ is performed. The Green function $G_m(y,x|A)$ for the scalar particle in the external gauge field is determined from the equation

$$\left[\left(i\frac{\partial}{\partial x_\alpha}+\frac{g}{c\hbar}\cdot A_\alpha(x)\right)^2+\frac{c^2m^2}{\hbar^2}\right]G_m(x,y|A)=\delta(x-y)\ , \qquad (2)$$

where m is the mass of the scalar particle, and g is the coupling constant. In averaging over the external gauge field $A_\alpha(x)$, let us consider only the lowest order or only the two-point Gauss correlator

$$<\exp\left\{i\int dxA_\alpha(x)J_\alpha(x)\right\}>_A=\exp\left\{-\frac{1}{2}\int\int dxdyJ_\alpha(x)D_{\alpha\beta}(x-y)J_\beta(y)\right\}$$

where $J_\alpha(x)$ is the real current. The propagator of the gauge field has the following form:

$$D_{\alpha\beta}(x-y)=<A_\alpha(x)A_\beta(y)>_A\ \ . \qquad (4)$$

The mass of the bound state is usually defined through the loop function in the following way:

$$M = - \lim_{|x-y| \to \infty} \frac{\ln \Pi(x-y)}{|x-y|} \ . \tag{5}$$

Thus, if we know the loop function, then we can determine the bound state mass. The solution of (2) can be represented as a functional integral in the following way:

$$G_m(x, y|A) = \int_0^\infty \frac{ds}{(4s\pi)^2} \exp\left\{ -sm^2 - \frac{(x-y)^2}{4s} \right\} \tag{6}$$

$$\int d\sigma_\beta \exp\left\{ ig \int_0^1 d\xi \frac{\partial Z_\alpha(\xi)}{\partial \xi} A_\alpha(\xi) \right\} \ ,$$

where the following notation is used:

$$Z_\alpha(\xi) = (x-y)_\alpha \xi + y_\alpha - 2\sqrt{s} B_\alpha(\xi); \tag{7}$$

$$d\sigma_\beta = N \delta B_\beta \exp\left\{ -\frac{1}{2} \int_0^1 d\xi B'^2(\xi) \right\}$$

with the normalization $B_\alpha(0) = B_\alpha(1) = 0$; $\int d\sigma_\beta = 1$, where N is the normalization constant. Substituting (6) into (1) and performing averaging over the external gauge field one can obtain for the loop function

$$\Pi(x) = \iint_0^\infty \cdots \int_0^\infty \frac{d\mu_1 d\mu_2 \cdots d\mu_n}{(8\pi^2 x)^n} \cdot J(\mu_1, \mu_2, \cdots, \mu_n) \times \tag{8}$$

$$\times \exp\left\{ -\frac{|x|}{2}\left(\frac{m_1^2}{\mu_1} + \mu_1\right) - \frac{|x|}{2}\left(\frac{m_2^2}{\mu_2} + \mu_2\right) - \cdots - \frac{|x|}{2}\left(\frac{m_n^2}{\mu_n} + \mu_n\right) \right\} .$$

Here

$$J(\mu_1, \mu_2, \cdots, \mu_n) = N_1 N_2 \cdots N_n \iint \cdots \int \delta\mathbf{r}_1 \delta\mathbf{r}_2 \cdots \delta\mathbf{r}_n \times \tag{9}$$

$$\times \exp\left\{ -\frac{1}{2} \int_0^x d\tau \left[\mu_1 \dot{\mathbf{r}}_1^2(\tau) + \mu_2 \dot{\mathbf{r}}_2^2(\tau) + \cdots + \mu_n \dot{\mathbf{r}}_n^2(\tau) \right] \right\} \times$$

$$\times \exp\left\{ -W_{1,1} - W_{2,2} - \cdots - W_{n,n} + 2 \sum_{i,j=1; i \neq j}^{n} W_{i,j} \right\} ,$$

$$W_{i,j} = \frac{g^2}{2}(-1)^{i+j} \int\limits_0^x \int\limits_0^x d\tau_1 d\tau_2 Z'^{(i)}{}_{\alpha}(\tau_1) D_{\alpha_\beta}\left(Z^{(\tau_1)-Z(\tau_2)}\right) Z'^{(j)}{}_{\beta}(\tau_2) \ . \quad (10)$$

We determine polarization loop function for n charged scalar particle in the external gauge field with masses m_1, m_2, $\cdots$, m_n. In the other had the functional integral represented in (9) is analogous to the Feynman path integral for the motion of n particles with masses μ_1, μ_2, $\cdots$, μ_n in the non-relativistic quantum mechanics [18]. The interaction between these particles is described by expression (10) which contains the potential and nonpotential parts, in particular, $W_{1,1}$, $W_{2,2}$, $\ldots$, $W_{n,n}$ define nonpotential interactions, and $W_{i,j}$ $(j \neq i)$, defines potential interactions of a nonlocal nature. Then the interaction Hamiltonian can be represented in the form

$$H = \frac{1}{2\mu_1}\mathbf{P}_1^2 + \frac{1}{2\mu_2}\mathbf{P}_2^2 + \cdots + \frac{1}{2\mu_n}\mathbf{P}_n^2 + V(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_n) \; . \tag{11}$$

From the the Schrodinger equation (SE)

$$H\Psi(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_n) = E(\mu_1, \mu_2, \cdots, \mu_n)\Psi(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_n) \; , \tag{12}$$

can be determine the $E(\mu)$- eigenvalues of the Hamiltonian (11). According to (12) the functional integral in (9) in the $|x-y| \to \infty$ limit can be represented as:

$$\lim_{|x|\to\infty} J(\mu_1, \mu_2, \cdots, \mu_n) \Longrightarrow \exp\{-x \cdot E(\mu_1, \mu_2, \cdots, \mu_n)\} \; , \tag{13}$$

where $E(\mu_1, \mu_2, \cdots, \mu_n)$ is a eigenvalues of the Hamiltonian and depending only on the reduced mass of the bound state and on the coupling constant g. From (5) it follows that knowing the loop function one can determine the mass of the bound state as well. However, the functional integrals represented in (8) and (9) cannot be evaluated in a general way. According to (5), one needs to derive the loop function in asymptotics. In this approximation the integral in (9) is evaluated by the saddle-point technique and, hence, for the bound state mass we obtain

$$M = \frac{1}{2} \min_{\mu_1,\mu_2,\cdots,\mu_n} \left\{ \frac{m_1^2}{\mu_1} + \mu_1 + \cdots + \frac{m_n^2}{\mu_n} + \mu_n + 2E(\mu_1,\mu_2,\cdots,\mu_n) \right\} , \quad (14)$$

and for the μ_j we get following system equations

$$\mu_j - \frac{m_j^2}{\mu_j} + 2\mu_j \frac{dE(\mu_1,\mu_2,\cdots,\mu_n)}{d\mu_j} = 0 \; ; \quad j = 1,\ 2,\cdots,n \, . \quad (15)$$

We will consider the parameters μ_1, μ_2, $\cdots$, μ_n as masses of the constituent particles in the bound state. These masses differ from m_1, m_2, $\cdots$, m_n which represent the masses of a free state. For the further calculation introduced the reduced mass two-, three- and n- body systems

$$\begin{aligned} \frac{1}{M_2} &= \frac{1}{\mu_1} + \frac{1}{\mu_2} \; ; \\ \frac{1}{M_3} &= \frac{1}{\mu_1 + \mu_2} + \frac{1}{\mu_3} \; ; \\ \frac{1}{M_4} &= \frac{1}{\mu_1 + \mu_2 + \mu_3} + \frac{1}{\mu_4} \; ; \\ &\vdots \qquad \vdots \qquad \vdots \\ \frac{1}{M_n} &= \frac{1}{\mu_1 + \mu_2 + \cdots + \mu_{n-1}} + \frac{1}{\mu_n} \; . \end{aligned} \tag{16}$$

Then for the mass of bound state we obtained:

$$M = \mu_1 + \mu_2 + \cdots + \mu_n + M_2 \frac{dE}{dM_2} + M_3 \frac{dE}{dM_3} + \cdots + M_n \frac{dE}{dM_n} \; . \tag{17}$$

In the this case the constituent masses $\mu_1, \mu_2, \cdots, \mu_n$ determined from the system equations:

$$
\begin{aligned}
1 &- \frac{m_1^2}{\mu_1^2} + \frac{2M_2^2 dE}{\mu_1^2 dM_2} + \cdots + \frac{2M_n^2 \qquad dE}{(\mu_1 + \mu_2 + \cdots + \mu_{n-1})^2 dM_n} = 0; \\
1 &- \frac{m_2^2}{\mu_2^2} + \frac{2M_2^2 dE}{\mu_2^2 dM_2} + \cdots + \frac{2M_n^2 \qquad dE}{(\mu_1 + \mu_2 + \cdots + \mu_{n-1})^2 dM_n} = 0; \\
1 &- \frac{m_3^2}{\mu_3^2} + \frac{2M_3^2 dE}{\mu_3^2 dM_3} + \cdots + \frac{2M_n^2 \qquad dE}{(\mu_1 + \mu_2 + \cdots + \mu_{n-1})^2 dM_n} = 0; \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
1 &- \frac{m_{n-1}^2}{\mu_{n-1}^2} + \frac{2M_{n-1}^2 dE}{\mu_{n-1}^2 dM_{n-1}} + \frac{2M_n^2 \qquad dE}{(\mu_1 + \mu_2 + \cdots + \mu_{n-1})^2 dM_n} = 0; \\
1 &- \frac{m_n^2}{\mu_n^2} + \frac{2M_n^2 dE}{\mu_n^2 dM_n} = 0 \; . \qquad (18)
\end{aligned}
$$

The system equation represented in (18) should be calculated analytically for the given value of n. In particular for $n = 2$ we have:

$$
\begin{aligned}
M &= \mu_1 + \mu_2 + M_2 \frac{dE}{dM_2} + E(M_2) \; ; \qquad (19) \\
\mu_1 &= \sqrt{m_1^2 - 2M_2^2 \frac{dE}{dM_2}} \; ; \quad \mu_2 = \sqrt{m_2^2 - 2M_2^2 \frac{dE}{dM_2}} \; ;
\end{aligned}
$$

and from (18) the case $n = 3$ we get

$$
\mathrm{M}{=}\mu_1 + \mu_2 + \mu_3 + M_2 \tfrac{dE}{dM_2} + M_3 \tfrac{dE}{dM_3} + E(M_2, M_3) \; ; (20)
$$

$$
\mu_3 = \sqrt{m_3^2 - 2M_3^2 \tfrac{dE}{dM_3}} \; ; \quad \mu_j = \tfrac{1}{\sqrt{2}} \sqrt{m_j^2 - 2M_2^2 \tfrac{dE}{dM_2}} \times
$$

$$
\times \sqrt{1 + \sqrt{1 - \tfrac{8M_2^2 M_3^2 (dE/dM_3)}{(m_1^2 - 2M_2^2(dE/dM_2))(m_2^2 - 2M_2^2(dE/dM_2))}}} \; ; \qquad j = 1, \; 2 \; .
$$

Thus we can determent the mass constituent mass bound state systems taking into account relativistical corrections. The quantity $E(M_2, M_3, \cdots, M_n)$ is defined as eigenvalues of the interaction Hamiltonian.

The energy spectrum hydrogen atom

In the N.Brambilla, A.Vairo, Phys. Lett., **359B**,p.133(1995) starting from the Hamiltonian

$$H = \sqrt{\vec{P}^2 + m^2} - \frac{\alpha}{r}, \tag{21}$$

in the framework of perturbation theory (limited of the order $\sim \alpha_{em}^6$) for the state $n = 2, \ell = 1$ the energy spectrum calculated.

Table 1. The energy spectrum hydrogen atom at the any values of coupling constant.

α	$E_{bin}/m_e(TB)$	$E_{bin}/m_e(our)$
0.0155522	0.999969765	0.999969766
0.142460	0.997452	0.997457
0.2599358	0.99147	0.99152
0.3566678	0.4838	0.9840
0.4359255	0.975	0.976
0.5	0.967	0.9682

W.Lucha and F.F.Schoberl; Phys. Rev.**A54**,p.3790.

The nonperturbative correction for the interaction hamiltonian

$$
\begin{aligned}
W_{i,j} &= \frac{g^2}{2}(-1)^{i+j} \cdot \int_0^x \int_0^x d\tau_1 d\tau_2 \cdot \left(\vec{n} + \frac{1}{c}\vec{r}_i^{\,\prime}(\tau_1)\right) \cdot \left(\vec{n} + \frac{1}{c}\vec{r}_j^{\,\prime}(\tau_2)\right) \\
&\times \int \frac{d\vec{q}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{ds}{2\pi} \widehat{D}\left(\vec{q}^{\,2} + \frac{s^2}{c^2}\right) \qquad (22) \\
&\times \exp\left\{ is(\tau_1 - \tau_2) + \frac{is}{c}\left(r_i^{(4)}(\tau_1) - r_j^{(4)}(\tau_2)\right) + i\vec{q}(\vec{r}_i(\tau_1) - \vec{r}_j(\tau_2)) \right\} .
\end{aligned}
$$

where $\vec{n} = \vec{r}/r$, $\vec{r} = \vec{r}_1(\tau) - \vec{r}_2(\tau)$, $r = |\vec{r}|$ and $\widehat{D}$ is the Fourier image of the D function. The interaction between constituent particles is caused by the interchange of the gauge field quanta so let us write the propagator in the standard way:

$$
\widehat{D}\left(\vec{q}^{\,2} + \frac{s^2}{c^2}\right) \simeq \frac{1}{\vec{q}^{\,2} + \frac{s^2}{c^2}} = \int_0^{\infty} d\eta \cdot \exp\left\{-\eta \cdot \left(\vec{q}^{\,2} + \frac{s^2}{c^2}\right)\right\} . \qquad (23)
$$

According to (22), after integration over $d\vec{q}$ we have for the interaction potential

$$W_{i,j} = \frac{2}{3} g^2 (-1)^{i+j} \int\limits_0^t \int\limits_0^t d\tau_1 d\tau_2 \int\limits_{-\infty}^{\infty} \frac{ds}{2\pi} \int\limits_0^{\infty} \frac{d\eta}{(2\sqrt{\pi\eta})^3} \tag{24}$$

$$\times \quad \exp\left\{-\frac{\vec{r}^2}{4\eta}\right\} \sum_{k=0}^{\infty} \sum_{n=0}^{k} \frac{(-1)^{n+k}}{n!(k-n)!} \eta^n r^{(4)(k-n)} \left(\frac{is}{c}\right)^{n+k} e^{is\tau} \Theta_{ij} \ ,$$

where the following notation is introduced:

$$\tau = (\tau_1 - \tau_2) \ ; \tag{25}$$

$$r^{(4)} = r_i^{(4)}(\tau_1) - r_j^{(4)}(\tau_2) \ ;$$

$$\Theta_{ij} = 1 + \frac{\vec{n}}{c} \cdot \left(\vec{r}_i'(\tau_1) + + \vec{r}_j'(\tau_2)\right) + \frac{\vec{r}_i'(\tau_1)\vec{r}_j'(\tau_2)}{c^2} \ .$$

Here τ_1 and τ_2 are considered as the proper time of the constituent particles 1 and 2, respectively.

Thus, we choose the dependence of the Euclid time $r^{(4)}$ on τ in the following manner:

$$r^{(4)} = c(\tau_1 - \tau_2)u \equiv c\tau u \ . \qquad (26)$$

where u is a new variable. Taking into account (25) and (26) and integrating over ds and du, after some simplification, from (24) we obtain

$$\begin{aligned} W_{i,j} &= (-1)^{i+j} \cdot \frac{g^2}{6\pi} \cdot \int\limits_0^t \int\limits_0^t d\tau_1 d\tau_2 \frac{\delta(\tau_1 - \tau_2)}{|\vec{r}_j(\tau_1) - \vec{r}_j(\tau_2)|} \\ &+ (-1)^{i+j} \cdot \frac{g^2}{6\pi} \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)! c^{2k}} \cdot \int\limits_0^t d\tau \frac{\partial^{2k}}{\partial \tau^{2k}} \\ &\times (|\vec{r}_i(\tau) - \vec{r}_j(\tau)|)^{2k-1} \equiv W_{i,j}^{(1)} + W_{i,j}^{(2)} \ . \end{aligned} \qquad (27)$$

Let us consider each term in (27) separately.

$$-W_{1,1}^{(1)} + 2W_{1,2}^{(1)} - W_{2,2}^{(1)} = \int\limits_0^t d\tau \left\{ -\frac{\alpha_{em}}{r(\tau)} + V(0) \right\} , \qquad (28)$$

where

$$r(\tau) = |\vec{r}_1(\tau) - \vec{r}_2(\tau)|; \tag{29}$$

$$V(0) = \alpha_{em} \int \frac{d\vec{q}}{(2\pi)^2} \cdot \frac{1}{\vec{q}^2} \ ,$$

In this case, from (27) and (28) we can see that the diagonal interaction, when $i = j$, defines the mass renormalization.

$$H = H^0 + \triangle H^0_{nonper} \ , \tag{30}$$

where H^0 is the nonrelativistic Hamiltonian and $\triangle H^0_{nonper}$ is the nonperturbative correction

$$H^0 = \frac{1}{2\mu} \cdot \vec{P}^2 - \frac{\alpha_{em}}{r} \ ;$$

$$\triangle H^0_{nonper} = -\frac{\alpha_{em}}{r} \cdot \left[\frac{1}{\sqrt{1 + \hbar^2 \ell(\ell+1)/(c^2 r^2 M_2^2)}} - 1 \right] . \tag{31}$$

Thus, we have obtained the nonperturbative correction to the interaction Hamiltonian which is related to the relativistic nature of the system. We have ascertained that in the nonrelativistic limit ($c \to \infty$) this term is absent. At the same time, in particular, our system consists of quarks and antiquarks which move with a constant speed to each other.

The Glueball Mass Spectrum

The mass spectrum of consisting of scalar gluons

Let us now begin to calculate the mass spectrum of the two-gluon bound state taking into account the one-gluon exchange and the nonperturbative character of interaction. The interaction Hamiltonian can be rewritten as

$$H = \frac{1}{2\mu}\vec{P}^2 + \sigma_{ad} r - \frac{4}{3}\frac{\alpha_s}{r} - \frac{4}{3}\frac{\alpha_s}{r}\left[\frac{1}{\sqrt{1+\ell(\ell+1)/(r^2\mu^2)}} - 1\right] . \quad (32)$$

After some simplifications we determine the glueball mass and the constituent mass of the gluon. The numerical results are in Table 2. From Table 2 it can be seen that with increasing coupling constant the bound state mass decreases but the constituent mass of the particles grows.
Kaidalov, A. B., Simonov, Yu. A.: Yad. Fiz **63**, 1428 (2000)

Table 2. Mass spectrum of the glueball and gluons in the bound state when the nonperturbative character of interaction is taken into consideration; the results of [7] are in the parentheses

	α_s	0	0.1	0.2	0.3	.39
$\ell = 1$	ρ	0.574	0.58	0.59	0.595	0.60
	$M_2, (Gev)$	0.346	0.355	0.364	0.374	0.383
	$\mu_G, (Gev)$	0.693	0.711	0.729	0.748	0.765
	$M, (Gev)$	2.772	2.684	2.594	2.500	2.418
		(2.77)		(2.56)	(2.45)	(2.36)
$\ell = 2$	ρ	0.574	0.575	0.58	0.58	0.585
	$M_2, (Gev)$	0.413	0.418	0.424	0.430	0.435
	$\mu_G, (Gev)$	0.826	0.837	0.849	0.860	0.871
	$M, (Gev)$	3.304	3.239	3.173	3.106	3.045
		(3.30)		(3.14)	(3.05)	(2.97)
$\ell = 3$	ρ	0.575	0.575	0.576	0.576	0.585
	$M_2, (Gev)$	0.470	0.474	0.478	0.482	0.486
	$\mu_G, (Gev)$	0.941	0.949	0.955	0.965	0.972
	$M, (Gev)$	3.762	3.710	3.656	3.603	3.555

Calculation of the glueball mass spectrum taking into account the spin-orbit interaction

In this section, we will determine the mass spectrum of the two-gluon bound state when all effects of the gluon-gluon interaction such as the one-gluon exchange, nonperturbative character, and spin-orbit corrections are taken into account. The total Hamiltonian can be written as a sum of two parts. The first one is the central Hamiltonian which corresponds to the conditions of the one-gluon exchange and nonperturbative character of interaction and also to the confinement. The second one is the Hamiltonian of the spin-orbit interaction

$$H = H_c + H_{spin} \ , \qquad (33)$$

where H_c is the central part

$$H_c = \frac{1}{2\mu}\vec{P}^2 + \sigma_{ad} r - \frac{4}{3}\frac{\alpha_s}{r} - \frac{4}{3}\frac{\alpha_s}{r}\left[\frac{1}{\sqrt{1+\ell(\ell+1)/(r^2\mu^2)}} - 1\right] . \qquad (34)$$

The second part of the Hamiltonian is defined in the standard form (for details see [10, 31])

$$H_{spin} = H_{SS} + H_{LS} + H_{TT} \ . \tag{35}$$

where H_{SS} is the spin-spin interaction Hamiltonian

$$H_{SS} = \frac{(\mathbf{S_1 S_2})}{\mu^2} \Delta V_v \ , \tag{36}$$

and also the spin-orbit interaction Hamiltonian

$$H_{LS} = \frac{(\mathbf{LS})}{8\mu^2} \left[\frac{3}{r} \frac{\partial}{\partial r} V_v - \frac{1}{r} \frac{\partial}{\partial r} V_s \right] \ , \tag{37}$$

and, at last, the tensor interaction Hamiltonian

$$H_{TT} = \frac{S_{12}}{48\mu^2} \left[\frac{1}{r} \frac{\partial}{\partial r} V_v - \frac{\partial^2}{\partial r^2} V_v \right] \ . \tag{38}$$

Here V_v is the vector potential corresponding to the one-gluon exchange

$$V_v = -\frac{4\alpha_s}{3} \frac{1}{\sqrt{r^2 + \ell(\ell+1)/\mu^2}} \ ; \tag{39}$$

V_s is the confinement potential

$$V_s = r\sigma_{ad} \ ; \tag{40}$$

and also the following notation is used:

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} \tag{41}$$
$$S_{12} = \frac{4}{(2\ell+3)(2\ell-1)}\left[\mathbf{L}^2\mathbf{S}^2 - \frac{3}{2}(\mathbf{LS}) - 3(\mathbf{LS})^2\right] \ .$$

Using the explicit form of the Hamiltonian introduced in equations (35)-(41), let us start to determine the mass of the glueball with the spin-spin interactions when $\ell = 0$.
Our numerical results are presented in Table 3.

Table 3. The mass spectrum of the glueball with taking into account the nonperturbative character of interaction and spin-spin interaction for the case of $\ell = 0$. In GeV units. $\sigma_{ad} = 0.45\ GeV^2$, $\alpha_s = 0.3$

J^{PC}	*our result*	*lattice data*	*Exp.*	*other works*
0^{++}	1.64	1.73 [32]	1.50 [36]	
		1.63 [33]	2.11 [36]	1.98 [38]
		1.61 [34]	2.32 [37]	2.69 [39]
		1.75 [35]		
2^{++}	1.97	2.40 [41]	2.02 [40]	2.42 [38]
		2.35 [33]		2.70 [39]
		2.26 [34]		
		2.42 [35]		

From Table 3, we can see that our results are in good agreement with the results of other authors. Let us now consider the general case when $\ell \neq 0$. Using the OR method we obtain the energy spectrum $E(\mu)$ for the total Hamiltonian from SE

$$E = E^{(C)} + E^{(SS)} + E^{(LS)} + E^{(TT)} . \quad (42)$$

Here $E^{(C)}$ is the contribution of the central interaction Hamiltonian

$$E^{(C)} = \frac{x^2\sqrt{\sigma_{ad}}}{8\rho^2}\frac{\Gamma(2+\rho+2\rho\ell)}{\Gamma(3\rho+2\rho\ell)} + \frac{\sqrt{\sigma_{ad}}}{xz}\frac{\Gamma(4\rho+2\rho\ell)}{\Gamma(3\rho+2\rho\ell)} - \qquad (43)$$

$$- \frac{4\alpha_s xz\sqrt{\sigma_{ad}}}{3\Gamma(3\rho+2\rho\ell)}\int_0^\infty du \frac{u^{3\rho+2\rho\ell-1}e^{-u}}{\sqrt{u^{2\rho}+z^2\ell(\ell+1)}} \;;$$

$E^{(LS)}$ is the spin-orbit interaction contribution

$$E^{(LS)} = \frac{z^2\sqrt{\sigma_{ad}}(\mathbf{LS})}{8\Gamma(3\rho+2\rho\ell)}\left\{-\frac{\Gamma(2\rho+2\rho\ell)}{xz} + 4\alpha_s xz\int_0^\infty du\frac{u^{3\rho+2\rho\ell-1}e^{-u}}{[u^{2\rho}+z^2\ell(\ell+1)]^{3/2}}\right\} \;; ($$

$E^{(TT)}$ is the inclusion for the tensor interaction

$$E^{(TT)} = \frac{\alpha_s xz^3\sqrt{\sigma_{ad}}S_{12}}{12\Gamma(3\rho+2\rho\ell)}\int_0^\infty du\frac{u^{5\rho+2\rho\ell-1}e^{-u}}{[u^{2\rho}+z^2\ell(\ell+1)]^{5/2}} \;; \qquad (45)$$

and $E^{(SS)}$ is the contribution of the spin-spin interaction

$$E^{(SS)} = \frac{\alpha_s \ell \sqrt{\sigma_{ad}} (\mathbf{S_1 S_2})}{18\Gamma(3\rho + 2\rho\ell)} \cdot x z^3 \rho^2 \int_0^\infty du \frac{u^{3\rho+2\rho\ell-1} e^{-u}}{[u^{2\rho} + z^2\ell(\ell+1)]^{5/2}} \times \left[u^{2\rho} + \frac{z^2}{2}(3+2\ell)(1+\ell) \right] . \quad (46)$$

The parameter x is derived from the equation

$$2 + \frac{1}{\sqrt{\sigma_{ad}}} \frac{\partial E}{\partial x} - 0 \,, \quad (47)$$

and then the energy spectrum is determined in the following form:

$$E(\mu) = \min_{\{\rho, Z\}} [E(x,\ \rho,\ z)] \ . \quad (48)$$

The numerical results are in Table 4.

Table 4. The mass spectrum of the glueball for the general case. In *GeV* units. $\sigma_{ad} = 0.45\ GeV^2$, $\alpha_s = 0.3$

	$\ell = 1$			$\ell = 2$			$\ell = 3$		
	J^{PC}	our result	other work	J^{PC}	our result	other work	J^{PC}	our result	other work
$S = 0$	0^{--}	2.95		0^{++}	3.39	1.72	0^{--}	3.95	
	1^{--}	2.99		1^{++}	3.42		1^{--}	3.97	3.81
				2^{++}	3.47	3.50	2^{--}	4.00	3.90
							3^{--}	4.05	4.10
$S = 1$	0^{-+}	2.92	2.59	0^{+-}	3.36	4.82	0^{-+}	3.90	3.64
	1^{-+}	2.95		1^{+-}	3.39	2.95	1^{-+}	3.95	
	2^{-+}	3.02	3.10	2^{+-}	3.44	4.10	2^{-+}	3.99	3.89
				3^{+-}	3.52	3.53	3^{-+}	4.03	
							4^{-+}	4.10	
$S = 2$	0^{--}	2.86		0^{++}	3.31	2.67	0^{--}	3.90	
	1^{--}	2.89		1^{++}	3.33		1^{--}	3.92	
	2^{--}	2.95		2^{++}	3.38	2.38	2^{--}	3.95	
	3^{--}	3.05		3^{++}	3.46	3.69	3^{--}	4.00	

Calculation of the three-gluon mass spectrum

In this section, we will determine the mass spectrum of the three -gluon bound state system.

$$H = \frac{1}{2}\sum_{j=1}^{3}\frac{1}{\mu_j}\vec{P}_j^2 - \frac{3\alpha_s}{4}\left(\frac{1}{|\vec{r}_1-\vec{r}_2|}+\frac{1}{|\vec{r}_1-\vec{r}_3|}+\frac{1}{|\vec{r}_2-\vec{r}_3|}\right)+ + \sigma\left(|\vec{r}_1-\vec{r}_2|+|\vec{r}_1-\vec{r}_3|+|\vec{r}_2-\vec{r}_3|\right). \quad (49)$$

After some simplification, this Hamiltonian can be expressed in the centre of mass system as

$$H = \frac{1}{2M_2}\vec{P}_x^2+\frac{1}{2M_3}\vec{P}_y^2-\frac{3\alpha_s}{4}\left(\frac{1}{x}+\frac{1}{|\vec{x}M_2/\mu_1+\vec{y}|}+\frac{1}{|\vec{x}M_2/\mu_2-\vec{y}|}\right)+ + \sigma\left(x+|\vec{x}M_2/\mu_1+\vec{y}|+|\vec{x}M_2/\mu_2-\vec{y}|\right)\ , \quad (50)$$

where M_2 and M_3 is represented in (16). Let us introduced new variables $(\vec{R},\vec{r})$:

$$\vec{x}=\frac{1}{M_2\ e^2}\vec{R}\ ; \quad \vec{y}=\frac{1}{\sqrt{M_2M_3}\ e^2}\vec{r}\ , \quad (51)$$

and from (50) we get for SE

$$\left[\frac{1}{2}\vec{P}_r^2 + \frac{1}{2}\vec{P}_R^2 - \frac{3\alpha_s}{4}\left(\frac{1}{R} + \frac{\lambda}{|\vec{r} + c_1\vec{R}|} + \frac{\lambda}{|\vec{r} - c_2\vec{R}|}\right) + \frac{U}{2} + \right.$$
$$\left. + \frac{\sigma}{\lambda}\left(R + |\vec{r} + c_1\vec{R}| + |\vec{r} - c_2\vec{R}|\right)\right]\Psi(\vec{R},\vec{r}) = 0 \ .$$

and the following notations is used:

$$\lambda = c_1 + c_2 \ ; \qquad c_j = \frac{1}{\mu_j}\sqrt{M_2 M_3} \ ; \qquad j = 1, 2 \ , \tag{52}$$

$$E = -\frac{1}{2} \cdot M_2 \cdot U(M_2, M_3) \ , \tag{53}$$

The numerical results are given in Table 5.

Table 5. Spin-averaged masses (in GeV) of ggg states with $\ell = 0,\ 1,\ 2,\ 3,\ 4$ and $\sigma = 0.18 Gev^2$

ℓ	$M(our)$	$M\ (lat)$
0	3.22	3.4 ± 0.21
1	3.76	3.46 ± 0.2
2	4.2	
3	4.5	
4	5.2	

Conclusion

$*$ The mass spectrum of the bound state is derived analytically and it was shown that the constituent mass differs from the free particle mass and, in particular, gluons become massive when in the bound state.
$*$ We have determined the mass spectrum of the glueball taking spin-orbit and spin-spin interactions into account. In our approach for the case of $\ell = 0$ the nonperturbative correction is equal to zero and the vector potential is defined by the Coulomb interaction only. When $\ell \neq 0$ the vector potential connected to the one-gluon exchange differs from the ordinary Coulomb potential. In this case, our numerically calculated mass spectrum agrees satisfactorily with the results of other researchers.
$*$ The mass spectrum scalar three gluon bound state system