

Recent progress in transverse-momentum dependent PDF's

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Recent results concerning the understanding of gauge-invariant **transverse-momentum dependent** (unintegrated) parton distribution functions (TMD PDF's) are presented. The **renormalization-group properties** of these objects are addressed, and analysis of the leading-order **UV anomalous dimensions** is given in the light-cone gauge. A **generalized definition** of the TMD PDF, based on the renormalization procedure for the **Wilson exponentials** with obstructions, is proposed.

- **Integrated PDF's:** definition; gauge invariance; RG properties
- **Unintegrated (TMD) PDF's:** shortcomings (complete gauge invariance, undesirable divergences)
- **Towards a solution:** gauge invariance; factorization; reduction to the integrated case
- On the “**completely correct**” TMD PDF's: calculation of AD in the light-cone gauge; generalized renormalization and cancelation of extra divergences
- Conclusions and outlook

Integrated PDF's: definition; gauge invariance; RG properties

$$q_{i/h}(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle h(P) | \bar{\psi}_i(\xi^-, \mathbf{0}_\perp) \gamma^+ \psi_i(0^-, \mathbf{0}_\perp) | h(P) \rangle \sim \langle h(P) | a_i^\dagger(x) | h(P) \rangle$$

Corresponds to the number of partons (probabilistic interpretation)

Gauge invariance is saved by the insertion of the **gauge link**:

$$[y, x|\Gamma] = \mathcal{P} \exp \left[-ig \int_{x[\Gamma]}^y dz_\mu A_a^\mu(z) t_a \right]$$

so that

$$\hat{q}_{i/h}(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle h(P) | \bar{\psi}_i(\xi^-, \mathbf{0}_\perp) [\xi^-, 0^-] \gamma^+ \psi_i(0^-, \mathbf{0}_\perp) | h(P) \rangle$$

Renormalization properties are described by the DGLAP equation:

$$\mu \frac{d}{d\mu} \hat{q}_{i/h}(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij} \left(\frac{x}{z} \right) \hat{q}_{j/h}(x, \mu)$$

where $P_{ij} \left(\frac{x}{z} \right)$ is the DGLAP integral kernel, which controls the dependence from the UV scale μ

Formally, this approach can be applied to the more complicated case—uPDF's (unintegrated, or TMD)...

Unintegrated (TMD) PDF's

“Naive” definition:

$$f_i(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp}.$$

$$\cdot \langle p | \bar{\psi}_i(\xi^-, \xi_\perp) [\xi^-, \xi_\perp; \infty^-, \xi_\perp;]^\dagger \gamma^+ [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi_i(0^-, \mathbf{0}_\perp) | p \rangle \Big|_{\xi^+ = 0}$$

Formally:

$$\int d^2k_\perp f_i(x, \mathbf{k}_\perp) = q_i(x)$$

$f_i(x, \mathbf{k}_\perp)$ accumulates a lot of the phenomenologically important quantities. E.g., T -odd functions, responsible for the single-spin asymmetries in SIDIS and DY (see the talk by O. Teryaev)

However: this definition suffers from several shortcomings.

- **gauge invariance** is not complete: in the light-cone gauge, dependence on the pole prescription in the gluon propagator still takes place
- **extra (rapidity) divergences** associated with the features of the light-cone gauge, or the light-like Wilson lines (*in the integrated case, these divergences cancel*)
- **reduction to the integrated case:** formal integration doesn't produce correct result because of additional uncanceled UV divergences

Looking for the solution:

- **gauge invariance** is completely restored by means of the additional transverse Wilson line at light-cone infinity (Belitsky, Ji, Yuan). This gauge link contributes only in the light-cone gauge and cancels the pole-prescription dependence
- **extra divergences** can be avoided by using the non-light-like gauge connectors in covariant gauges, or an axial gauge off the light cone (Collins, Soper). This entails the introduction of an additional rapidity parameter $\zeta = (p \cdot n)^2/n^2$ (with $n^2 \neq 0$) to encode the deviation from the light cone; the calculations become more complicated; problems with factorization could arise.
- **generalized renormalization** procedure for the light-like Wilson lines (or a subtractive method) (Collins, Hautmann): extra divergences cancel by the additional “soft” factor, defined by the vacuum average of particular Wilson lines (demonstrated explicitly in the covariant gauge, in the 1-loop order)

Towards the “completely correct” definition:

- calculate the **anomalous dimension** of the TMD PDF in the light-cone gauge and identify extra divergences in terms of the *defect* of anomalous dimension
- perform the **generalized renormalization** of TMD PDF, in analogue to the renormalization of the Wilson contours with cusps or self-intersection
- propose the **modified definition** of the TMD PDF, free of the above mentioned disadvantages

In the **tree** approximation, the TMD PDF reads

$$\begin{aligned}
 f^{(0)}(x, \mathbf{k}_\perp) &= \\
 &= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathbf{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle p | \bar{\psi}(\xi^-, \xi_\perp) \gamma^+ \psi(0^-, 0_\perp) | p \rangle = \\
 &= \delta(1-x) \delta^{(2)}(\mathbf{k}_\perp)
 \end{aligned}$$

The one-gluon exchanges, contributing to the UV-divergences, are described by the diagrams:

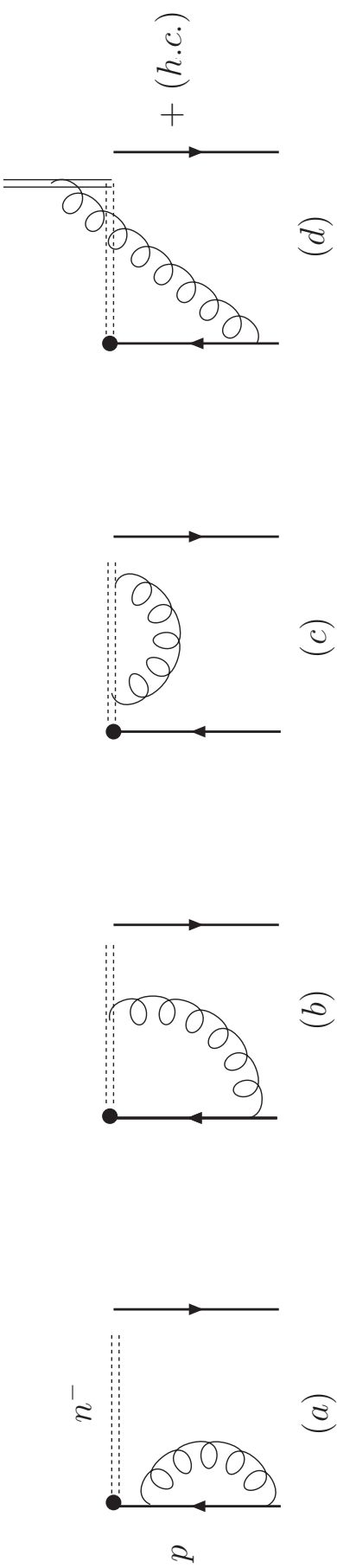


Figure 1: One-gluon exchanges for the TMD PDF: diagrams producing UV divergences. Only (a) and (d) contribute in the light-cone gauge

Source of the uncertainties and extra divergences: pole in the gluon propagator

$$D_{\text{LC}}^{\mu\nu}(q) = \frac{1}{q^2} \left[g^{\mu\nu} - \frac{q^\mu n^{-\nu}}{[q^+]} - \frac{q^\nu n^{-\mu}}{[q^+]} \right]$$

Possible pole prescriptions:

$$d_{\text{PV}}^{\mu\nu}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu}) \frac{1}{2} \left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta} \right)$$

$$d_{\text{Adv/Ret}}^{\mu\nu}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu}) \frac{1}{q^+ \mp i\eta}$$

In what follows, we keep η small, but finite. Dimensional regularization is used to control UV singularities.

The UV divergent part read:

$$\begin{aligned} \Sigma_{\text{left}}^{UV}(p, \alpha_s; \epsilon) &= \\ &= -\frac{\alpha_s C_F}{\pi} \frac{1}{\epsilon} \left[-\frac{3}{4} - \ln \frac{\eta}{p^+} + \frac{i\pi}{2} + i\pi C_\infty \right] + \alpha_s C_F \frac{1}{\epsilon} [iC_\infty] = \\ &= -\frac{\alpha_s C_F}{\pi} \frac{1}{\epsilon} \left[-\frac{3}{4} - \ln \frac{\eta}{p^+} + \frac{i\pi}{2} \right] \end{aligned}$$

Imaginary part of AD — gluons in the Glauber regime (Idilbi, Ma junder)

prescription dependence is canceled

$$C_\infty = \begin{cases} 0, & \text{Advanced: } \frac{1}{[q^+]} = \frac{1}{q^+ - i\eta} \\ -1, & \text{Retarded: } \frac{1}{[q^+]} = \frac{1}{q^+ + i\eta} \\ -\frac{1}{2}, & \text{Principal Value: } \frac{1}{[q^+]} = \frac{1}{2} \left(\frac{1}{q^+ - i\eta} + \frac{1}{q^+ + i\eta} \right) \end{cases}$$

Taking into account (*h.c.*) contributions, one gets total real UV divergent part:

$$\begin{aligned}\Sigma_{\text{tot}}(p, \alpha_s(\mu); \epsilon) &= \Sigma_{\text{left}} + \Sigma_{\text{right}} = \\ &= -\frac{\alpha_s C_F}{4\pi} \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} \right)\end{aligned}$$

Dependence on η remains:

- gauge invariance is not complete
- AD doesn't coincide with AD_{2q}

One-loop **anomalous dimension** is defined via the renormalization constant

$$\gamma = \frac{1}{2} \frac{1}{Z^{(1)}} \mu \frac{\partial \alpha_s(\mu)}{\partial \mu} \frac{\partial Z^{(1)}(\mu, \alpha_s(\mu); \epsilon)}{\partial \alpha_s}$$

and reads

$$\gamma_{\text{LC}} = \gamma_{\text{smooth}} - \delta\gamma, \quad \gamma_{\text{smooth}} = \frac{3\alpha_s}{4\pi} C_F + O(\alpha_s^2)$$

Defect of anomalous dimension

$$\delta\gamma = -\frac{\alpha_s}{\pi} C_F \ln \frac{\eta}{p^+}$$

contains undesirable p^+ -dependent term which should be removed by a consistent procedure.

Note, that $\delta\gamma$ is nothing else, but the **cusp anomalous dimension**:

$$p^+ = (p \cdot n^-) \sim \cosh \chi$$

defines an angle χ between the direction of the quark momentum p_μ and the light-like vector n^- . In the large χ limit:

$$\ln p^+ \rightarrow \chi, \quad \chi \rightarrow \infty$$

Renormalization of the Wilson operators with obstructions (cusps, self-intersections) requires additional renormalization factor depending on the cusp angle (Korchemsky, Radyushkin)

$$Z_\chi = \left[\langle 0 | \mathcal{P} \exp \left[ig \int_\chi d\zeta^\mu \hat{A}_\mu^a(\zeta) \right] | 0 \rangle \right]^{-1}$$

Generalized renormalization:

$$\mathcal{O}_{\text{ren}}(\chi, \dots) = Z_\chi Z_R \mathcal{O}(\chi, \dots)$$

Choose the integration path as follows

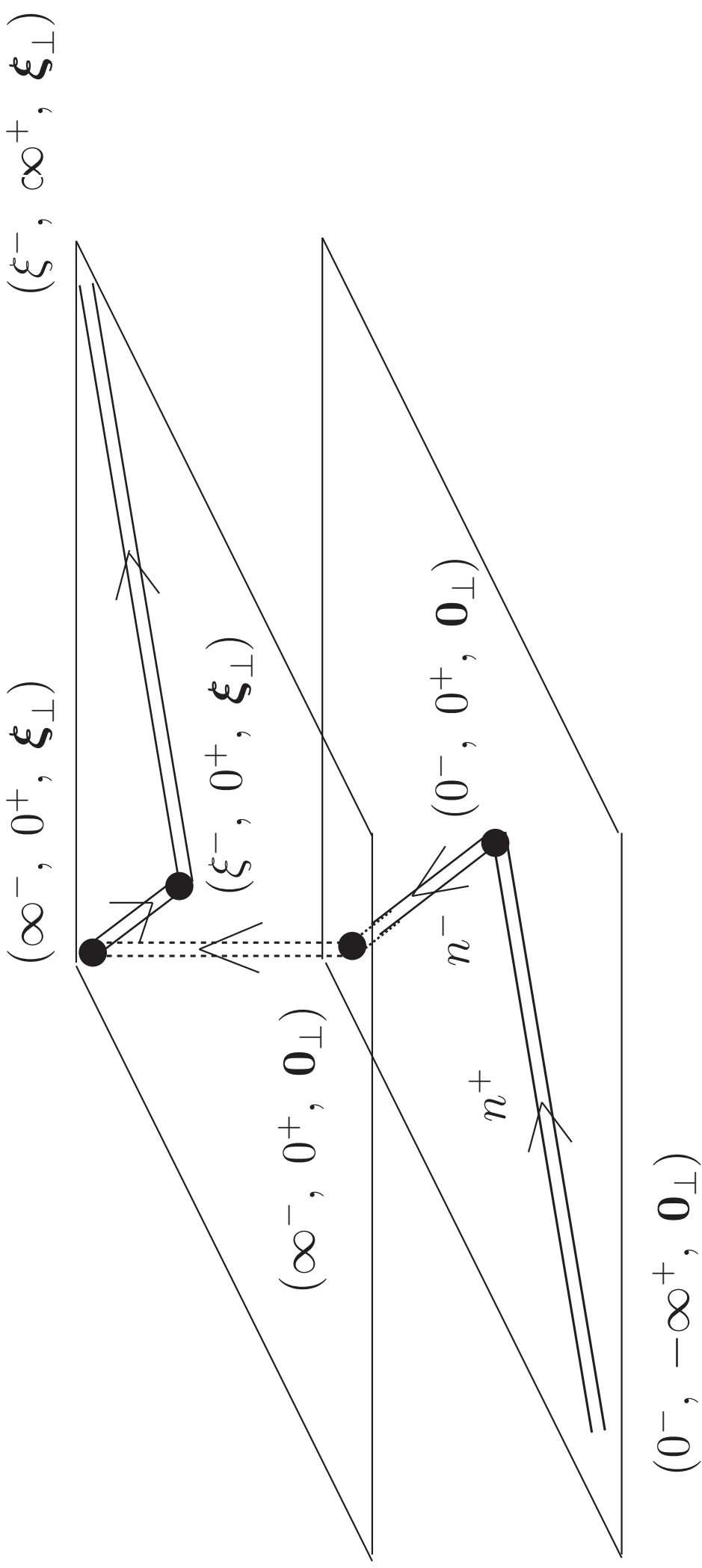


Figure 2: Space-time picture: integration trajectory for the additional cusp-dependent renormalization factor

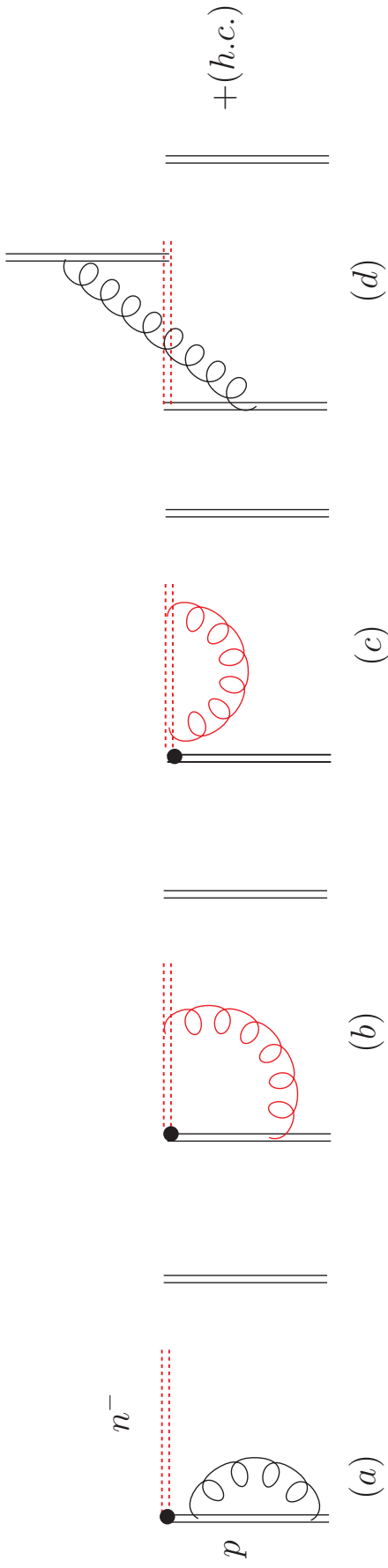


Figure 3: One-gluon exchanges for the generalized multiplicative renormalization factor

The generalized **renormalization constant** reads

$$\hat{Z}_{\text{mod}} = 1 + \frac{\alpha_s}{4\pi} C_F \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} + 4 \ln \frac{\eta}{p^+} \right) = 1 - \frac{3\alpha_s}{4\pi} C_F \frac{2}{\epsilon}$$

so that

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \hat{Z}_{\text{mod}}(\mu, \alpha_s, p^+) = \frac{3\alpha_s}{4\pi} C_F + O(\alpha_s^2)$$

i.e., equal to the anomalous dimension of the corresponding operator with the **smooth gauge connector**, according to the anomalous dimensions sum rule.

ADSR can be formulated in the following form

$$\begin{aligned}
 \text{AD} \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle p | \bar{\psi}(\xi) \gamma^+ [\xi, 0]_{\text{direct link}} \psi(0) | p \rangle = \\
 = \text{AD} \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \\
 \cdot \langle p | \bar{\Psi}(\xi | \infty) \gamma^+ \Psi(0 | \infty) | p \rangle \cdot \Phi(p^+, n^- | 0^-, \mathbf{0}_\perp) \Phi^\dagger(p^+, n^- | \xi^-, \xi_\perp)
 \end{aligned}$$

Generalized definition of TMD PDF:

$$\begin{aligned}
 & f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = \\
 & \frac{1}{2} \int \frac{d\xi^- d^2 \boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} e^{-ik^+ \xi^- + ik_\perp \cdot \boldsymbol{\xi}_\perp} \langle q(p) | \bar{\psi}(\xi^-, \mathbf{k}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \\
 & \quad \times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \infty_\perp]^\dagger \gamma^+ [\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp] [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \\
 & \quad \times \psi(0^-, \mathbf{0}_\perp) | q(p) \rangle \left[\Phi(p^+, n^- | 0^-, \mathbf{0}_\perp) \Phi^\dagger(p^+, n^- | \xi^-, \boldsymbol{\xi}_\perp) \right]
 \end{aligned}$$

Soft factor:

$$\Phi(p^+, n^- | 0) = \left\langle 0 \left| \mathcal{P} \exp \left[ig \int_{\mathcal{C}_{\text{cusp}}} d\zeta^\mu t^a A_\mu^a(\zeta) \right] \right| 0 \right\rangle$$

$$\Phi^\dagger(p^+, n^- | \xi) = \left\langle 0 \left| \mathcal{P} \exp \left[-ig \int_{\mathcal{C}_{\text{cusp}}} d\zeta^\mu t^a A_\mu^a(\xi + \zeta) \right] \right| 0 \right\rangle$$

η -evolution equation:

$$\eta \frac{d}{d\eta} f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = [\mathcal{K}(\mu) + \mathcal{G}(\mu)] \otimes f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta)$$

$$\mathcal{K}(\mu) + \mathcal{G}(\mu) =$$

$$= \frac{\alpha_s}{\pi} C_F \delta(1-x) \left[\delta^{(2)}(\mathbf{k}_\perp) \left(\ln \frac{\mu^2}{p^2} - \ln \frac{\mu^2}{\lambda^2} \right) - \frac{1}{\pi} \left(\frac{1}{\mathbf{k}_\perp^2 + x\lambda^2 - x(1-x)p^2} + \frac{1}{\mathbf{k}_\perp^2 + \lambda^2} \right) \right]$$

The **renormalization-group behavior** of the functions $\mathcal{K}(\mu)$ and $\mathcal{G}(\mu)$ is determined by the universal cusp anomalous dimension

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \mathcal{K}(\mu) = -\frac{1}{2} \mu \frac{d}{d\mu} \ln \mathcal{G}(\mu) = \gamma^{\text{cusp}} = \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2)$$

Dependence on the dimensional regularization scale μ of the re-defined TMD PDF:

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = \frac{3\alpha_s}{4\pi} C_F + O(\alpha_s^2)$$

Consistency equation:

$$\mu \frac{d}{d\mu} \left[\eta \frac{d}{d\eta} f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta) \right] = 0$$

Reduction to the integrated PDF

$$\int d^{\omega-2} \mathbf{k}_{\perp} f_{i/a}^{\text{mod}}(x, \mathbf{k}_{\perp}; \mu, \eta) = f_{i/a}(x, \mu)$$

restores the **DGLAP** evolution

Probabilistic interpretation

The distribution functions cannot be calculated from first principles, but their **evolution** can. The RG properties define the necessary condition for the unintegrated PDF to be a number density.

The requirement that the off-the-light-cone two-quark matrix element should have an anomalous dimension equal to that of the corresponding quantity with the **smooth gauge connector** in order to respect the probabilistic interpretation, is tantamount to the **anomalous dimensions sum rule**. Therefore, the RG properties can serve to define the necessary condition for the PDF to be a number density.

The generalized TMD PDF obeys this condition.

Conclusions

- The anomalous dimension of the TMD PDF in the **light-cone gauge** is calculated in 1-loop order; the contribution of the cusp anomalous dimension is obtained.
- The generalized completely gauge invariant **definition of TMD PDF** is proposed.
- The direct connection to the **integrated PDF** is established.
- The **parton interpretation** of the distribution functions in terms of the RG properties is proposed.

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