EXTENDED SIEGERT THEOREM IN THE
RELATIVISTIC INVESTIGATION OF THE DEUTERON
PHOTODISINTEGRATION REACTION

S.B.¹, V. Burov¹, K. Kazakov² and D. Shulga²

(1) BLTP, Joint Institute for Nuclear Research, Dubna, Russia
(2) LTNP, Far Eastern National University, Vladivostok, Russia

XIX Baldin ISHEPP, October 2, 2008, Dubna
Photodisintegration of the deuteron - motivation:

- breakup reaction, includes effects of the bound state (deuteron), continuum states \((np\)-pair\)
- real photon \(-q^2 = 0\), investigation of the off-shell behavior of nucleon form factors
Investigations of the deuteron photodisintegration:

- plane-wave relativistic impulse approximation

- effects of negative energy components in deuteron

- relativistic contribution of the final state interaction

- effects of the two-particle current by using Siegert theorem
Let us consider the charge and current operators as a sum of $n$-particle operators:

\[
\hat{\rho} = \sum_{n \geq 1} \hat{\rho}_n, \quad \hat{j} = \sum_{n \geq 1} \hat{j}_n,
\]

Using the equation of current conservation in a form

\[
\nabla \cdot \hat{j} + i[\hat{H}, \hat{\rho}] = 0,
\]
one obtains the following decomposition on the $n$-particle operators

\[ \nabla \cdot \hat{j}_1 + i \left[ \hat{T}_1, \hat{\rho}_1 \right] \quad (n = 1) \]

\[ + \nabla \cdot \hat{j}_2 + i \left[ \hat{T}_1, \hat{\rho}_2 \right] + i \left[ \hat{V}_2, \hat{\rho}_1 \right] \quad (n = 2) \]

\[ + \ldots \]

Applying the Siegert ansatz for the two-body system

\[ \hat{\rho}_2 = 0 \]

one obtain

\[ \nabla \cdot (\hat{j}_1 + \hat{j}_2) + i \left[ \hat{T}_1, \hat{\rho}_1 \right] + i \left[ \hat{V}_2, \hat{\rho}_1 \right] = 0 \]
After the Fourier transformation and some algebra the equation
\[ \nabla \cdot \hat{j} + i \left[ \hat{H}, \hat{\rho}_{[1]} \right] = 0 \]
can be rewritten in a limit \( q \to 0 \) as original Siegert theorem
\[ \hat{j}(q = 0) = i[\hat{H}, \hat{D}] = \dot{\hat{D}} \]
with electric dipole moment
\[ D = \sum e_i x_i. \]
The transversal components are not included into consideration.
To take into account the transversal components of the current, one need to use the following expression

\[(1 + \mathbf{q} \cdot \nabla_q) \mathbf{j}(q) = -i \nabla_q \dot{\rho}(q) - 2i \mathbf{q} \times \hat{\mathbf{\mu}}(q).\]

After applying some algebra one can obtain for the current operator

\[\mathbf{j}(q) = \mathbf{j}_C(q) + \mathbf{j}_M(q),\]

where the generalized electric dipole and magnetic moments are introduced

\[\mathbf{j}_C(q) = i[\hat{H}, \hat{D}(q)] \quad \mathbf{j}_M(q) = -i\mathbf{q} \times \hat{\mathbf{M}}(q)\]

\[\hat{D}(q) = -i \int_0^1 \frac{d\lambda}{\lambda} \nabla_q \hat{\rho}_1(\lambda \mathbf{q}) \quad \hat{M}(q) = -i \int_0^1 d\lambda \nabla_q \times \hat{\mathbf{j}}_1(\lambda \mathbf{q})\]

This is the extended Siegert theorem.
To find expression for the amplitude of the deuteron photodisintegration

\[ \langle \mathbf{P}_f, f \mid \hat{S} \mid \mathbf{P}_i, i \rangle = \hat{A}^\mu (P_f - P_i) \langle \mathbf{P}_f, f \mid \hat{j}^\mu (0) \mid \mathbf{P}_i, i \rangle, \]

with

\[ \hat{A}^\mu (P_f - P_i) = \int d^4 x e^{i(P_f - P_i) \cdot x} \hat{A}^\mu (x) \]

Using Foldy identity for the Fourier transformed expressions

\[ \int d^3 R \ e^{i K \mathbf{R}} j(\mathbf{R}) = \]

\[ - \int d^3 R \int_0^1 d\lambda \ [e^{i\lambda K \mathbf{R}} \mathbf{R} \ (\nabla \cdot j(\mathbf{R})) + i\lambda e^{i\lambda K \mathbf{R}} \mathbf{K} \times (\mathbf{R} \times j(\mathbf{R}))], \]
the amplitude becomes

$$\langle P_f, f | \hat{S} | P_i, i \rangle = (2\pi)^4 \delta^{(4)}(P_f - P_i - q)T_{fi}$$

with

$$T_{fi} = -\langle 0_\gamma | \hat{E}(0) | 1^q_\gamma \rangle \cdot D_{fi}(K) - \langle 0_\gamma | \hat{H}(0) | 1^q_\gamma \rangle \times M_{fi}(K),$$

and photon matrix elements

$$\langle 0_\gamma | \hat{E}(0) | 1^q_\gamma \rangle = \frac{i}{\sqrt{2E_\gamma}} (E_\gamma \epsilon - q \epsilon_0),$$

$$\langle 0_\gamma | \hat{H}(0) | 1^q_\gamma \rangle = \frac{i}{\sqrt{2E_\gamma}} q \times \epsilon.$$. 
The generalized electric dipole and magnetic moments are introduced in terms of matrix elements of 1-particle current

\[ \hat{D}(q) = -i \int_{0}^{1} \frac{d\lambda}{\lambda} \nabla_q \langle P_i + \lambda q, f | \hat{\rho}(1)(0) | P_i, i \rangle, \]

\[ \hat{M}(q) = -i \int_{0}^{1} d\lambda \nabla_q \times \langle P_i + \lambda q, f | \hat{j}(1)(0) | P_i, i \rangle. \]

Finally

\[ T_{fi} = \frac{-1}{\sqrt{2E\gamma}} \left[ E\gamma \int_{0}^{1} \frac{d\lambda}{\lambda} (\varepsilon \cdot \nabla_q) \rho(1)(\lambda q) + \int_{0}^{1} d\lambda (q \times \varepsilon) \cdot (\nabla_q \times j(1)(\lambda q)) \right]. \]
\[ j^{(1)}_{\mu}(q) = \langle K + \lambda q, f | \hat{\jmath}^{(0)}_{\mu} | K, i \rangle = \]
\[ = S_p \left\{ \bar{\chi}^{(0)}_{S_m S}(p) \Gamma^{(1)}_{\mu}(q, \lambda) \Lambda_{\lambda q} G^{(1)} \left( \mathcal{L}^{-1}_{\lambda q} \left( \frac{K - q\lambda}{2} + p \right) \right) \right. \]
\[ \left. \Gamma_{m_d} \left( \mathcal{L}^{-1}_{\lambda q} \left( p - \frac{q\lambda}{2} \right); K^{(0)} \right) \Lambda_{\lambda q}^{-1} \right\} \]
\[ + (-1)^{I+1} S_p \left\{ \bar{\chi}^{(0)}_{S_m S}(p) \Gamma^{(2)}_{\mu}(q, \lambda) \Lambda_{\lambda q} G^{(2)} \left( \mathcal{L}^{-1}_{\lambda q} \left( \frac{K - q\lambda}{2} - p \right) \right) \right. \]
\[ \left. \Gamma_{m_d} \left( \mathcal{L}^{-1}_{\lambda q} \left( -p - \frac{q\lambda}{2} \right); K^{(0)} \right) \Lambda_{\lambda q}^{-1} \right\} \]

Here \( \mathcal{L}^{\pm 1}_{q} \) corresponds to Lorenz transformation from the center of mass system to the laboratory system:

\[ k_0 = \frac{1}{M_d} \left( E_d k'_0 = q \cdot k' \right) \quad k = k' + \frac{q}{M_d} \left( \frac{q \cdot k'}{M_d + E_d} = k'_0 \right), \]

and BS vertex functions for the deuteron are transformed as

\[ \Lambda^{\pm 1} = \sqrt{\frac{E_d + M_d}{2M_d}} \left( 1 \mp \frac{\gamma^0 \gamma \cdot q}{E_d + M_d} \right), \]
BS amplitude for the $np$-pair

$$
\chi_{S_{m_s}}^{(0)}(p_f, P_f(0)) = \sum_{m_1 m_2} C^{S_{m_s}}_{\frac{1}{2} m_1 \frac{1}{2} m_2} u^{(1)}_{m_1}(\hat{p}) u^{(2)}_{m_2}(-\hat{p}).
$$

The deuteron BS vertex function $\Gamma_{m_d}(k, K_i)$ calculated by using the Graz II kernel separable kernel.

The $\gamma NN$ electromagnetic vertex is in on-mass-shell form

$$
\Gamma^{(1,2)}_{\mu} = \gamma^{(1,2)}_{\mu} F^{(1,2)}_1 - 1/4m(\gamma_{\mu} \hat{q} - \hat{q} \gamma_{\mu})^{(1,2)} F^{(1,2)}_2.
$$
Angular distribution with $E_\gamma = 20$ MeV
Angular distribution with $E_\gamma = 100$ MeV
Angular distribution with $E_\gamma = 200$ MeV
Component $T_{22}$ of the deuteron tensor asymmetry with $E_\gamma = 20$ MeV
Component $T_{22}$ of the deuteron tensor asymmetry with $E_\gamma = 100$ MeV
Component $T_{22}$ of the deuteron tensor asymmetry with $E_\gamma = 200$ MeV
Conclusion:

- effects of the two-particle current are investigated for reaction of the deuteron photodisintegration by applying the extended Siegert theorem;
- the contributions are found to be positive and rather large for the cross section and deuteron tensor asymmetry.