Landau gauge gluon and ghost propagators in 4d SU(3) gluodynamics on large lattices: recent results

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Abstract: We present recent results for the Landau gauge gluon and ghost propagators in $SU(3)$ pure gauge theory at Wilson $\beta = 5.7$ for lattice sizes up to 96^4 corresponding to physical volumes up to $(15.8 \text{ fm})^4$. Considerable attention is paid to finite-volume effects. We employ a gauge-fixing method that combines a simulated annealing algorithm with finalizing overrelaxation. In the infrared region $q^2 \leq 0.01 \text{ Gev}^2$ we find the gluon propagator to become flat as a function of q^2 with a weak volume dependence. Also the ghost dressing function seems to tend to a constant value in the deep infrared.

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Introduction

- Nonperturbative studies of Landau gauge gluon and ghost propagators

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z_{gl}(q^2)}{q^2}\,, \quad G^{ab} = \delta^{ab}\frac{Z_{gh}(q^2)}{q^2}$$

with continuum Dyson-Schwinger (DS) or Funct. Renorm. Group (FRG) Eqs. and within the lattice approach hopefully will provide consistent results.

- DS Eqs. have power-like conformal solution
[von Smekal, Hauck, Alkofer '98; Zwanziger '02; Lerche, von Smekal '02]

$$Z_{gh}(q^2) \propto (q^2)^{\alpha_{gh}} \quad \text{and} \quad Z_{gl}(q^2) \propto (q^2)^{\alpha_{gl}}$$

with $\alpha_{gl} + 2\alpha_{gh} = 0,$

$D(q^2) = \frac{Z_{gl}}{q^2} \rightarrow 0, \quad Z_{gh}(q^2) \rightarrow \infty$ for $q^2 \rightarrow 0$.

It is argued to be unique, when DS combined with FRG Eqs. [Fischer, Pawlowski, '07]

- DS Eqs. provide also a regular solution
[Boucaud et al. '05 - '07; Aguilar et al. '04 - '08]

- We present some further steps towards IR limit for the Landau gauge gluon and ghost propagators in quenched QCD on very large lattices.

Gauge fixing: standard approach

In order to fix the Landau gauge we apply a gauge transformation $g(x)$ to link variables $U_{x,\mu} \in SU(3)$ such that the gauge functional is maximized

$$F_U[g] = \sum_{x,\mu} \frac{1}{3} \,\mathfrak{Re}\,\mathrm{Tr}\ {}^{g}U_{x,\mu}\,.$$

⇒ For $A_\mu(x+\hat{\mu}/2) := (1/2ig_0)\left(U_{x,\mu} - U^\dagger_{x,\mu}\right)_{\mathrm{traceless}}$
this is equivalent to $\Delta_\mu A_\mu = 0\,,$

⇒ but not unique: Gribov copies,

⇒ search for global maxima -
fundamental modular region (FMR).

Standard prescription:

i) $g(x)$ taken with periodic b.c.'s,

ii) maximize $F_U[g]$ with overrelaxation (OR) method.

Drawbacks of OR:

i) substantial slowing down of OR convergence with increasing lattice extension L,

ii) its possibilities to find global maximum of $F_U[g]$ are strongly limited.

Simulated annealing: the principle

- Simulated annealing (SA) is a "stochastic optimization method" – here with the statistical weight $W[g] \propto \exp\{F_U[g]/T\}$ –
allowing quasi-equilibrium tunnelings through functional barriers, in the course of a "temperature" T decrease.

- In principle - with infinitely slow cooling down - it allows to reach global extrema (contrary to OR, "tied" to the (initially chosen) local maximum).

- Control parameters at hand:

  i) N_{iter}, T_{max} and T_{min},

  ii) schedule for temperature steps
  $T_i, \; i = 1, ..., N_{iter}$ can be optimized.

⇒ The larger N_{iter} the higher the local maxima,
$N_{iter} \to \infty \quad \Longrightarrow \quad$ global maximum.

⇒ Schedule in practice: $T_{max} = 0.45, T_{min} = 0.01$,
$N_{iter} = O(5 \cdot 10^3 - 15 \cdot 10^3)$ with tiny (larger) T-steps close to T_{max} (close to T_{min}).

Lattice Faddeev-Popov operator

Lattice Faddeev-Popov operator can be written in terms of the (gauge-fixed) link variables $U_{x,\mu}$ as

$$M_{xy}^{ab} = \sum_{\mu} A_{x,\mu}^{ab}\, \delta_{x,y} - B_{x,\mu}^{ab}\, \delta_{x+\hat{\mu},y} - C_{x,\mu}^{ab}\, \delta_{x-\hat{\mu},y}$$

with

$$\begin{aligned} A_{x,\mu}^{ab} &= \quad \mathfrak{Re}\,\mathrm{Tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right], \\ B_{x,\mu}^{ab} &= 2 \cdot \mathfrak{Re}\,\mathrm{Tr}\left[T^b T^a\, U_{x,\mu}\right], \\ C_{x,\mu}^{ab} &= 2 \cdot \mathfrak{Re}\,\mathrm{Tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right] \end{aligned}$$

and T^a, $a = 1, \ldots, 8$ being the (hermitian) generators of the $\mathfrak{su}(3)$ Lie algebra satisfying $\mathrm{Tr}\left[T^a T^b\right] = \delta^{ab}/2$.

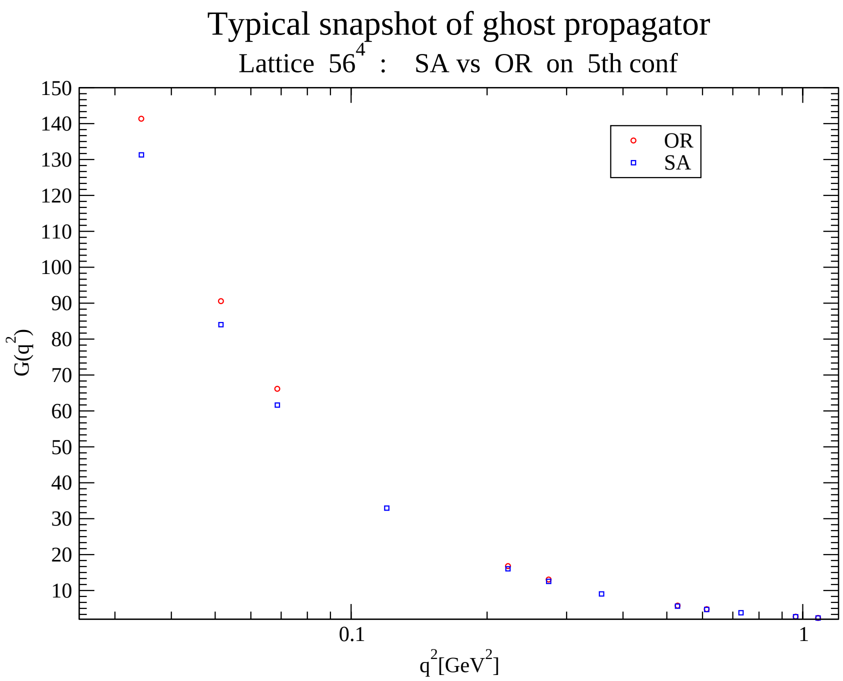
The ghost propagator is given by

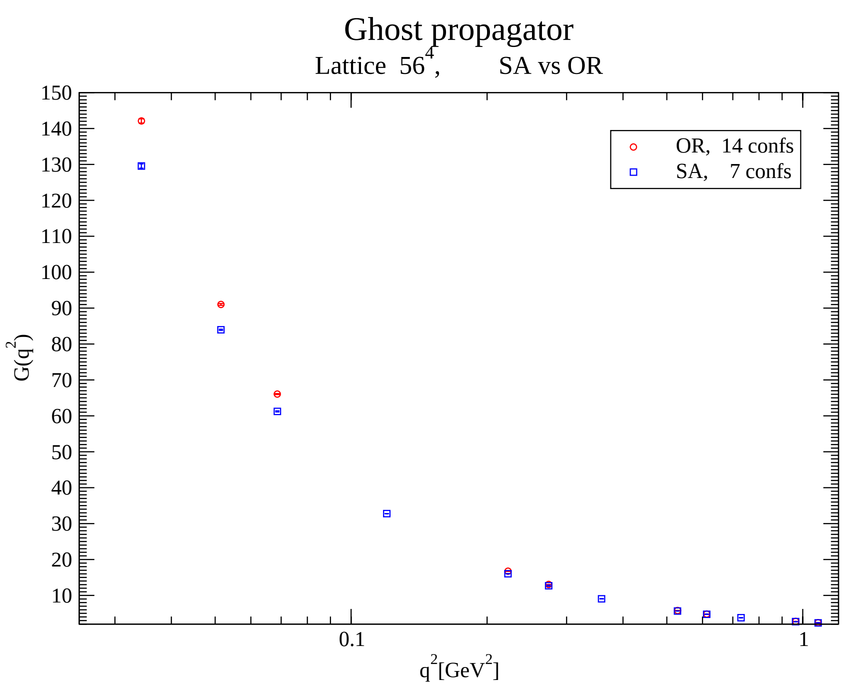
$$G^{ab} = \sum_{x,y} \left\langle \mathrm{e}^{-ik\cdot(x-y)} [M^{-1}]_{x,y}^{ab} \right\rangle$$

M-inversion with conjugate gradient method and plane wave sources.

Gauge fixing: SA vs. OR

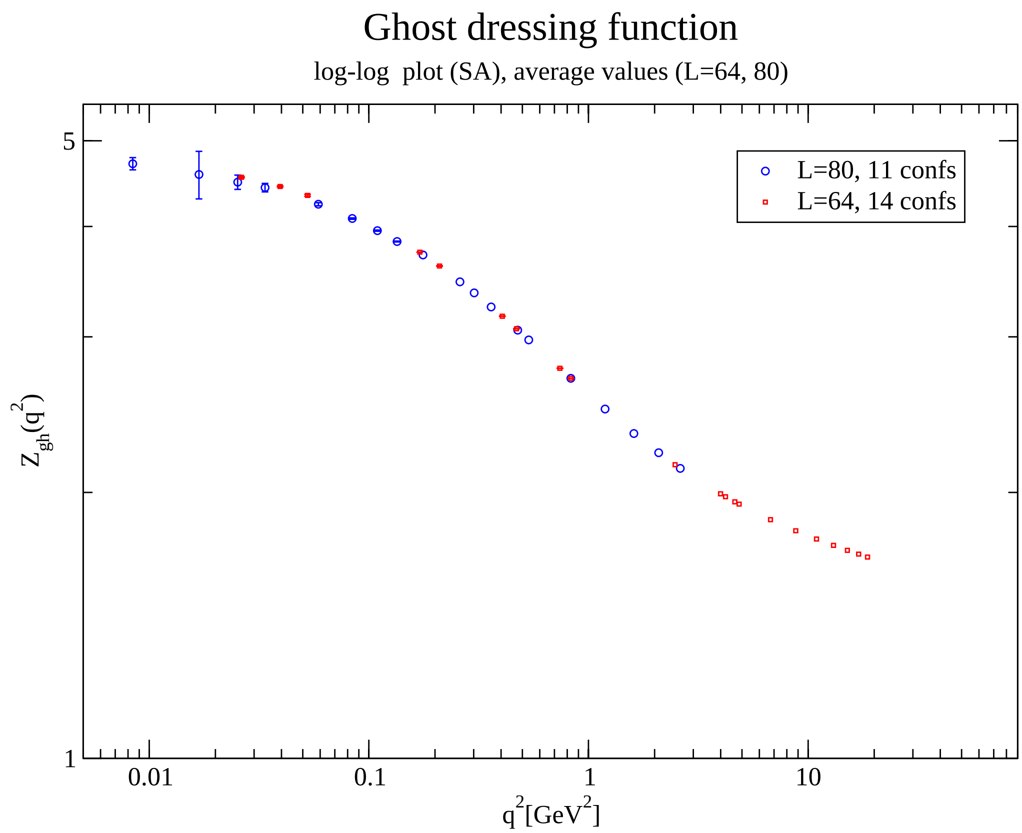
$SU(3)$ ghost propagator for $\beta = 5.70, \quad L = 56$





$\Rightarrow$ Influence of Gribov copies clearly visible, but seems to be moderate

Ghost : SA results, $\beta = 5.70$



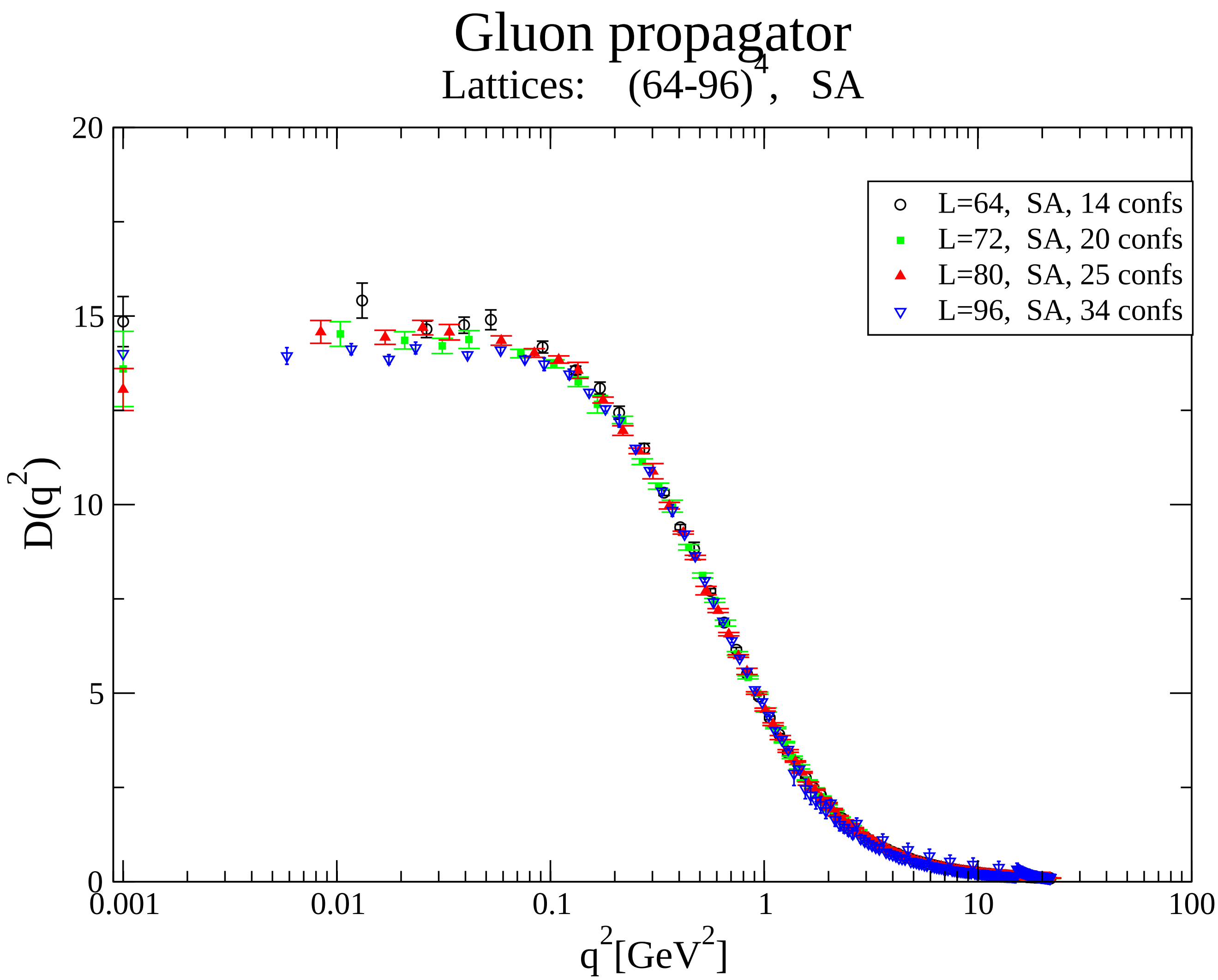
$\Rightarrow$ Weak estimator's dependence on MC configuration.

$\Rightarrow$ Finite-size effects of ghost propagator are very small and do not agree with finite-volume DS results [Fischer, Pawlowski '07].

$\Rightarrow$ No power-like asymptotics visible, i.e. differs from DS conformal solution with $\alpha_{gh} \approx 0.595$.

$\Rightarrow$ Lattice evidence for IR-regular ghost dressing function in agreement with the regular DS solutions ?

Gluon propagator: SA results, $\beta = 5.70$



$\Rightarrow$ Flattening is clearly seen.

$\Rightarrow$ Results seem to support plateau hypothesis with $\alpha_{gl} = 1$ and $\alpha_{gl} + 2\alpha_{gh} \neq 0$.

Conclusions and Questions

- Our lattice results seem to support the regular DS solution for the Landau gauge gluon and ghost propagators and to contradict the conformal one.

- Gribov copy effects seem to be moderate, but are still visible for the ghost propagator.
  Open question: Influence of enlargement of gauge orbits (e.g. with $Z(N)$ flips) and its influence on the finite-size behaviour.

- Weaknesses of the lattice approach:
  – in the IR the continuum limit not under control,
  – BRST invariance not properly treated
    (see talk by L. von Smekal),
  – choice of the potential A_μ not unique,
  – choice of the boundary conditions not unique
    (here always periodic).

- Rôle of zero-momentum modes? Can they be suppressed by proper choice of A_μ and/or boundary conditions with non-periodic gauge transformations as shown in the $U(1)$ case ?
  [Bogolubsky et al., '00]

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