Relativistic Multiparticle Interactions in Lobachevsky Geometry. Directed Nuclear Radiation.

Anton Baldin

29 September 2008

A.M.Baldin proposed a classification of applicability of the notion “elementary particle” on the basis of the variable b_{ik} introduced by him.

$$b_{ik} = -\left(U_i - U_k\right)^2 = 2\left[\left(U_i U_k\right) - 1\right] = 2\left[\frac{E_i E_k - \vec{p}_i \vec{p}_k}{m_i m_k} - 1\right]$$

$$b_{ik} = 2\left[\left(U_i U_k\right) - 1\right] = 2\left[\operatorname{ch} \rho_{ik} - 1\right]$$

- the region $0 \le b_{ik} \le 10^{-2}$

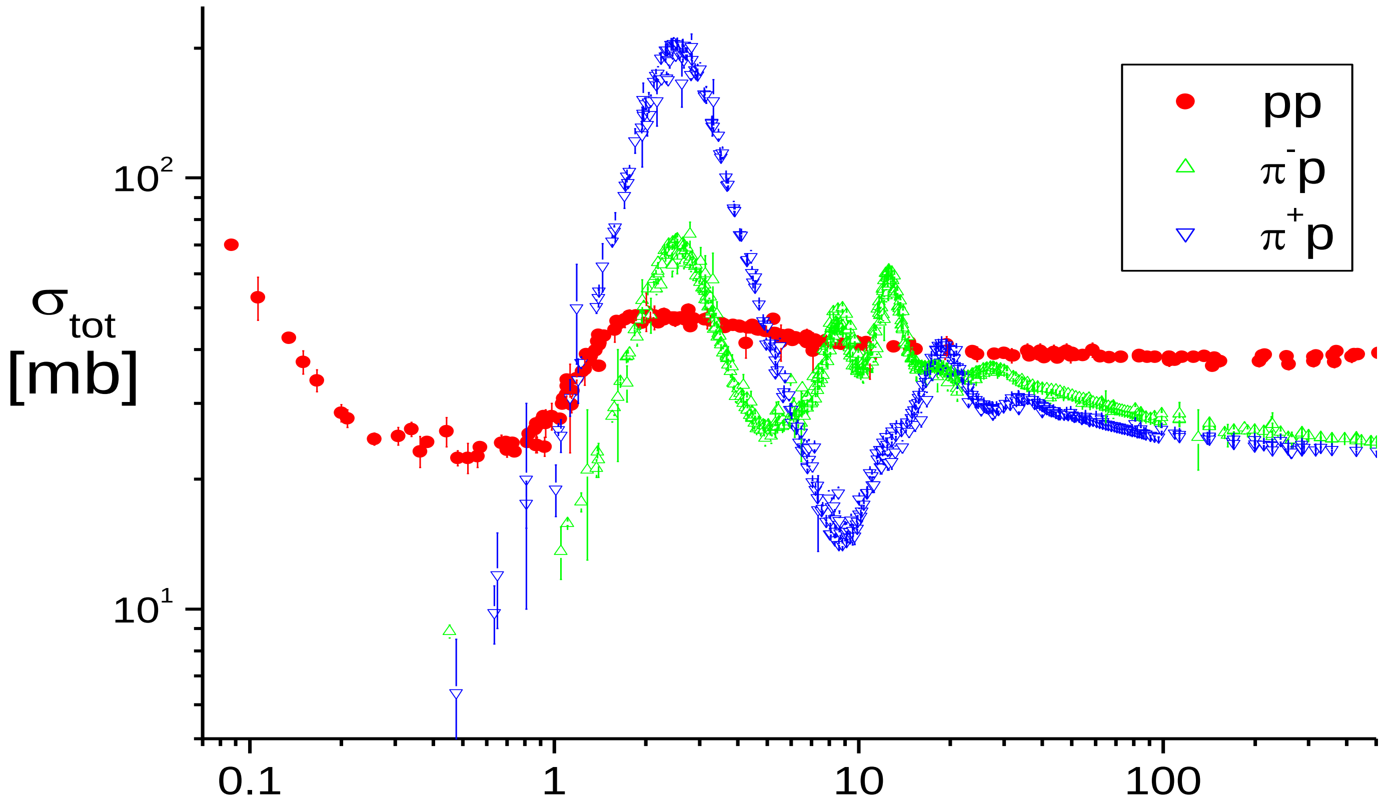
  relates to non-relativistic nuclear physics, where nucleons can be considered as point objects;

- the region $b_{ik} \sim 1$

  relates to excitation of internal degrees of freedom of hadrons;

- the region $b_{ik} >> 1$

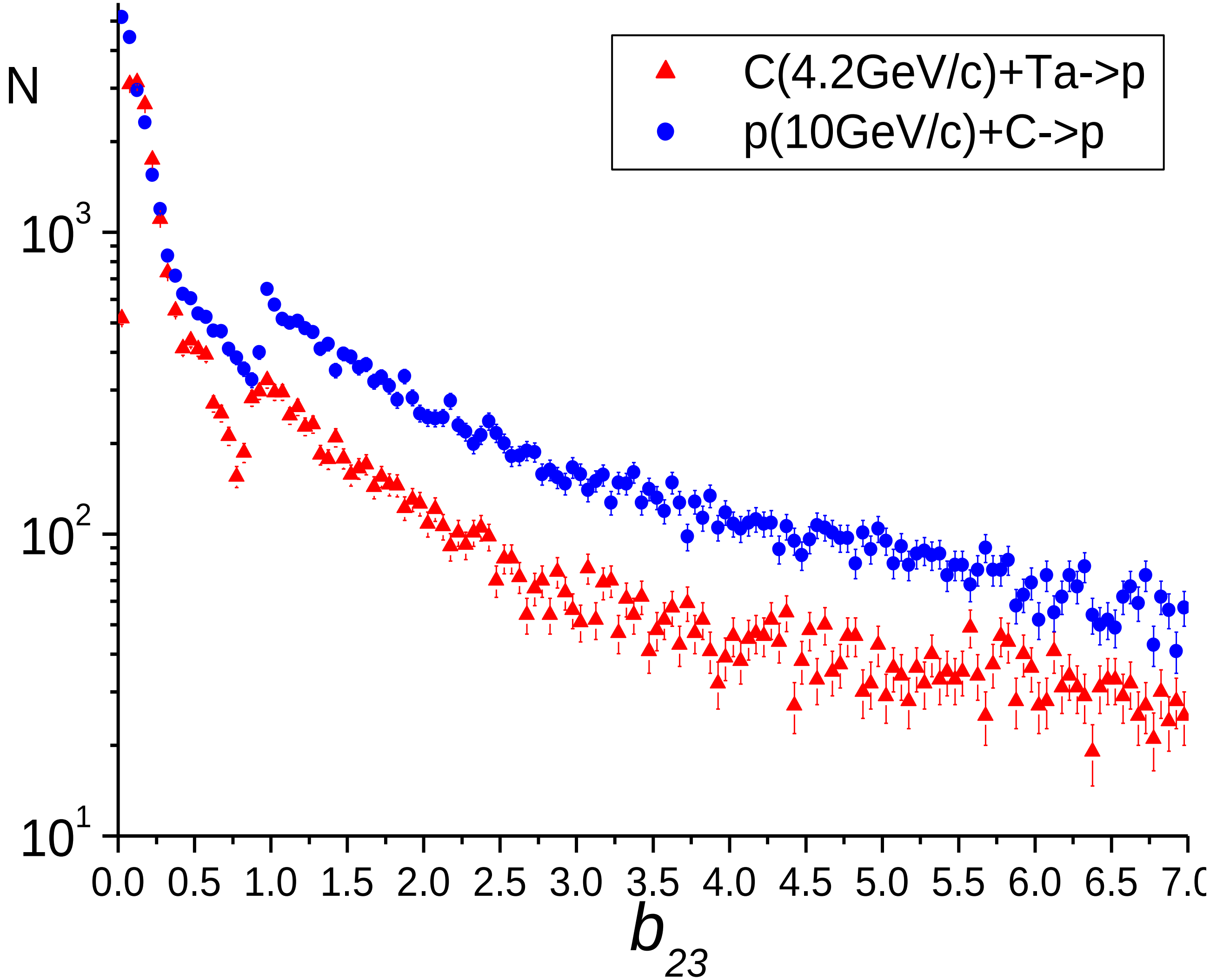
  should, in principle, be described by quantum chromodynamics.

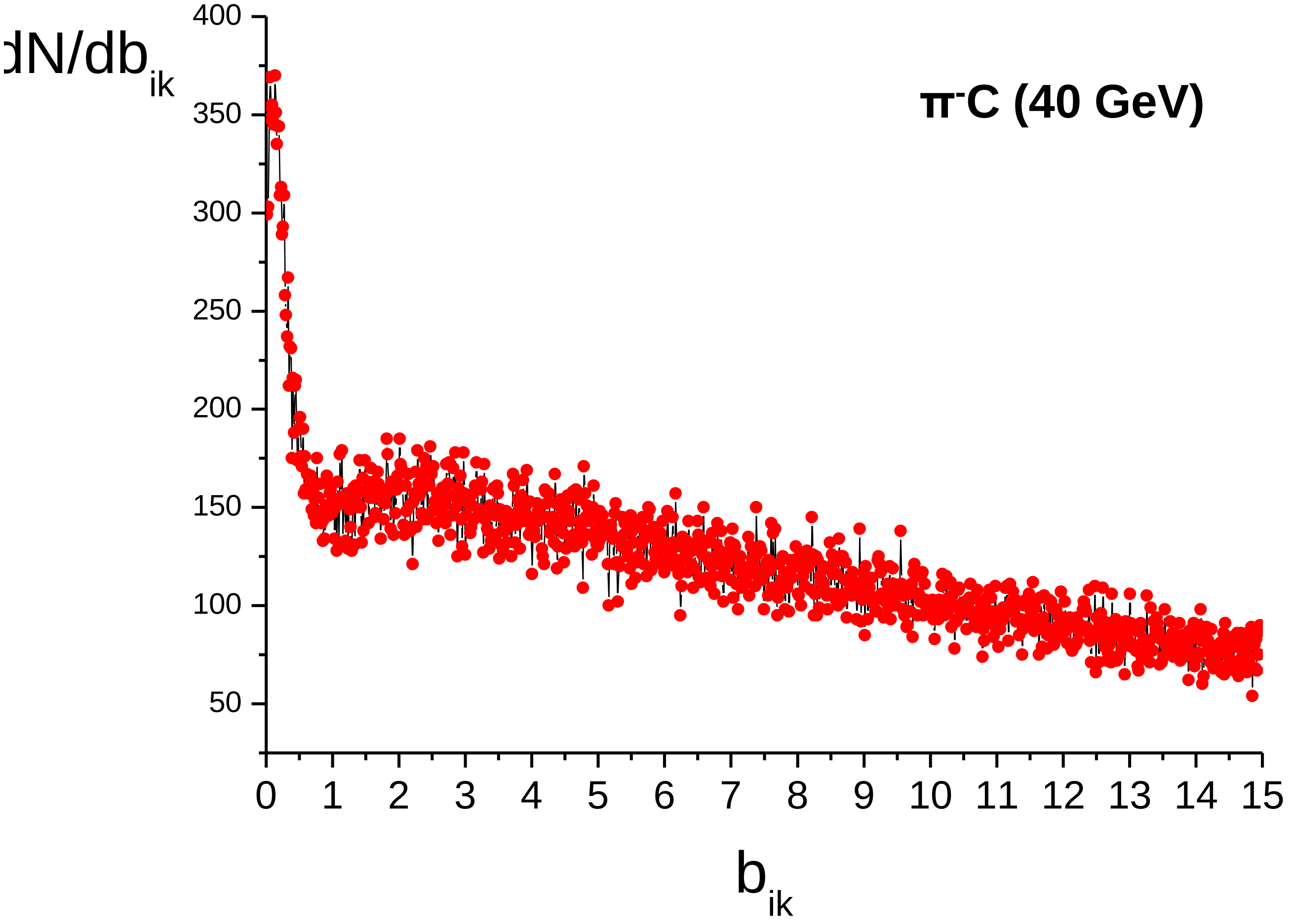


$$b_{ik} = 2\left(U_i U_k\right) - 1 = 2\left[\frac{P_i P_k}{m_i m_k} - 1\right]$$

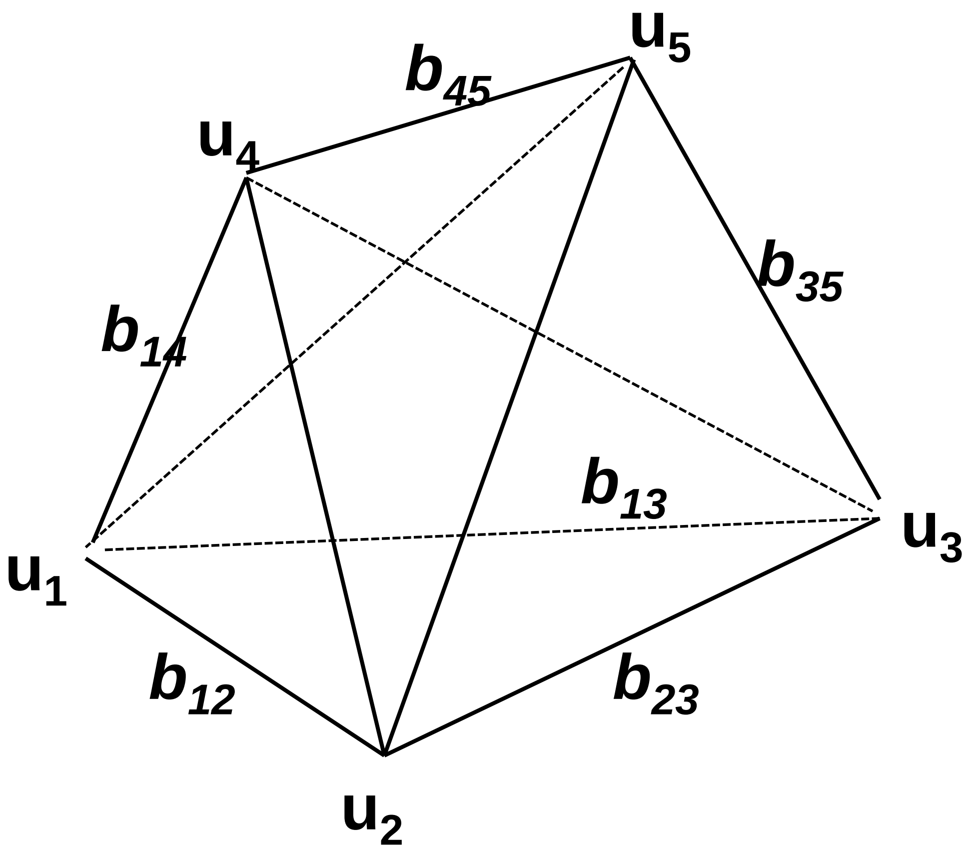
$$b_{12} = 2\left[\frac{E_{proj}}{m_{proj}} - 1\right]$$

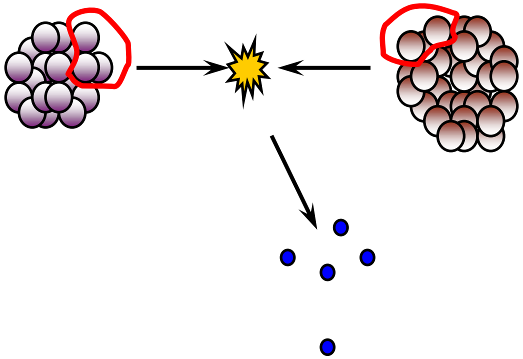
PDG K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002) (http://www-pdg.lbl.gov/)


N
C(4.2GeV/c)+Ta->p
p(10GeV/c)+C->p
10^3
10^2
10^1
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0
b_{23}

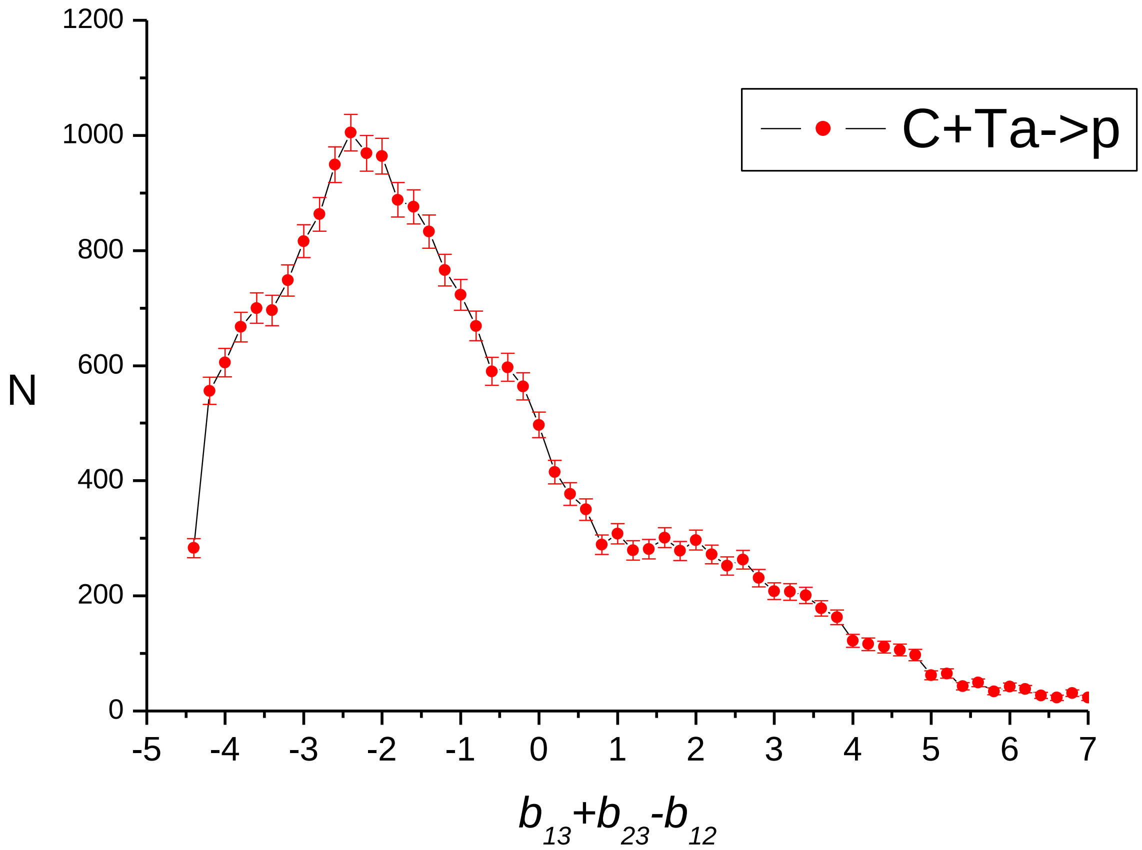


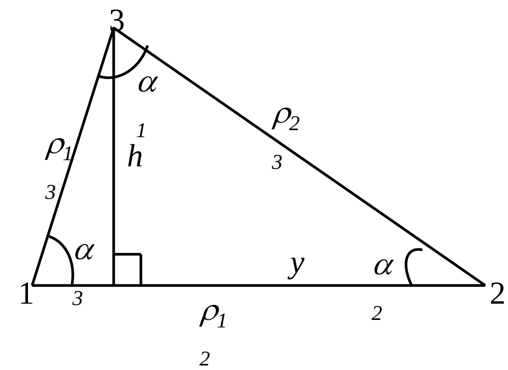
Here, all relative b_{ik} of particles produced in the reaction are considered


u_5
b_{45}
u_4
b_{35}
b_{14}
b_{13}
u_3
u_1
b_{12}
b_{23}
u_2

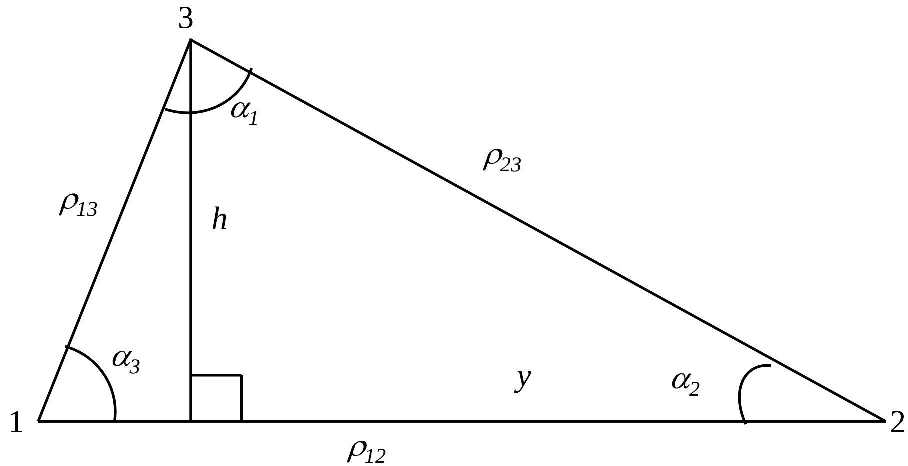


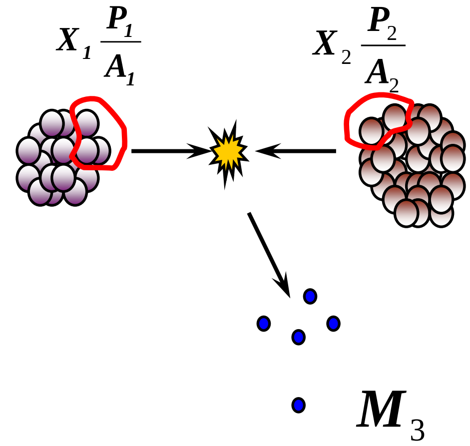
It should be noted that the variable b_{ik} does not form a metric space,
i.e. the relation $b_{12} + b_{13} \geq b_{23}$
is, generally speaking, wrong.





$$\min\left[-\sum_k \left(Y_\alpha - u_k^{\alpha}\right)^2 - \sum_i \left(Y_\beta - u_i^{\beta}\right)^2\right]$$





longitudinal rapidity $$y = \frac{1}{2}\ln\frac{E + p_{\parallel}}{E - p_{\parallel}}$$

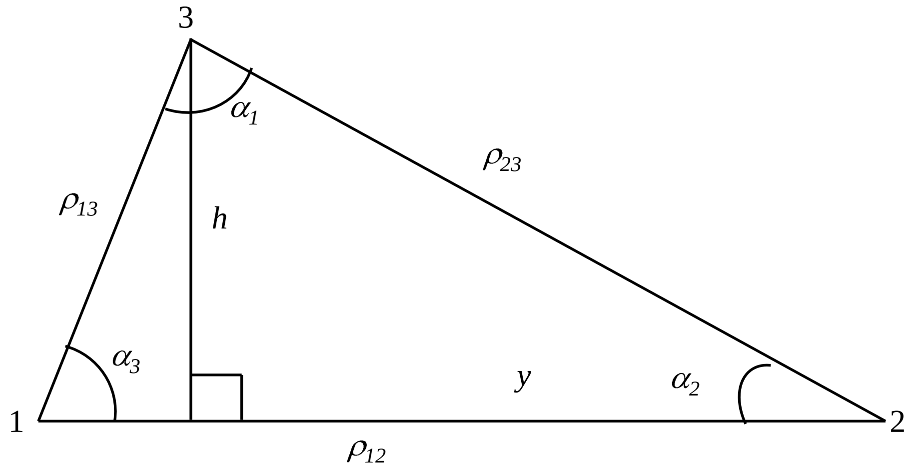
transverse mass : $$m_T = \sqrt{m^2 + p_T^2}$$

transverse rapidity $$\mathrm{ch}\, h = \frac{m_T}{m}$$

$$ch(\rho_{12}) = ch(\rho_{13}) \cdot ch(\rho_{23}) - sh(\rho_{13}) \cdot sh(\rho_{23}) \cdot \cos(\alpha_3)$$

$$\frac{sh(\rho_{12})}{\sin(\alpha_3)} = \frac{sh(\rho_{13})}{\sin(\alpha_2)} = \frac{sh(\rho_{23})}{\sin(\alpha_1)}$$

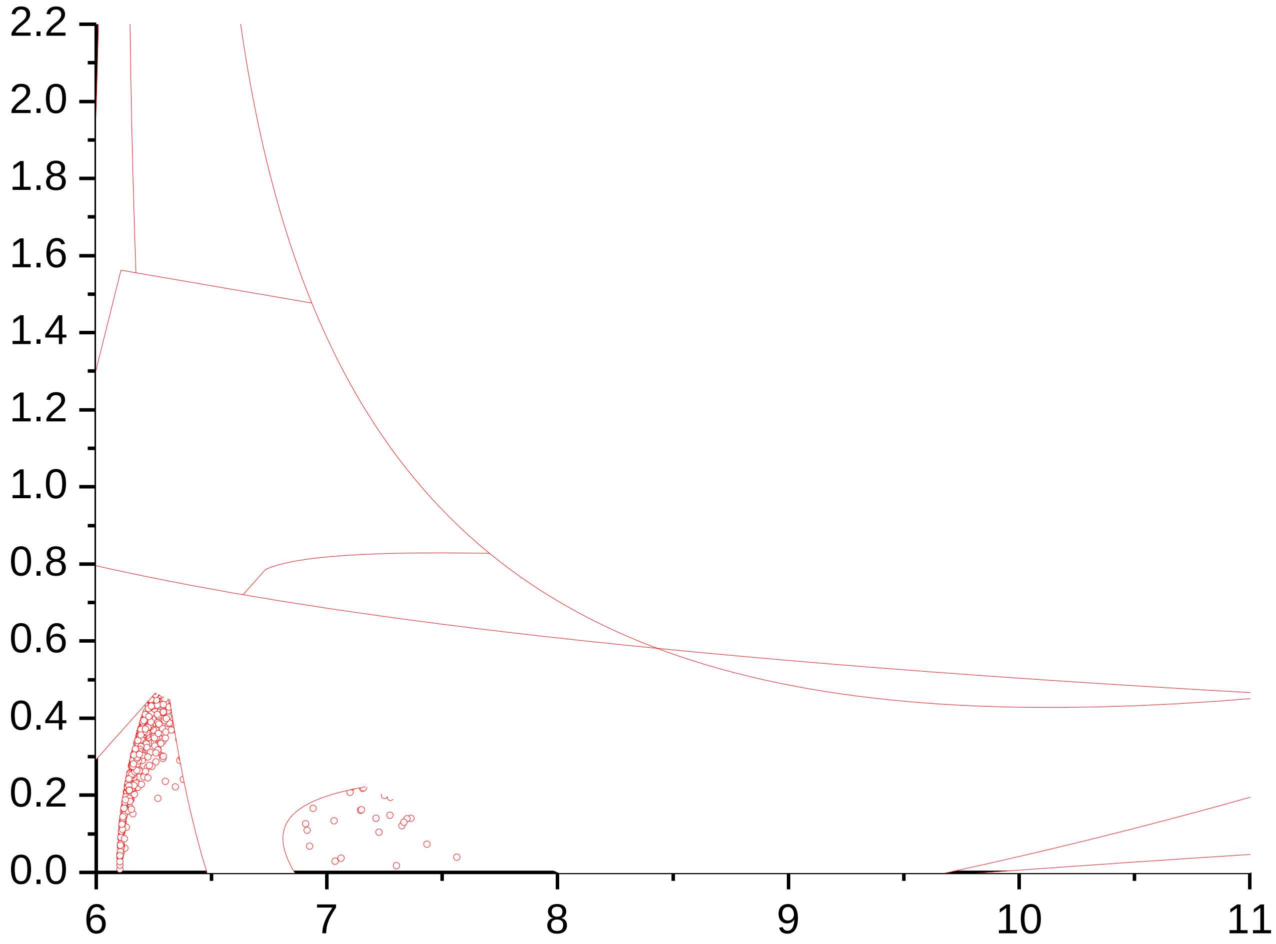
$$\mathrm{ch}\,\rho = \mathrm{ch}\, y \cdot \mathrm{ch}\, h$$



$$defect = \pi - \alpha_1 - \alpha_2 - \alpha_3$$

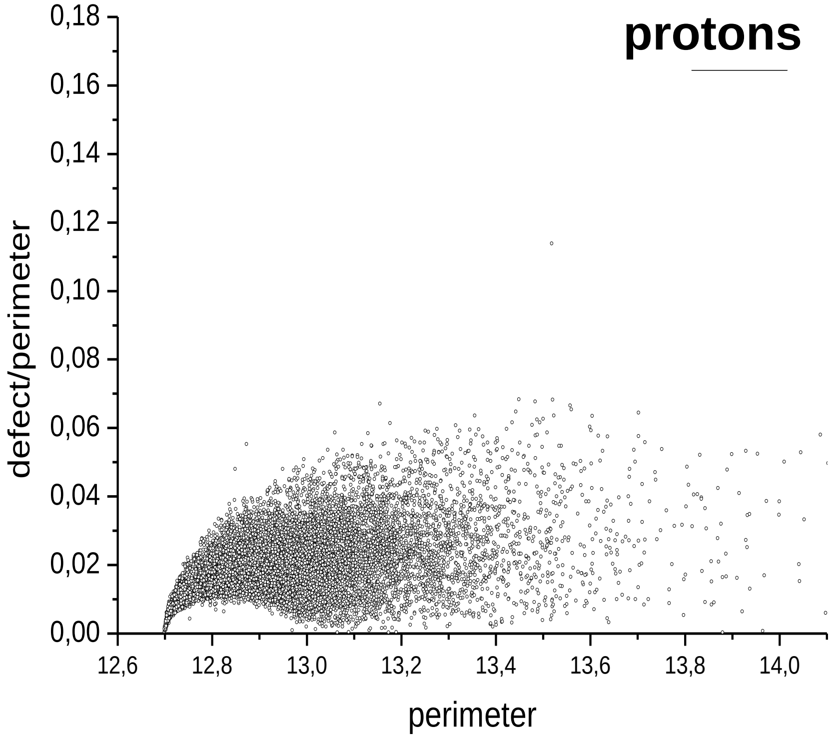
$$perimeter = \rho_1 + \rho_2 + \rho_3$$

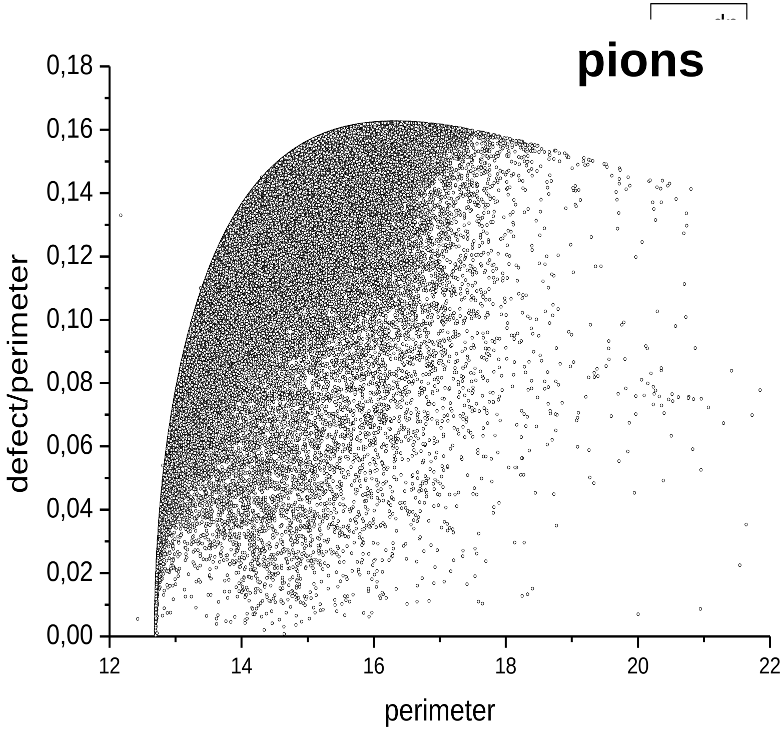
$$\Pi_L(h) = 2 \cdot arctg\left(e^{-h}\right)$$



2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
6
7
8
9
10
11

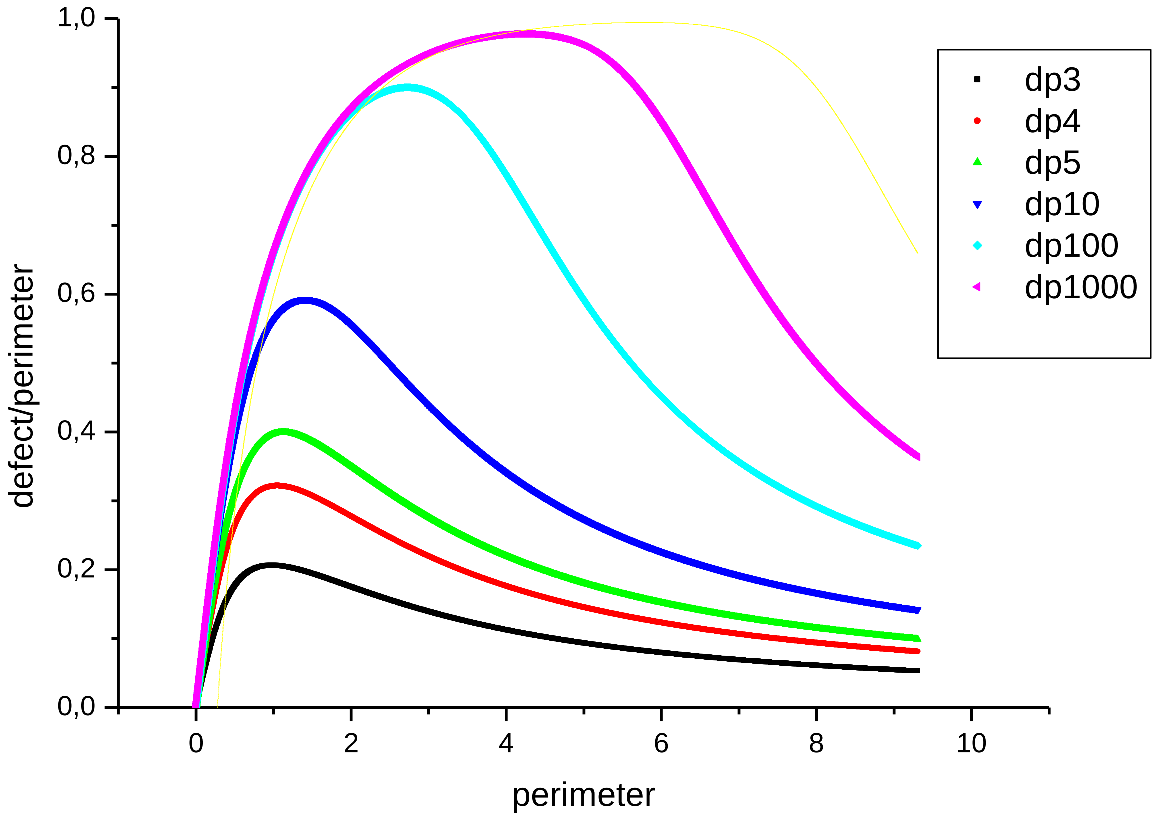
***It is important to stress that, unlike the Euclidean space, the area-to-perimeter ratio for triangles in the Lobachevski space is limited*.**

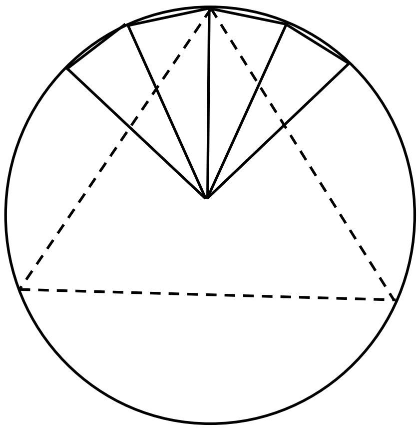




π^-C (40 GeV)

Analysis of Lobachevsky geometry

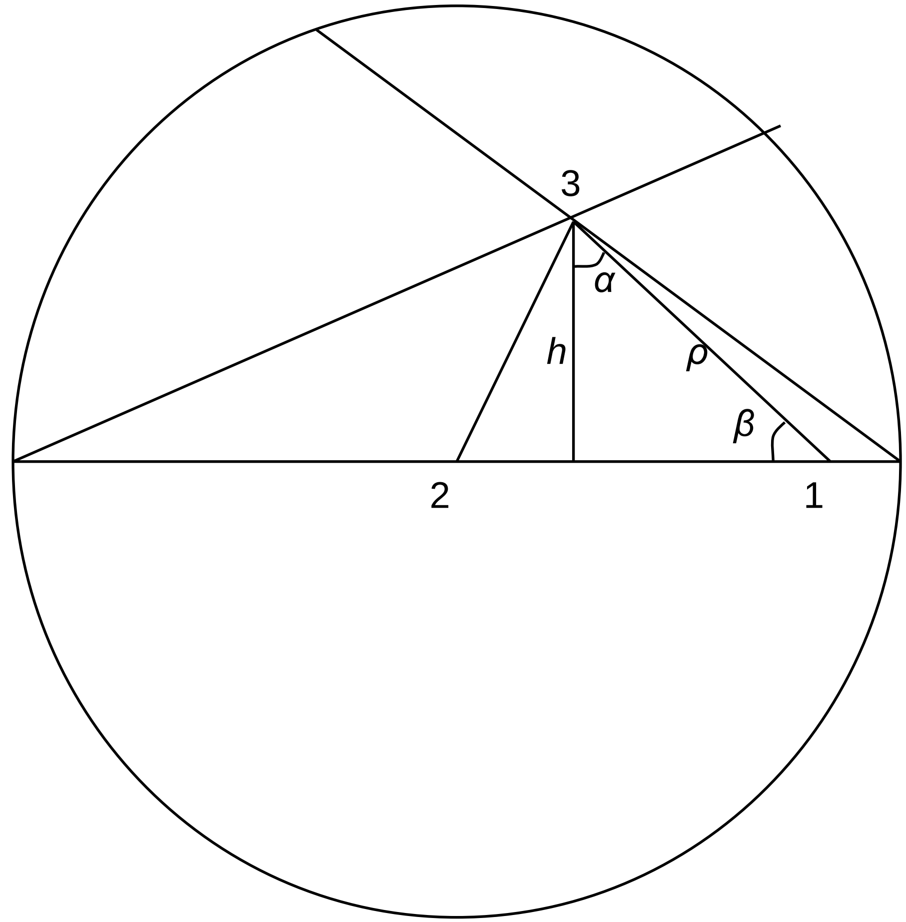




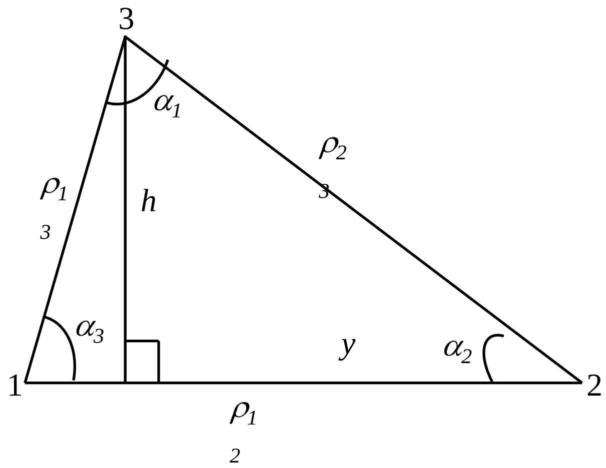
Regular polyhedrons with n=3, 4, 5, 10, 100, and 1000 inscribed in a circle with an increasing radius

$$tg\,\frac{\Pi_L(h)}{2} = e^{-h}$$

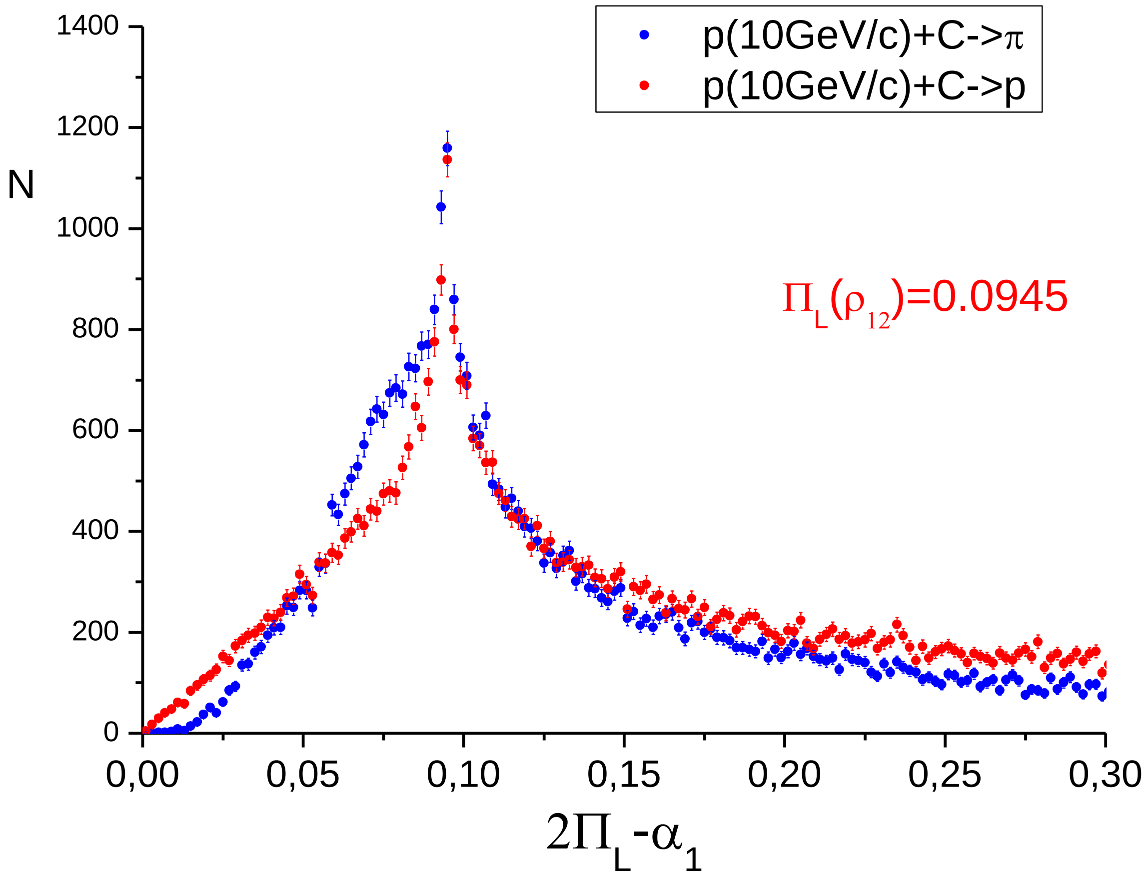
$$\Pi_L(h) = 2\cdot arctg\left(e^{-h}\right)$$



$$\Delta_{12}^{3} = 2\Pi_L\left(h_3\right) - \alpha_3$$

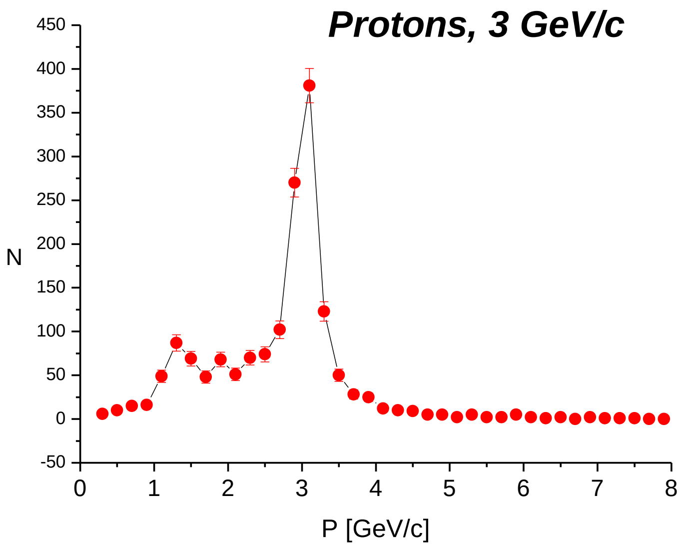


3
1
2
h
y
α_1
α_2
α_3
ρ_{13}
ρ_{23}
ρ_{12}

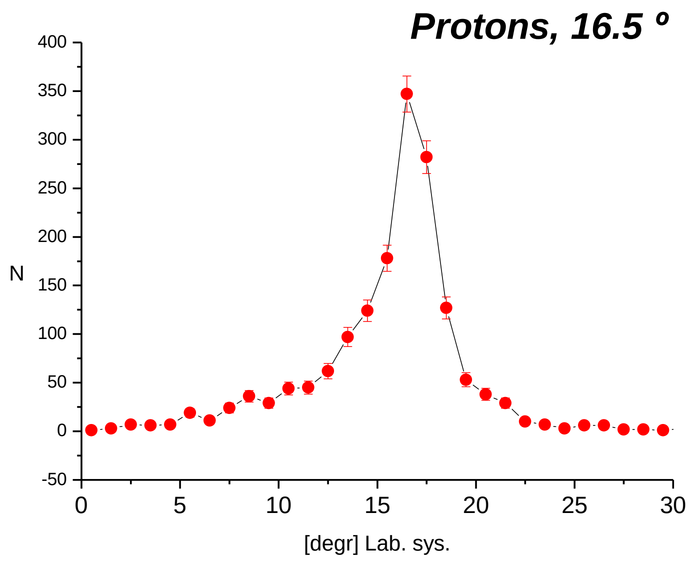


N
p(10GeV/c)+C->π
p(10GeV/c)+C->p
$\Pi_L(\rho_{12})$=0.0945
$2\Pi_L-\alpha_1$
0
200
400
600
800
1000
1200
1400
0,00
0,05
0,10
0,15
0,20
0,25
0,30

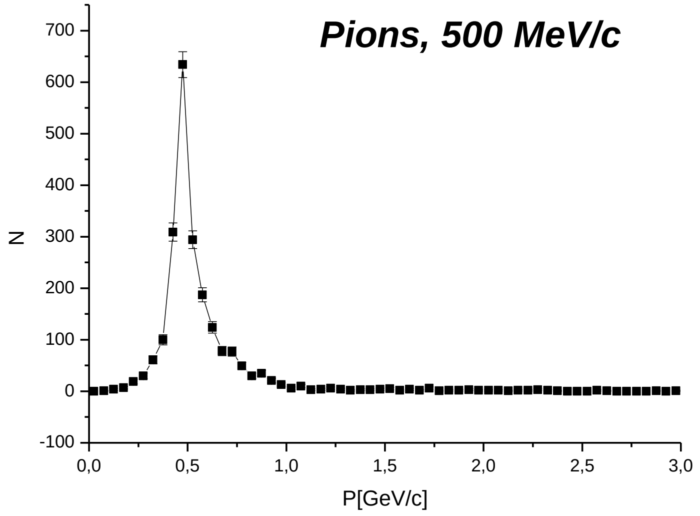
pC (10 GeV)



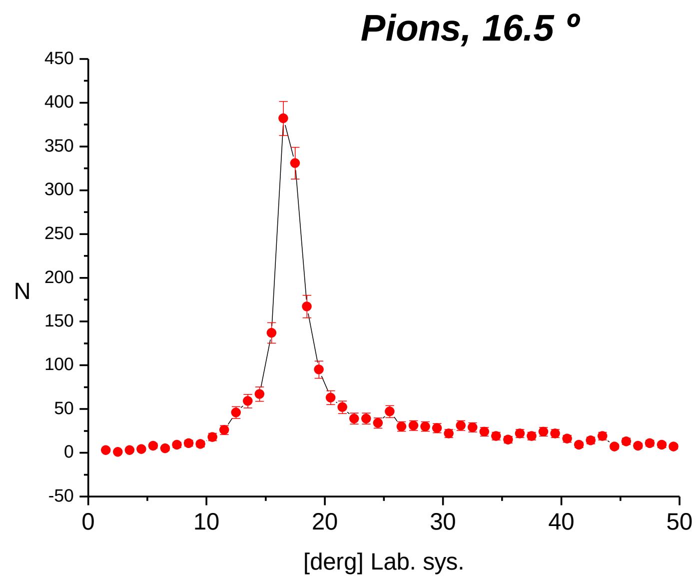
Protons, 3 GeV/c
N
P [GeV/c]



Protons, 16.5 º
N
[degr] Lab. sys.

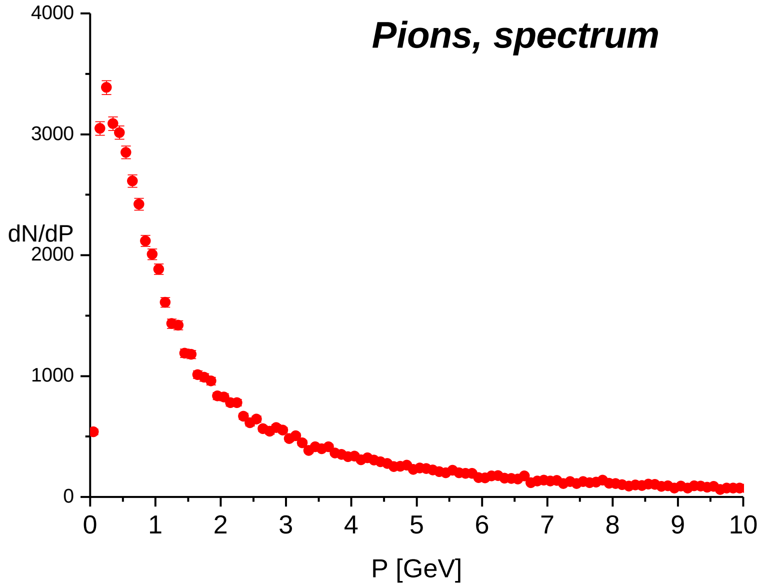


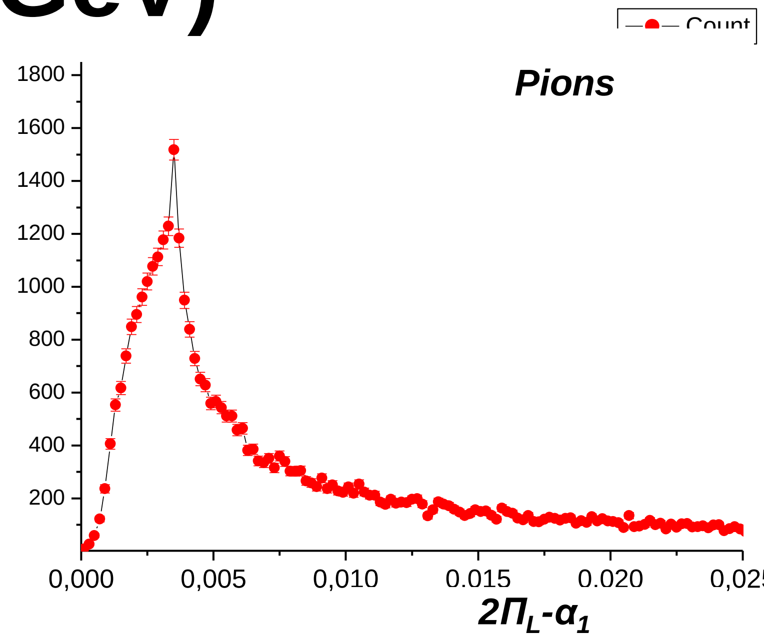
Pions, 500 MeV/c
N
P[GeV/c]

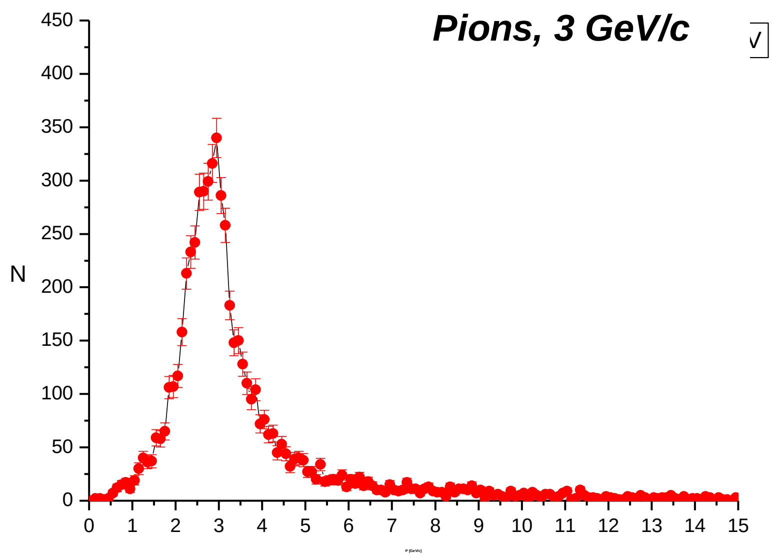


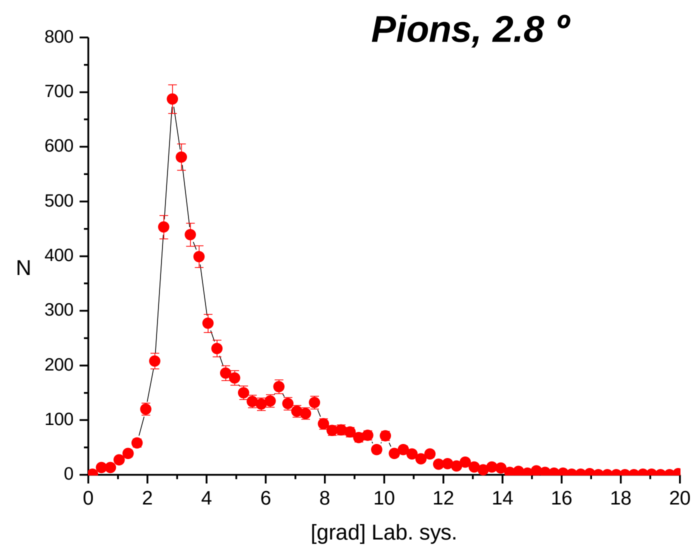
Pions, 16.5 º
N
[derg] Lab. sys.

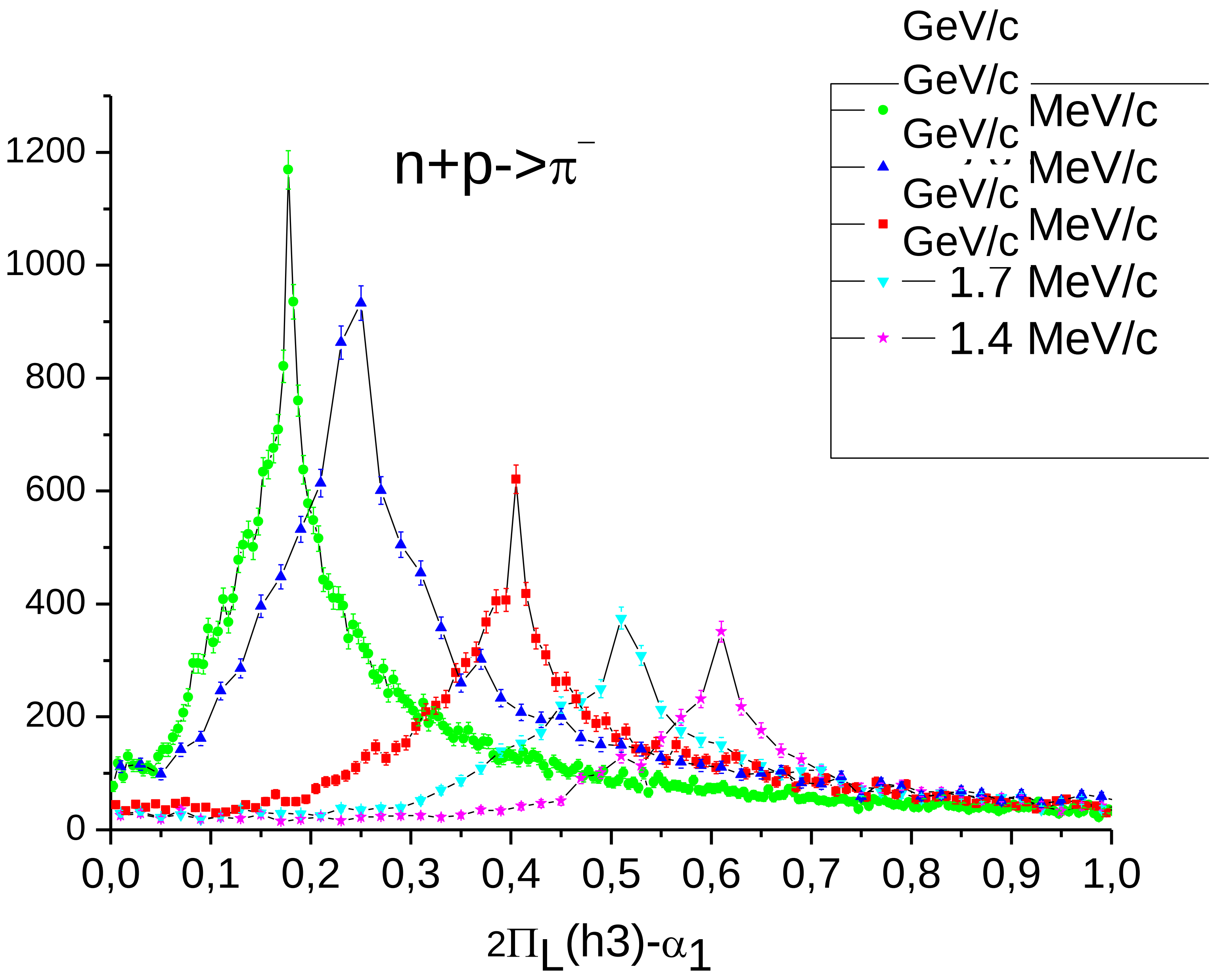
π^-C (40 GeV)


Pions, spectrum
dN/dP
P [GeV]
0
1000
2000
3000
4000
0 1 2 3 4 5 6 7 8 9 10


Count
Pions
200
400
600
800
1000
1200
1400
1600
1800
0,000 0,005 0,010 0.015 0.020
$2\Pi_L$-α_1

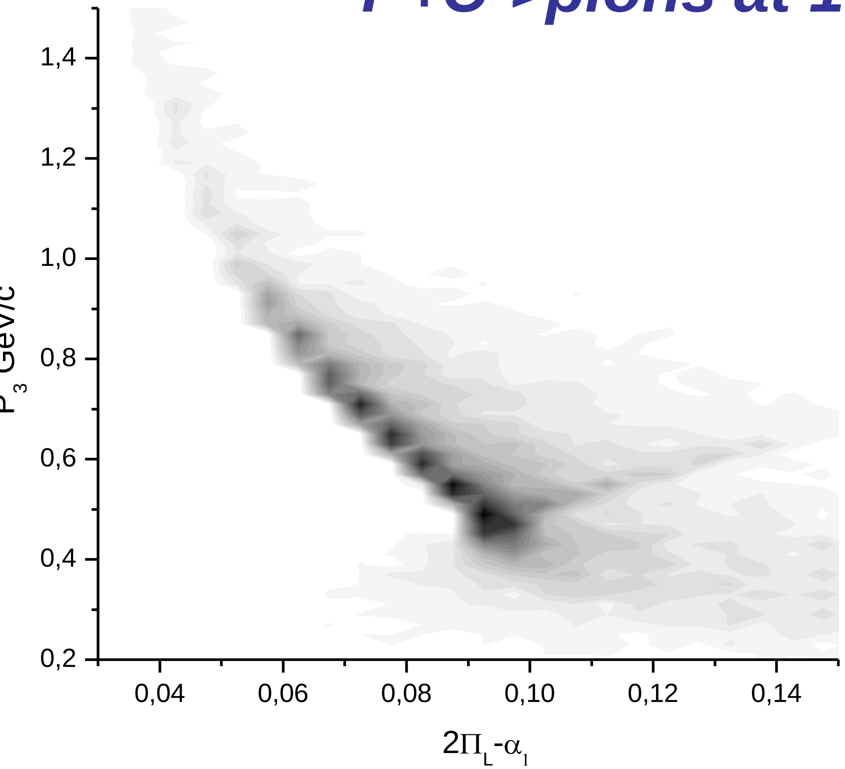

Pions, 3 GeV/c
N
0
50
100
150
200
250
300
350
400
450
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

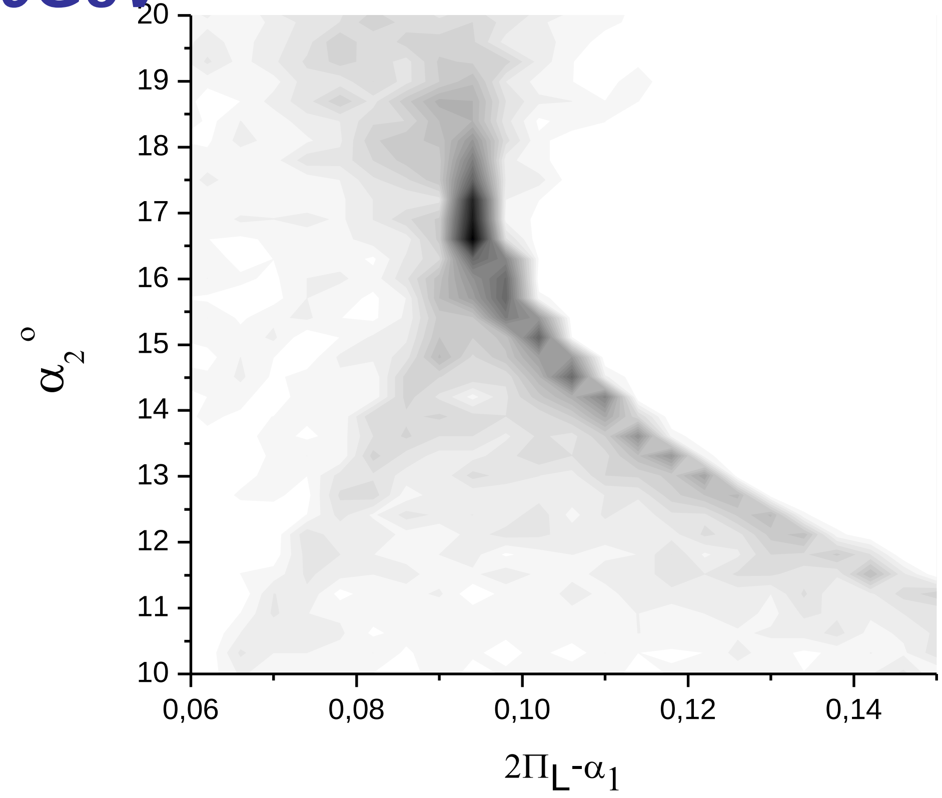

Pions, 2.8 º
N
0
100
200
300
400
500
600
700
800
0 2 4 6 8 10 12 14 16 18 20
[grad] Lab. sys.



n+p->π⁻
GeV/c
GeV/c
MeV/c
GeV/c
MeV/c
GeV/c
MeV/c
GeV/c
1.7 MeV/c
1.4 MeV/c
0
200
400
600
800
1000
1200
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
2ΠL(h3)-α1

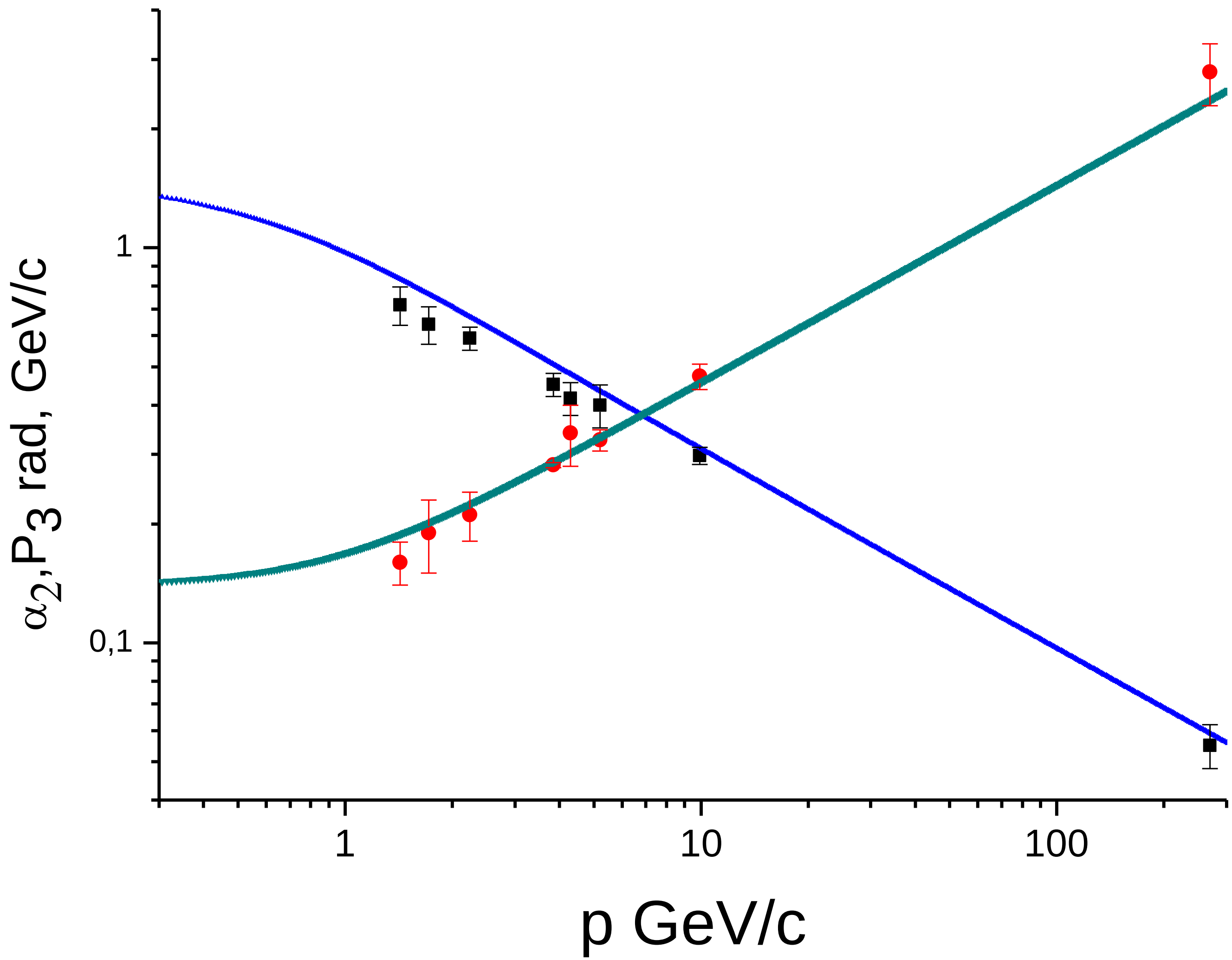
Directed Nuclear Radiation
P+C->pions at 10GeV

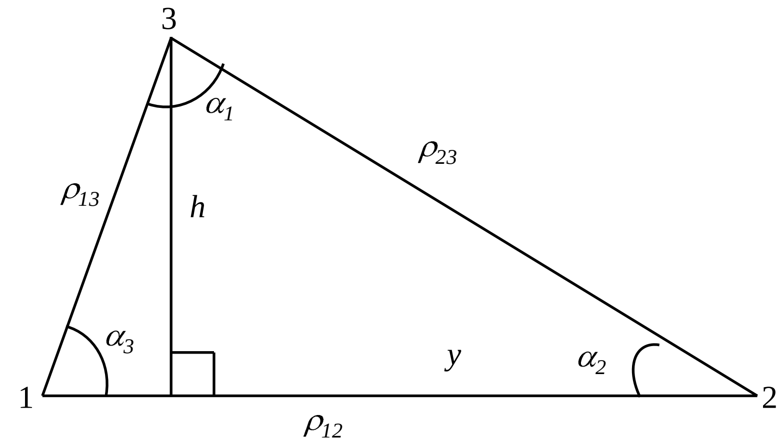




Directed Nuclear Radiation

$$\cos\alpha_2 = \sqrt{\frac{1+th(\rho_1)}{2}} - sh(\eta_3)\sqrt{\frac{1-th(\rho_1)}{2}}$$

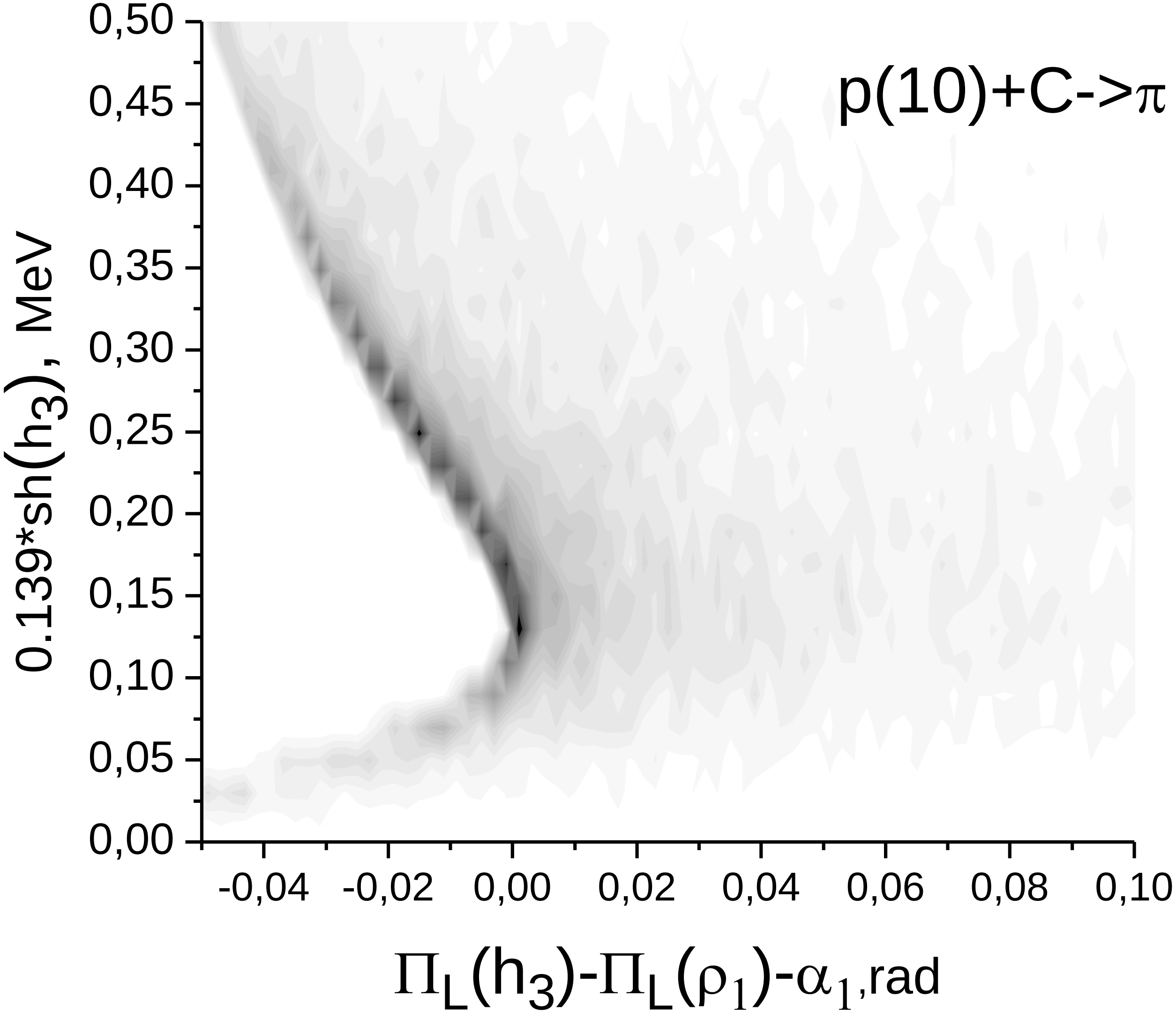




$$\cos\alpha_2 = \sqrt{\frac{1+th(\rho_1)}{2}} - sh(h_3)\sqrt{\frac{1-th(\rho_1)}{2}}$$

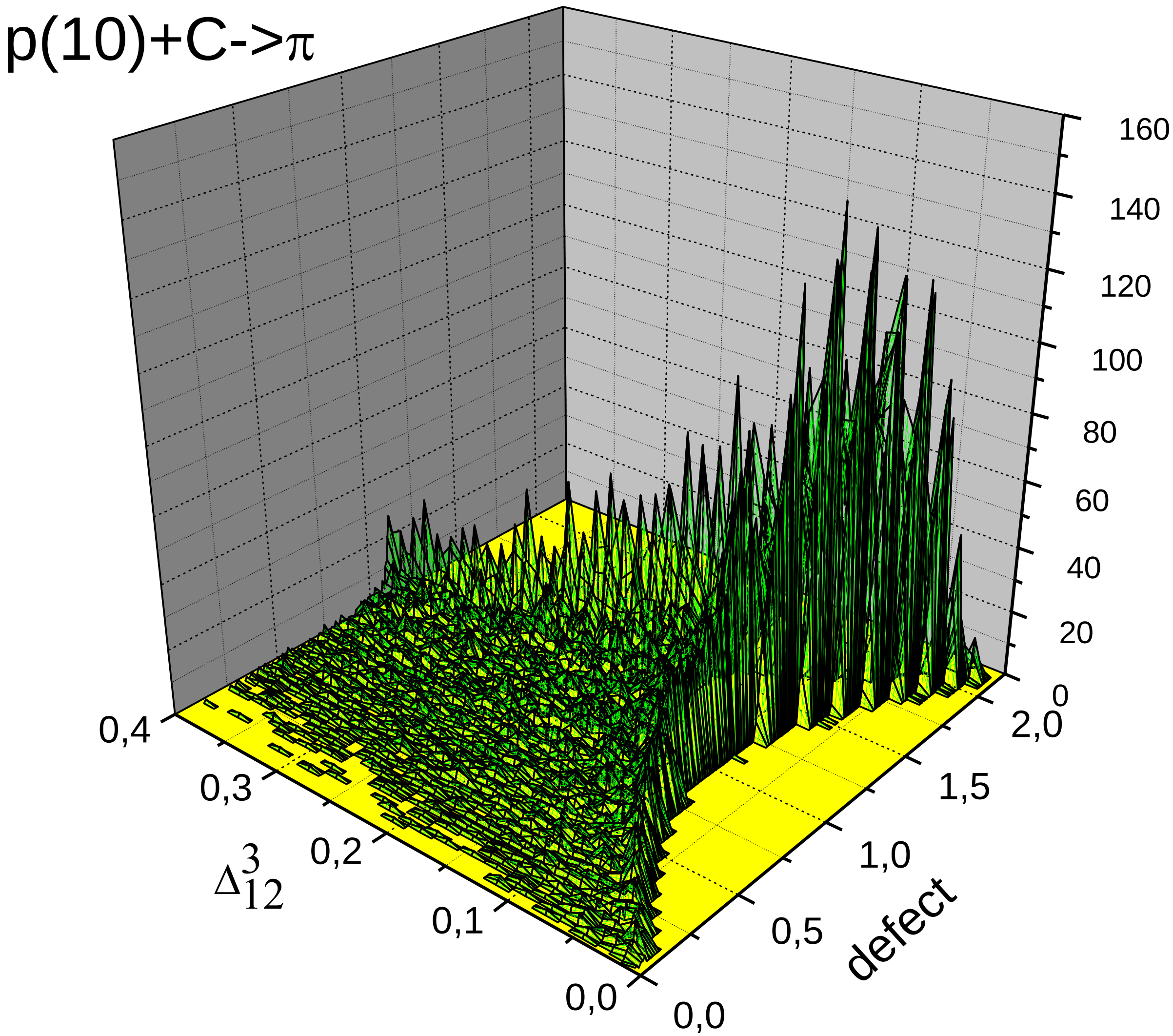
$$\alpha_1 = 2\arcsin\left\{\sqrt{\frac{1+th(\rho_1)}{2}}\sin(\Pi(h)) - \sqrt{\frac{1-th(\rho_1)}{2}}\cos(\Pi(h))\right\}$$

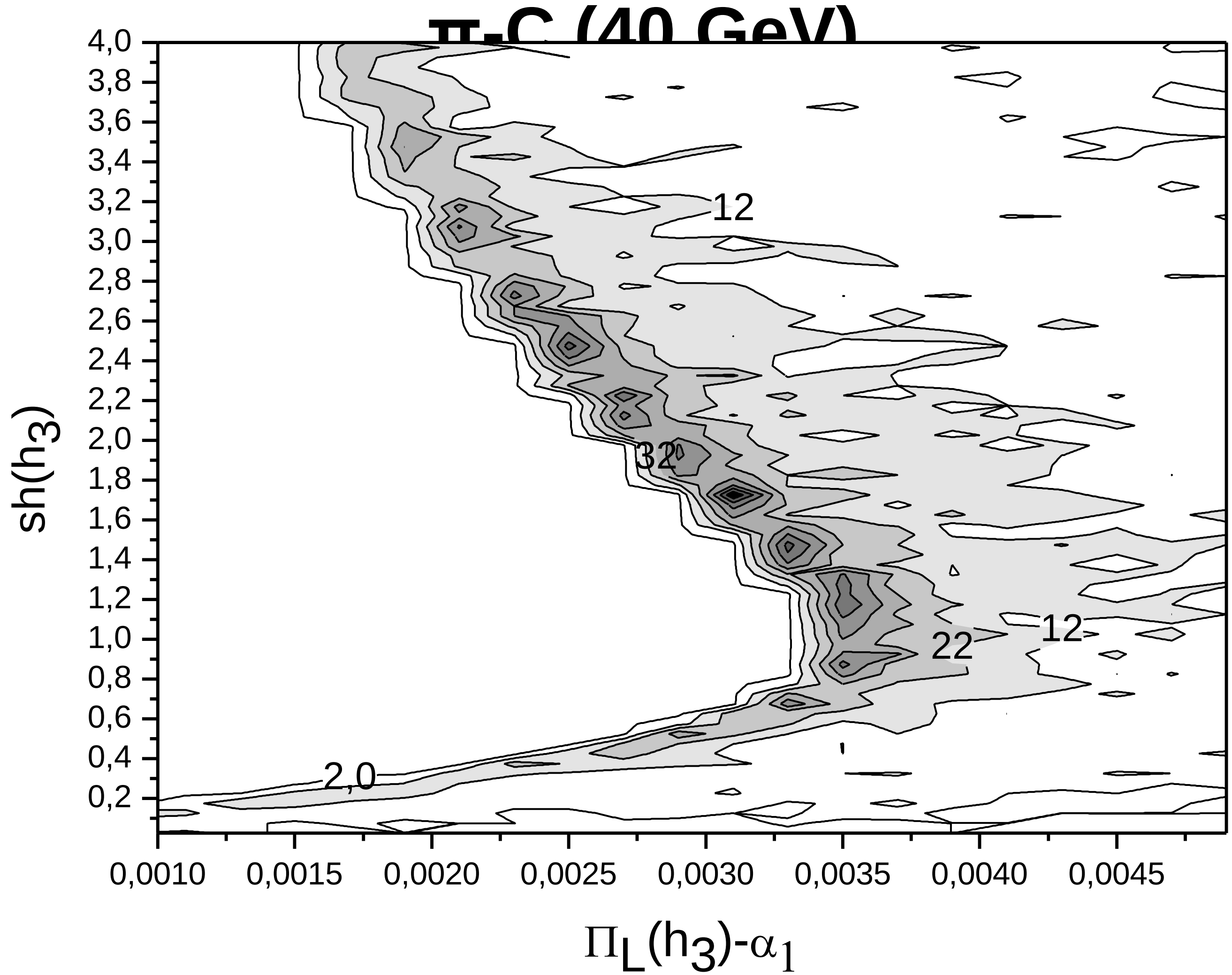
$$f(h) = 2\left(\Pi_L(h) - arctg\,\frac{th\left(\frac{\rho_1}{2}\right)}{sh(h)}\right) = 2arctg\left(\frac{\left(1-th\left(\frac{\rho_1}{2}\right)\right)\cdot sh(h)}{sh^2(h) + th\left(\frac{\rho_1}{2}\right)}\right)$$



p(10)+C->π
0.139*sh(h3), MeV
0,00
0,05
0,10
0,15
0,20
0,25
0,30
0,35
0,40
0,45
0,50
-0,04
-0,02
0,00
0,02
0,04
0,06
0,08
0,10
ΠL(h3)-ΠL(ρ1)-α1,rad

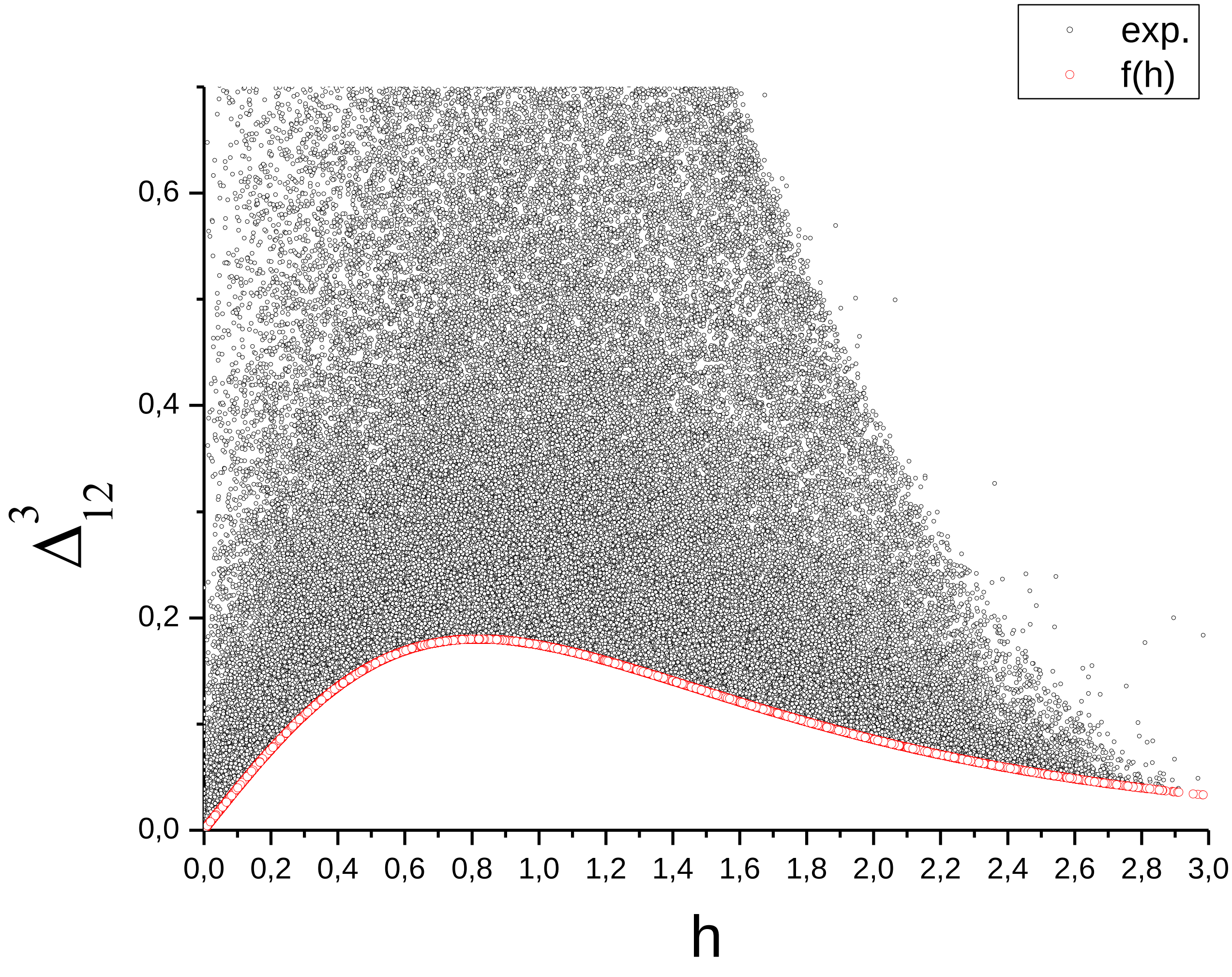
2


p(10)+C->π
160
140
120
100
80
60
40
20
0
0,4
0,3
0,2
0,1
0,0
Δ_{12}^{3}
0,0
0,5
1,0
1,5
2,0
defect



π-C (40 GeV)
sh(h3)
4,0
3,8
3,6
3,4
3,2
3,0
2,8
2,6
2,4
2,2
2,0
1,8
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0010
0,0015
0,0020
0,0025
0,0030
0,0035
0,0040
0,0045
ΠL(h3)-α1
12
32
22
12
2,0

n+p->π⁻ 5GeV



JOURNAL OF MATHEMATICAL PHYSICS VOLUME 39, NUMBER 9 SEPTEMBER 1998

A geometrical angle on Feynman integrals

A. I. Davydychev[a)]

Physics Department, University of Tasmania,
GPO Box 252-21, Hobart, Tasmania, 7001 Australia
and Institute for Nuclear Physics, Moscow State University, 119899, Moscow, Russia

R. Delbourgo[b)]

Physics Department, University of Tasmania,
GPO Box 252-21, Hobart, Tasmania, 7001 Australia

(Received 22 December 1997; accepted for publication 29 April 1998)

A direct link between a one-loop N-point Feynman diagram and a geometrical representation based on the N-dimensional simplex is established by relating the Feynman parametric representations to the integrals over contents of $(N-1)$-dimensional simplices in non-Euclidean geometry of constant curvature. In particular, the four-point function in four dimensions is proportional to the volume of a three-dimensional spherical (or hyperbolic) tetrahedron which can be calculated by splitting into birectangular ones. It is also shown that the known formula of reduction of the N-point function in $(N-1)$ dimensions corresponds to splitting the related N-dimensional simplex into N rectangular ones. [S0022-2488(98)00709-9]

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Nuclear Instruments and Methods in Physics Research A 559 (2006) 293–297

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

www.elsevier.com/locate/nima

Geometrical methods in loop calculations and the three-point function

Andrei I. Davydychev[a,b]

[a]Schlumberger, SPC, 155 Industrial Blvd., Sugar Land, TX 77478, USA
[b]Institute for Nuclear Physics, Moscow State University, 119992 Moscow, Russia

Available online 19 December 2005

Abstract

A geometrical way to calculate N-point Feynman diagrams is reviewed. As an example, the dimensionally regulated three-point function is considered, including all orders of its ε-expansion. Analytical continuation to other regions of the kinematical variables is discussed.

PACS: 11.15.Bt; 11.80.Cr; 11.55.Bq

Keywords: Feynman diagram; Three-point function; Dimensional regularization

ISSN 1472-2739 (on-line) 1472-2747 (printed)

Algebraic & Geometric Topology
Volume 3 (2003) 1–31
Published: 24 January 2002

The Regge symmetry is a scissors congruence in hyperbolic space

Yana Mohanty

Abstract We give a constructive proof that the Regge symmetry is a scissors congruence in hyperbolic space. The main tool is Leibon's construction for computing the volume of a general hyperbolic tetrahedron. The proof consists of identifying the key elements in Leibon's construction and permuting them.

Сибирский математический журнал
Сентябрь—октябрь, 2004. Том 45, № 5

УДК 514.13+514.132

ОБЪЕМ СИММЕТРИЧНОГО ТЕТРАЭДРА В ГИПЕРБОЛИЧЕСКОМ И СФЕРИЧЕСКОМ ПРОСТРАНСТВАХ

Д. А. Деревнин, А. Д. Медных, М. Г. Пашкевич

Аннотация: Получены элементарные формулы для вычисления объема симметричного тетраэдра в гиперболическом и сферическом пространствах.

Ключевые слова: гиперболический тетраэдр, сферический тетраэдр, формула объема, матрица Грама.

Сибирский математический журнал
Сентябрь—октябрь, 2004. Том 45, № 5

УДК 514.13+514.132

ОБЪЕМ СИММЕТРИЧНОГО ТЕТРАЭДРА В ГИПЕРБОЛИЧЕСКОМ И СФЕРИЧЕСКОМ ПРОСТРАНСТВАХ

Д. А. Деревнин, А. Д. Медных, М. Г. Пашкевич

Аннотация: Получены элементарные формулы для вычисления объема симметричного тетраэдра в гиперболическом и сферическом пространствах.

Ключевые слова: гиперболический тетраэдр, сферический тетраэдр, формула объема, матрица Грама.

Теорема 9. *Объем сферического симметричного тетраэдра T вычисляется по формуле*

$$-2\int_0^{\tau}(\operatorname{arsh}(\cos A\operatorname{sh}t)+\operatorname{arsh}(\cos B\operatorname{sh}t)+\operatorname{arsh}(\cos C\operatorname{sh}t)-t)\frac{dt}{\operatorname{sh}2t},$$

где τ — положительное число, которое находится из уравнения

$$\operatorname{cth}^2\tau=\frac{1-a^2-b^2-c^2-2abc}{\sqrt{(1-a+b+c)(1+a-b+c)(1+a+b-c)(1-a-b-c)}},$$

где $a=\cos A$, $b=\cos B$, $c=\cos C$.

CONCLUSIONS

- The new phenomenon of directed nuclear radiation was discovered based on analysis of experimental data obtained using bubble chambers.
- The connection between the basic notion of the Lobachevsky geometry, the angle of parallelism, and the experimentally observed directed nuclear radiation was established.
- The new variable $\Delta_{12}^{3} = 2\Pi_L(\alpha_3) - \alpha_3$ allows one to separate a particular class of events and can be used efficiently for analysis and selection of configurations (jets, e^+e^- identification, etc.) and correlations in multiparticle production. It also provides a new criterion for signal/background separation.
- Formulas describing directed nuclear radiation were obtained.
- The angle of directed nuclear radiation decreases with increasing relative velocity of interacting objects (energy), unlike Cherenkov radiation.
- The efficiency of the Lobachevsky space for analysis of experimental data on multiparticle production at relativistic energies, finding new effects, and planning future experiments was demonstrated.