# Combined Kinematical and Topological Method for Reconstruction of Neutral Strange Particles

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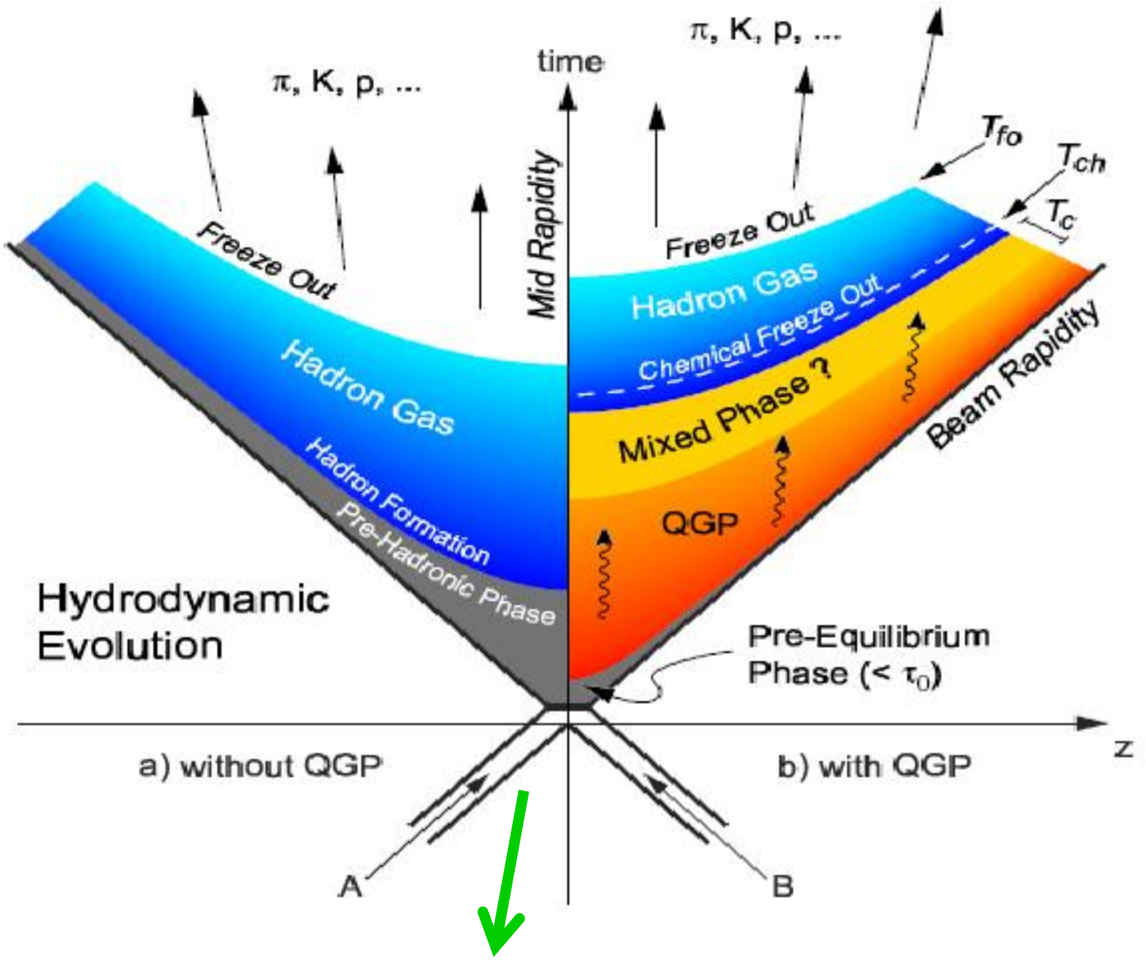
# Contents

# Phases of fireball evolution




New state of nuclear matter in heavy ion collisions is created (SPS,RHIC)

# Strange particles as probes of nuclear matter

Enhancement of strange particles yield and modification of resonance mass and width in QGP (T>150 MeV) are predicted by theory
*(Rafelski J,Muller B, Brown G., Rho M.,…)*

QGP

$$q+\bar{q}\rightarrow s+\bar{s}$$
$$g+g\rightarrow s+\bar{s}$$

$$E_{threshold}\approx 2m_S\approx 300MeV$$

Hadron gas

$$\pi+N\rightarrow\Lambda+K$$
$$K+\pi\rightarrow\bar{\Lambda}+N$$

$$E_{threshold}\approx 530MeV$$
$$E_{threshold}\approx 1420MeV$$

$$\sigma(s\bar{s})_{QGP}>\sigma(s\bar{s})_{HG}$$

Study of strange mesons and baryons is of special interest

$$K_S^0=(u\bar{s})\rightarrow\pi^+\pi^-(69\%)$$
$$\Lambda=(uds)\rightarrow p\pi^-(64\%)$$

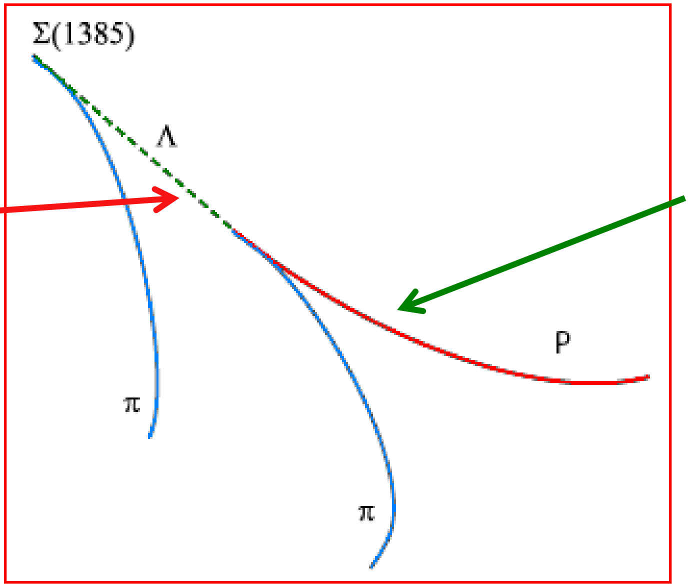
$$\Xi^-=(dss)\rightarrow\Lambda\pi^-(100\%)$$
$$\Omega^-=(sss)\rightarrow\Lambda K^-(68\%)$$

# Algorithm of V0 reconstruction

## V0 topology

Kinematic parameters of V0 particles are reconstructed by means of measurement of their daughter tracks parameters

Momentum of neutral strange particle is non directly measured




Momentum of charged particle is measured by the curvature of it trajectory

Topological cuts

- ✓(PV)- Primary vertex
- ✓(DV)- Decay vertex
- ✓Decay Length
- ✓DCA between V0 & PV
- ✓DCA between Pos & PV
- ✓DCA between Neg & PV
- ✓DCA between Pos & Neg

Standard topological cuts and kinematic fit method were used for neutral strange particle parameters reconstruction

# Method of Kinematic Fit

*The idea of kinematical fit is to use the known properties (constraints) of a given physical process to improve the measurements describing the process*

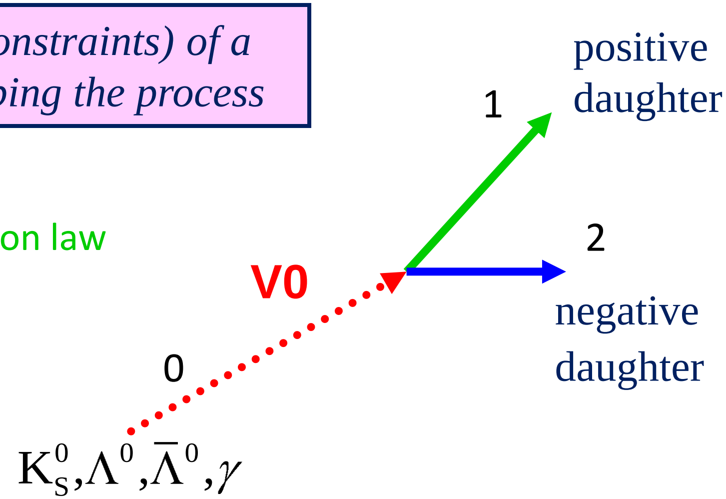
Minimization of $\chi^2$

$$\chi^2 = \min\sum_{i=2}^{9}\frac{(x_i - x_{mi})^2}{\delta x_i^2}$$

Constraints :

energy & momentum conservation law

$$\begin{cases} \vec{p}_0 = \vec{p}_1 + \vec{p}_2 \\ E_0 = E_1 + E_2 \end{cases}$$




**definition of kinematic parameters $x_i$**

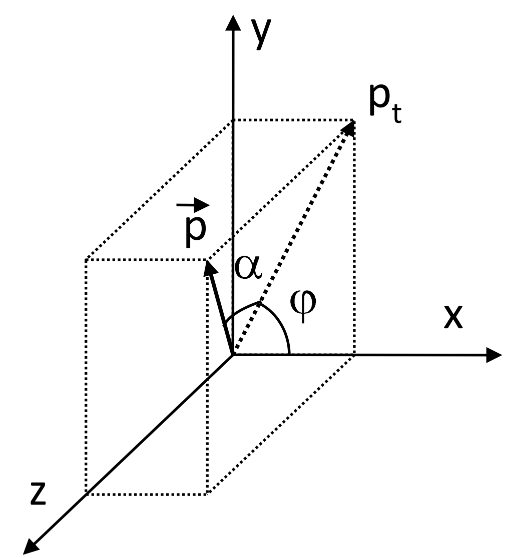
$x_1 = 1/p_{t0}$, $x_2 = \varphi_0$, $x_3 = \tan\alpha_0$ for V0

$x_4 = 1/p_{t1}$, $x_5 = \varphi_1$, $x_6 = \tan\alpha_1$ for positive daughter track

$x_7 = 1/p_{t2}$, $x_8 = \varphi_2$, $x_9 = \tan\alpha_2$ for negative daughter track

$x_{mi}$ – mesuared value of $x_i$

$\delta x_i$ – errors of $x_{mi}$




$$\begin{cases} p_{xi} = p_{ti} \cdot \cos\phi_i \\ p_{yi} = p_{ti} \cdot \sin\phi_i \\ p_{zi} = p_{ti} \cdot \tan\alpha_i \end{cases}$$

$$E_i = \sqrt{m_i^2 + p_{ti}^2 \cdot (1 + \tan^2\alpha_i)}$$

i=0,2,3

**Selection criteria for V0 particles: $\chi^2 < 10$**

# Numerical method of $\chi^2$ minimization

We need to find constraint minimum of the $\chi^2$ for reconstruction of V0 kinematic parameters:

$$\chi^2 = \min \sum_{i=1}^{8} \left( \frac{x_i - x_{mi}}{\delta x_i} \right)^2$$

where constraints are:

$$P_1 = p_{0x} - p_{1x} - p_{2x} = 0 \quad P_2 = p_{0y} - p_{1y} - p_{2y} = 0$$

$$P_3 = p_{0z} - p_{1z} - p_{2z} = 0 \quad P_4 = E_0 - E_1 - E_2 = 0$$

The Lagrange multipliers method is used to minimize $\chi^2$

$$F(\bar{x}) = \sum_{i=1}^{8} \left( \frac{x_i - x_{mi}}{\delta x_i} \right)^2 + \sum_{j=1}^{4} \lambda_j P_j, \quad \bar{x} = (x_0, \ldots, x_{12})$$

$$x_9 = \lambda_1, x_{10} = \lambda_2$$
$$x_{11} = \lambda_3, x_{12} = \lambda_4$$

$$\frac{\partial F(\bar{x})}{\partial x_i} = 0, \quad i = 0, \ldots, 12$$

System of nonlinear equations was solved using the Newton's method

$$\sum_{j=0}^{12} \left. \frac{\partial^2 F(\bar{x})}{\partial x_i \partial x_j} \right|_{\bar{x} = \bar{x}^k} \Delta x_j = -F(\bar{x}^k),$$

$\Delta x_j = x_j^{k+1} - x_j^k$, k - number of iteration step

# Computer simulation

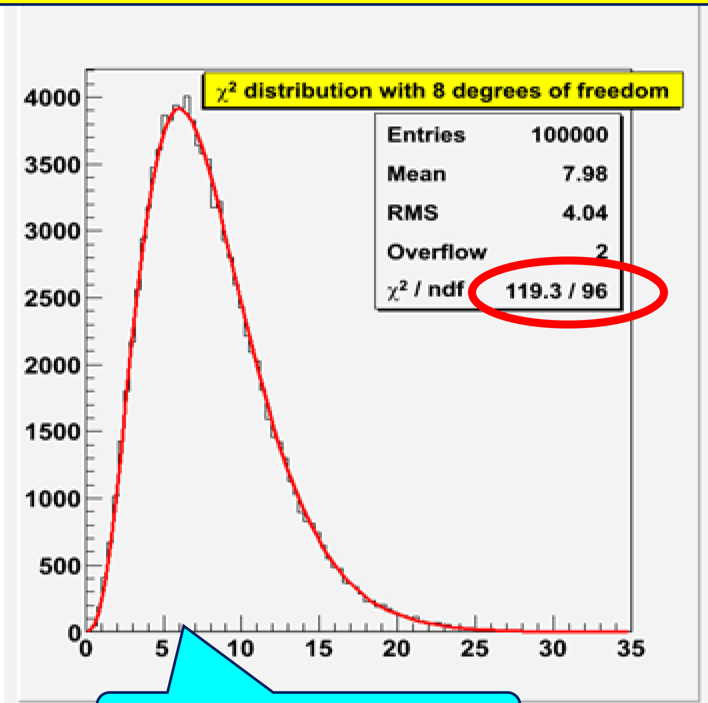
1. **C++ program for $\chi^2$ minimization was written**
2. **Simulated data were used for program testing**

⬇

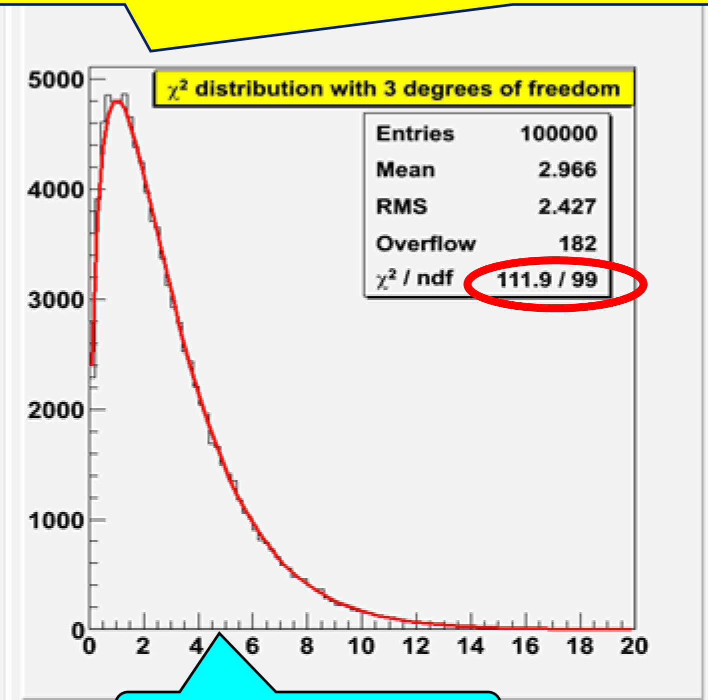
- ✓ Neutral strange particles were generated with exponential transverse momentum distribution
- ✓ 100 000 events of $\Lambda^0 \rightarrow p^+\pi^-$ and $K_S^0 \rightarrow \pi^+\pi^-$ decays were generated
- ✓ Measurement errors were simulated by Gauss distribution.
- ✓ Computer simulation was executed in ROOT kit using FOWL generator

# Test of kinematic fit method for $\chi^2$ distribution

$\chi^2$ distribution with 3 degrees of freedom was obtained after minimization




Before minimization




After minimization

**$\chi^2$ distribution with 3 degrees of freedom**

$$\varphi_{\chi^2(3)} = \frac{1}{\sqrt{2\pi}}\sqrt{x}e^{-x/2}$$

**Mean value and dispersion**

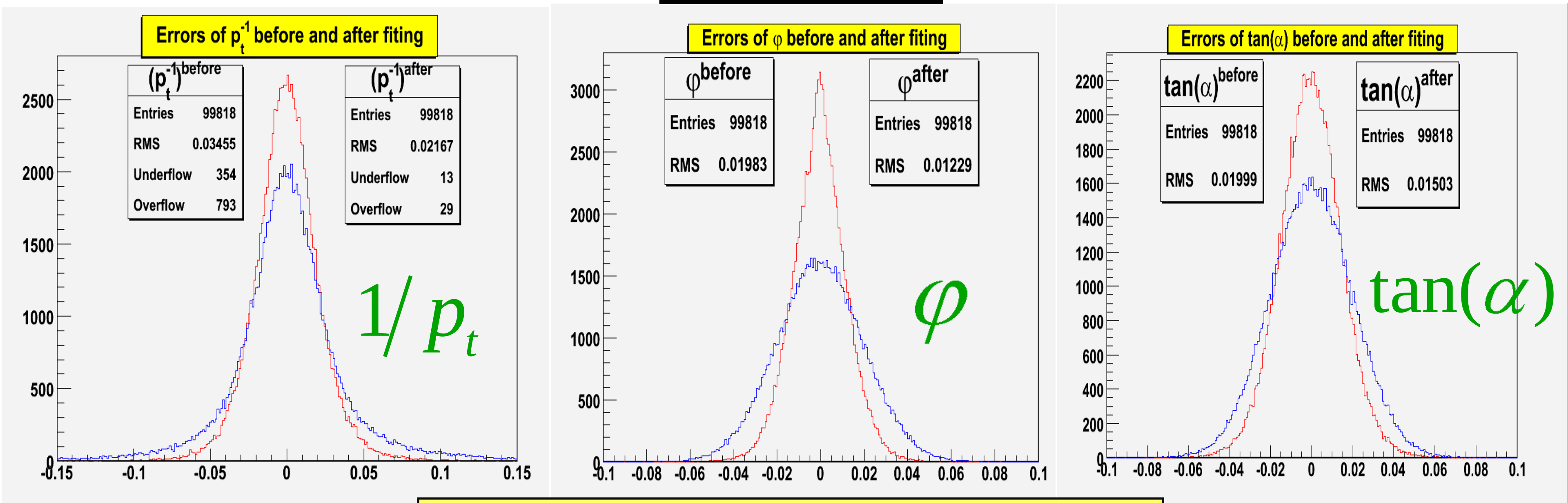
$$\overline{\chi^2(3)} = 3$$

$$\sigma\left(\chi^2(3)\right) = 2.45$$

➢ *Test results are in good agreement with expected distribution*

# Test of kinematic fit method for $K_S^0$ - parameters

$K_s^0 \rightarrow \pi^+ \pi^-$




- Gaussian distributions of generated errors

Errors of kinematic parameters before and after fit →

| | $p_t^{-1}$ | $\varphi$ | $\tan(\alpha)$ |
|---|---|---|---|
| before | 0.03455 | 0.01983 | 0.01999 |
| after | 0.02167 | 0.01229 | 0.01503 |

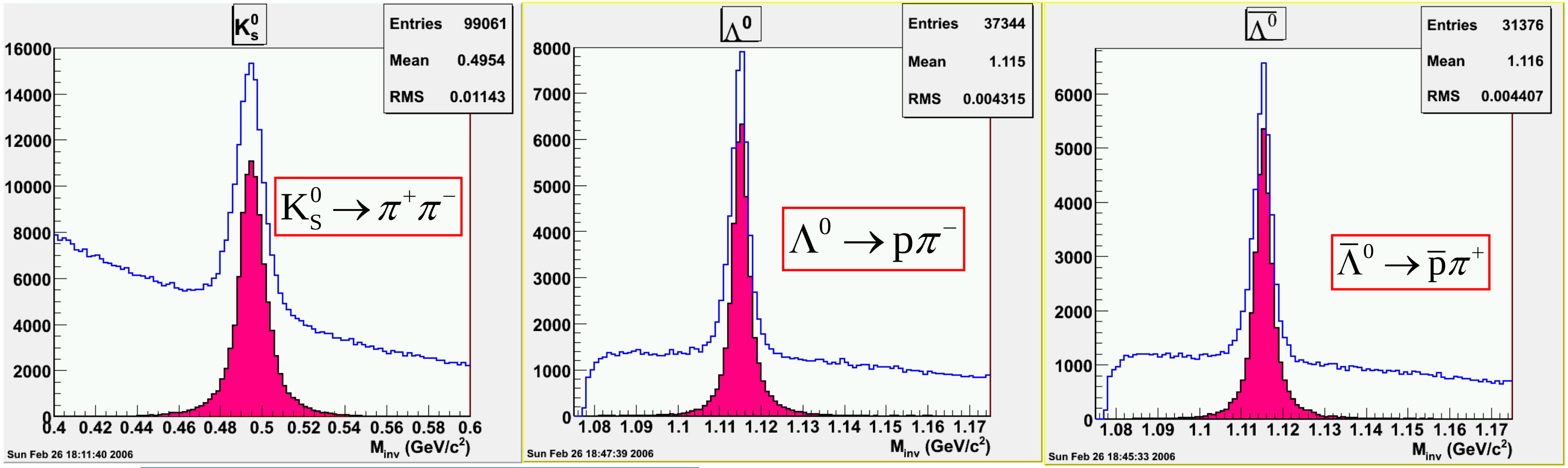
- *Errors of kinematic parameters after fit are less than generated ones*

# Analysis of d+Au sample

STAR data d+Au @ 200 GeV

P04if – production 13M events

$M_{inv}$ distributions of $K_S^0, \Lambda, \bar{\Lambda}$




— STAR standard method:

- topological cuts
- kinematical cut on $M_{inv}$

— Method of kinematical fit:

- cut on $\chi 2$ (<10)

- *Strong suppression of background without losses of statistics*
- *High quality of any distributions with strange particles and resonances*

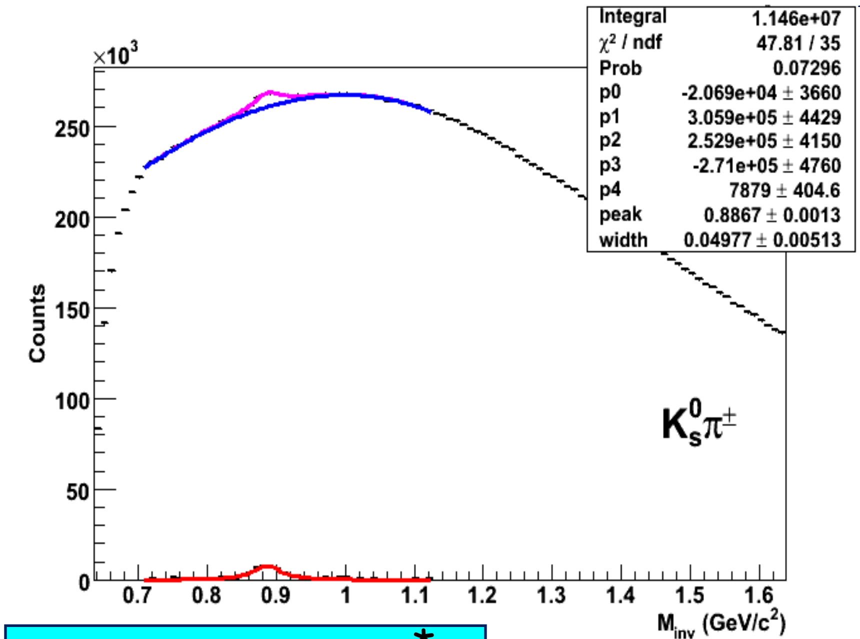
# $K^{*\pm}$ resonance reconstruction

principal mode of $K^{*\pm}$ decay

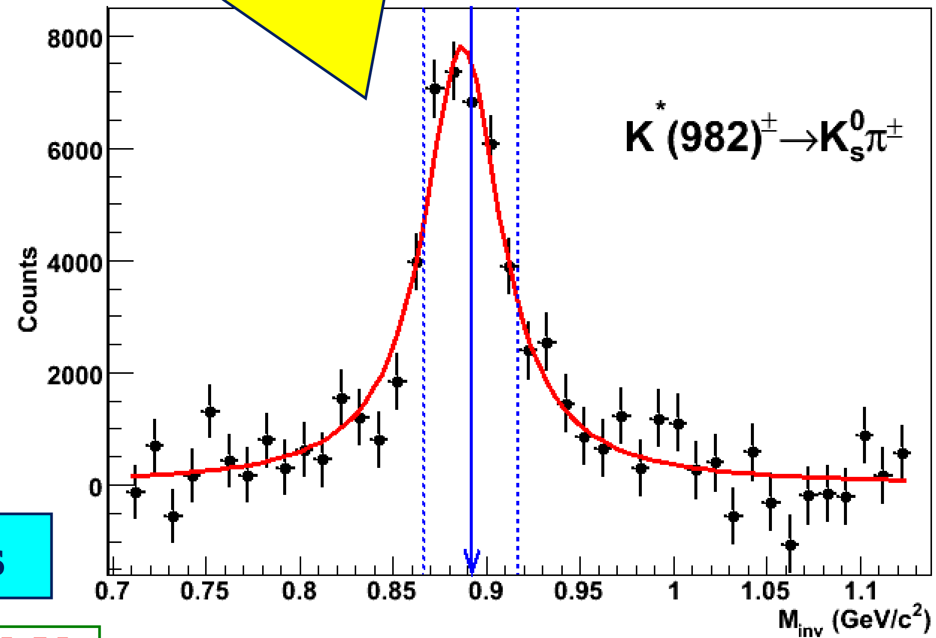
$K^{*\pm} \rightarrow K_S^0 \pi^\pm \sim 100\%$

data fit : $f(x) = BW + BG$

Background: $BG = p_3 x^3 + p_4 x^2 + p_5 x + p_6$




$K^{*\pm}$ distribution without background




**PDG values of $K^{*\pm}$**

$m_{K^*} = 891.66 \pm 0.26 MeV$

$\Gamma = 50.8 \pm 0.9 MeV$

**Fit values**

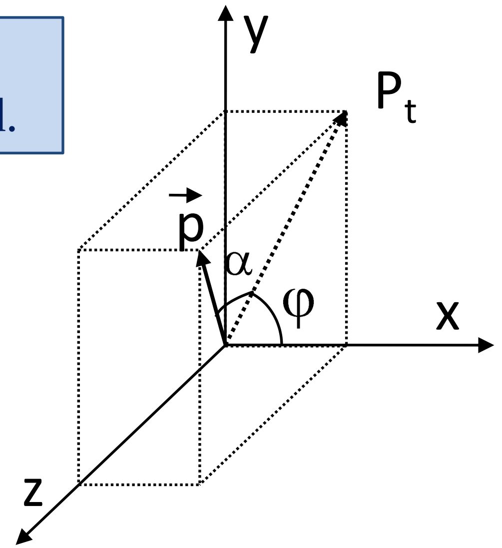
$m = 886.7 \pm 1.3 MeV$

$\Gamma = 49.77 \pm 5.13 MeV$

➢ *Kinematic fit allows to get true values of resonances mass and width*

# Method of combined kinematical and geometrical fit

New method allows us to reduce number of topological cuts.
Improvement of efficiency and statistics increase are expected.




Tracks of V0's daughter particles in TPC have helix topology

parametric equation of helix for i & j tracks

$$x(s_{i,j}) = x_{0_{i,j}} + \frac{1}{\gamma a_{i,j}}[\cos(\Phi_{0_{i,j}} + h_{i,j}s_{i,j}\gamma a_{i,j}\cos\lambda_{i,j}) - \cos\Phi_{0_{i,j}}]$$

$$y(s_{i,j}) = y_{0_{i,j}} + \frac{1}{\gamma a_{i,j}}[\sin(\Phi_{0_{i,j}} + h_{i,j}s_{i,j}\gamma a_{i,j}\cos\lambda_{i,j}) - \sin\Phi_{0_{i,j}}]$$

$$z(s_{i,j}) = z_{0_{i,j}} + s_{i,j}\sin\lambda_{i,j}$$

Measured kinematic parameters of particle's track

$$1/p_t = a, \quad \varphi, \quad tg\lambda$$

$$\Phi_0 = \varphi - h\pi/2$$

also we have coordinates of each track first point in TPC…

$$x_{0i}, y_{0i}, z_{0i}, x_{0j}, y_{0j}, z_{0j}$$

…and coordinates of primary vertex

$$x_s = 0, y_s = 0, z_s = 0$$

# Kinematical constraints

**Momentum-energy conservation law**

$$p_{t0}\cos\varphi_0 - p_{ti}\cos\varphi_i - p_{tj}\cos\varphi_j = 0 \qquad p_{t0}\sin\varphi_0 - p_{ti}\sin\varphi_i - p_{tj}\sin\varphi_j = 0$$

$$p_{t0}tg\lambda_0 - p_{ti}tg\lambda_i - p_{tj}tg\lambda_j = 0$$

$$\sqrt{m_0^2 + p_{t0}^2\left(1+tg^2\lambda_0\right)} - \sqrt{m_i^2 + p_{ti}^2\left(1+tg^2\lambda_i\right)} - \sqrt{m_j^2 + p_{tj}^2\left(1+tg^2\lambda_j\right)} = 0$$

Tangent, sine & cosine of V0 particle are expressed via coordinates of primary and secondary vertices

$$\sin\varphi_0 = \frac{y_{v0} - y_s}{\sqrt{\left(x_{v0} - x_s\right)^2 + \left(y_{v0} - y_s\right)^2}}$$

$$\cos\varphi_0 = \frac{x_{v0} - x_s}{\sqrt{\left(x_{v0} - x_s\right)^2 + \left(y_{v0} - y_s\right)^2}}$$

$$tg\lambda_0 = \frac{z_{v0} - z_s}{\sqrt{\left(x_{v0} - x_s\right)^2 + \left(y_{v0} - y_s\right)^2}}$$

$x_{v0}, y_{v0}, z_{v0}$ are coordinates of decay vertex

# Topological constraints

Helices intersection conditions

$$x_i - x_j = 0 \quad y_i - y_j = 0 \quad z_i - z_j = 0$$

This requirement is described by any two of three equations

$$x_i - x_j = x_{0i} - x_{0j} + \frac{1}{\gamma a_i}[\cos(\Phi_{0i} + \frac{h_i \gamma a_i (z_{V0} - z_{0i})}{tg\lambda_i}) - \cos\Phi_{0i}]$$
$$- \frac{1}{\gamma a_j}[\cos(\Phi_{0j} + \frac{h_j \gamma a_j (z_{V0} - z_{0j})}{tg\lambda_j}) - \cos\Phi_{0j}] = 0$$

$$y_i - y_j = y_{0i} - y_{0j} + \frac{1}{\gamma a_i}[\sin(\Phi_{0i} + \frac{h_i \gamma a_i (z_{V0} - z_{0i})}{tg\lambda_i}{}_i) - \sin\Phi_{0i}]$$
$$- \frac{1}{\gamma a_j}[\sin(\Phi_{0j} + \frac{h_j \gamma a_j (z_{V0} - z_{0j})}{tg\lambda_j}) - \sin\Phi_{0j}] = 0$$

➢ *It means that coordinates of decay vertex will be reconstructed in contrast to the kinematical fit method*

# Method of combined kinematical and geometrical fit

$\chi^2$ minimization of 12 experimentally measured quantities:

$$a_i, a_j, \varphi_i, \varphi_j, tg\lambda_i, tg\lambda_j$$

$$x_{0i}, y_{0i}, z_{0i}, x_{0j}, y_{0j}, z_{0j}$$

and 6 constraints:

$$P_1 = p_{0x} - p_{1x} - p_{2x} = 0 \quad P_2 = p_{0y} - p_{1y} - p_{2y} = 0$$
$$P_3 = p_{0z} - p_{1z} - p_{2z} = 0 \quad P_4 = E_0 - E_1 - E_2 = 0$$

$$x_i - x_j = 0$$
$$y_i - y_j = 0$$

↓

***This algorithm was tested using simulated data of*** $K_S^0 \to \pi^+\pi^-$ ***decay.***

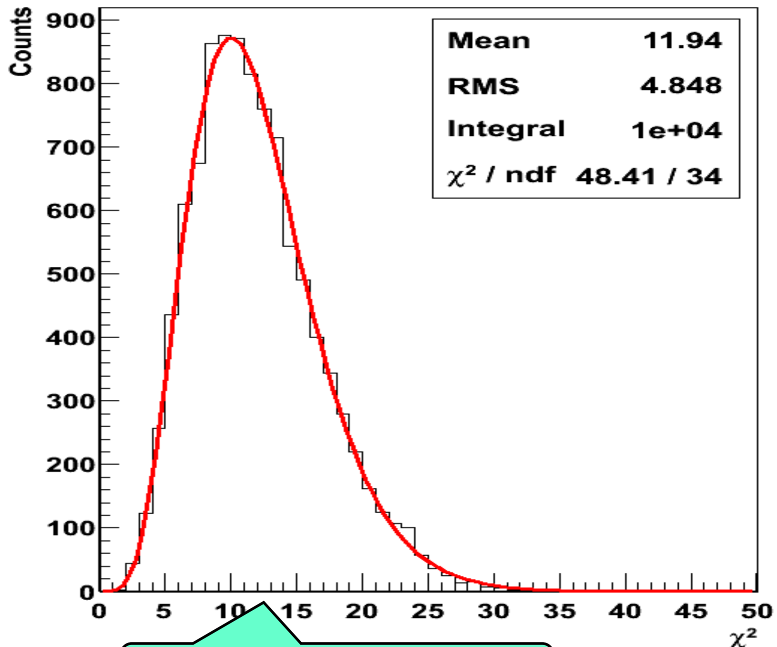
***10 000 events were simulated***

$\chi^2$ ***was minimized using C++ code based on this algorithm***

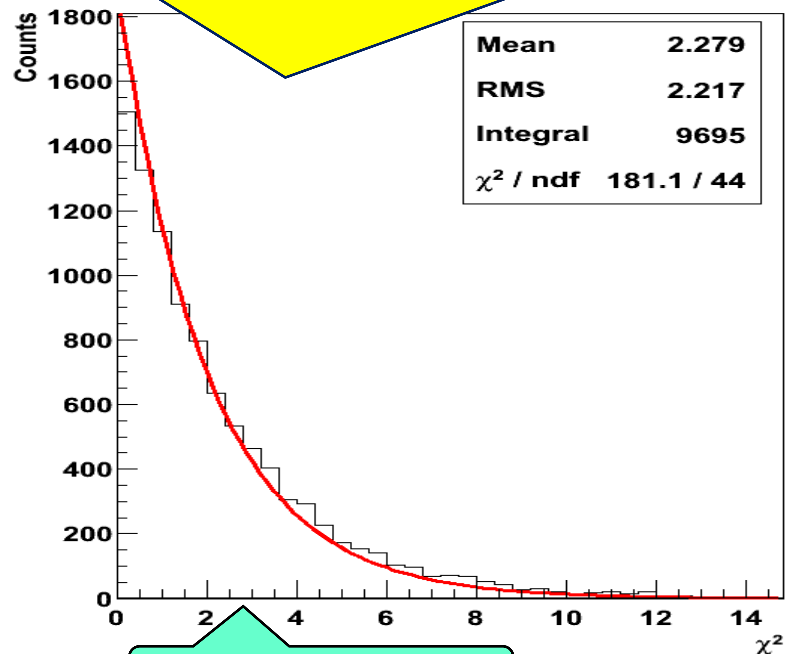
$\chi^2$ ***distribution with 2 degrees of freedom was obtained as expected.***

# Results of the testing

After minimization $\chi^2$ distribution with 2 degrees of freedom was obtained




Before minimization




After minimization

Mean value and dispersion of $\chi^2$ distribution with 2 degrees of freedom

$$\overline{\chi^2(2)} = 2$$

$$\sigma\ \chi^2(2) = 2$$

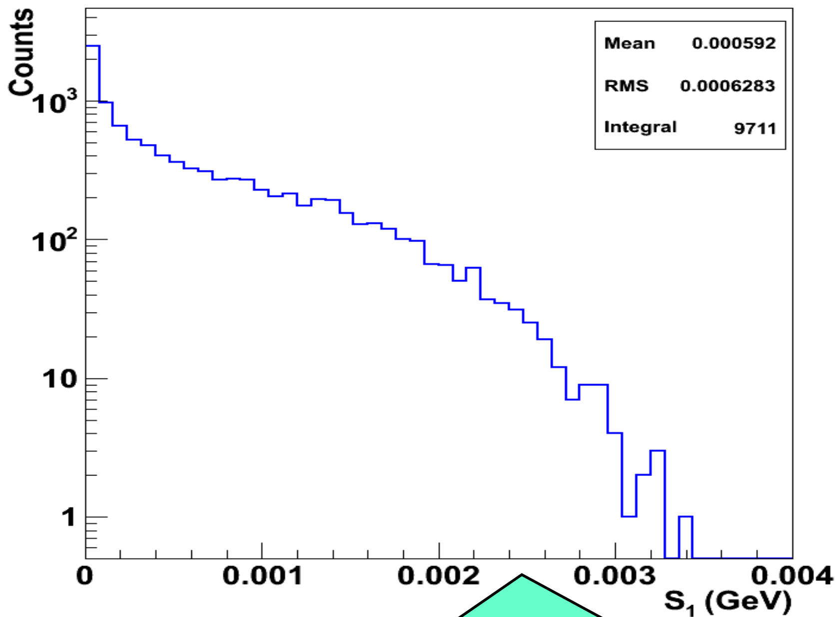
➤ *High reconstruction efficiency ≈ 97% is obtained*

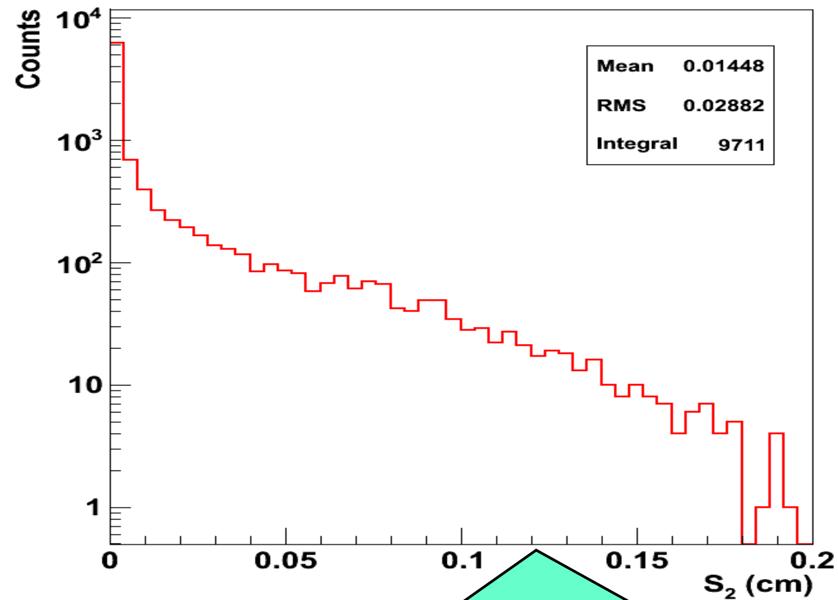
# Test of the kinematical and topological constraints

Accuracy of energy and momentum conservation laws and helices intersection

$$S_1 = \sum_{i=1}^{4} |P_i|$$

$$S_2 = |x_i - x_j| + |y_i - y_j|$$




Accuracy of conservation laws execution was set on $10^{-3}$GeV

Accuracy of helices intersection laws execution was set on 1mm

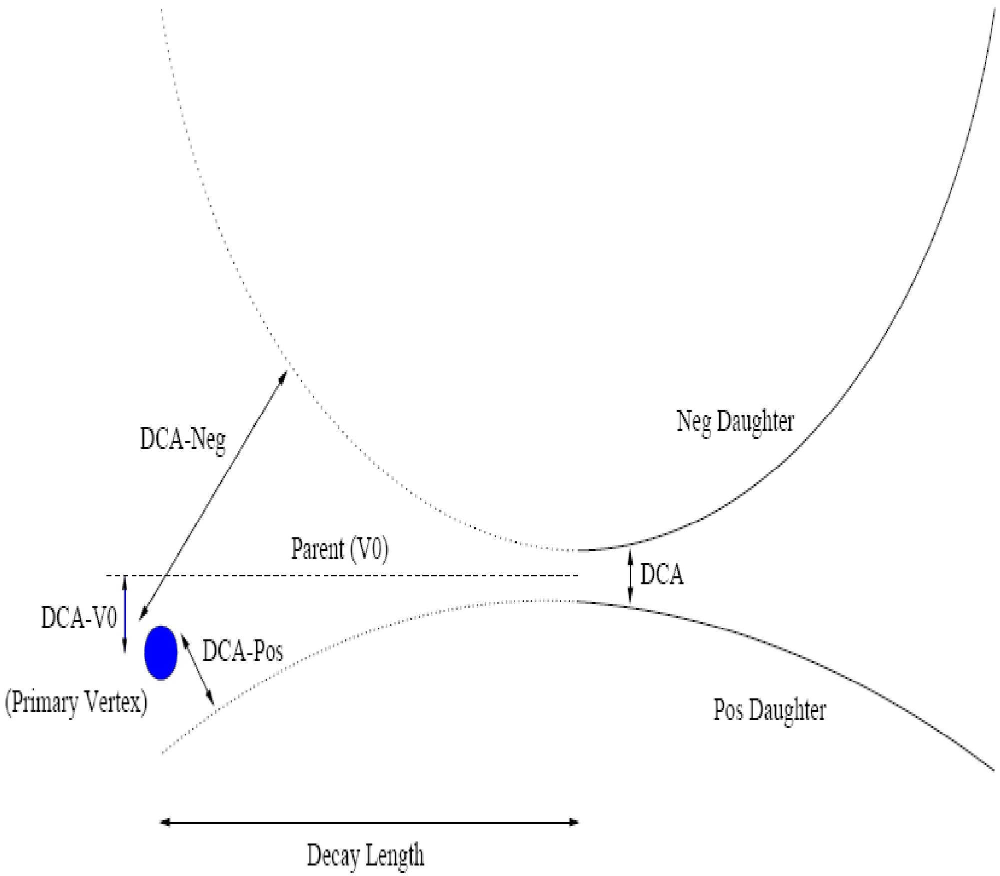
➢ *High precision of energy-momentum conservation laws and helices intersection after minimization was obtained*

# Summary

- ***Kinematical fit for neutral strange particle reconstruction was developed and tested using  simulated data.***
- ***Kinematical fit was applied for reconstruction*** $K_S^0, \Lambda^0, \bar{\Lambda}^0$ ***and finding mass and width of the*** $K^*(892)$ ***- resonance .***
- ***Combined method of topological and kinematical fit for neutral strange particles reconstruction was suggested and tested using  simulated data.***
- ***This method can be used for neutral strange (***$\Lambda^0, K_S^0, \bar{\Lambda}^0$***) and multistrange (***$\Xi^-, \Omega^-$***) particle reconstruction  in experiments with track detectors such as  STAR, MPD, ALICE…***

# Method of V0 selection using in STAR

## Topological cuts




## Parameters of selection

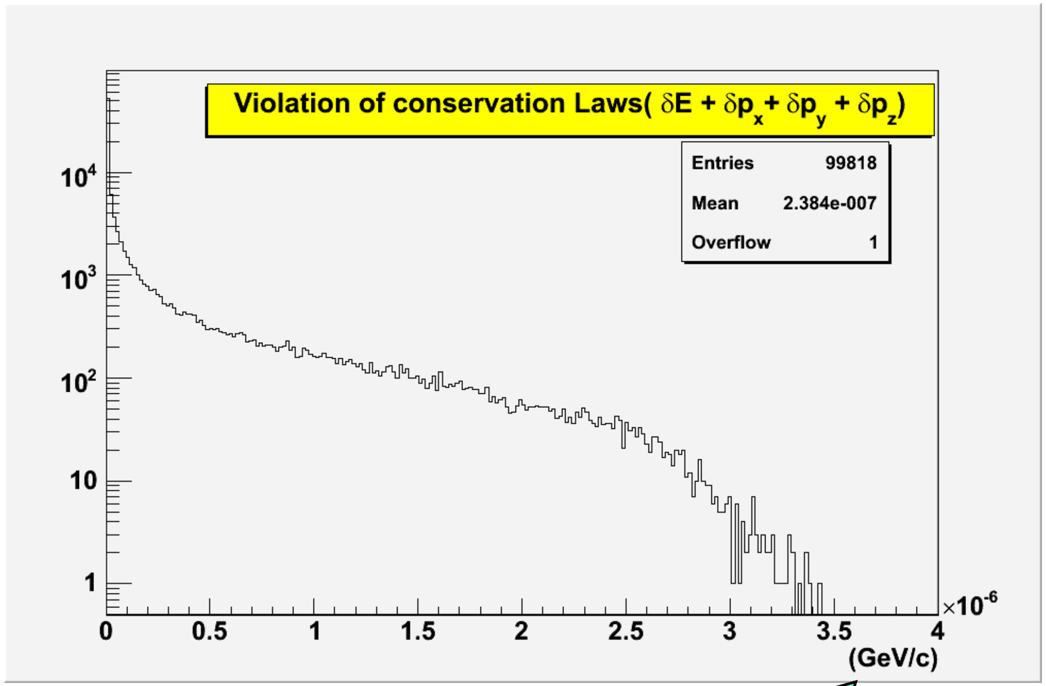
- ✓(PV)- Primary vertex
- ✓(DV)- Decay vertex
- ✓Decay Length
- ✓DCA(*) between V0 & PV
- ✓DCA between Pos & PV
- ✓DCA between Neg & PV
- ✓DCA between Pos & Neg

(*) DCA (Distance of Closest Approach)


30.09.2008

# Computer Simulation

## Accuracy of energy and momentum conservation laws




Strong execution of energy -momentum conservation laws after $\chi^2$ minimization was observed

Accuracy of conservation laws execution was set on $10^{-6}$ GeV



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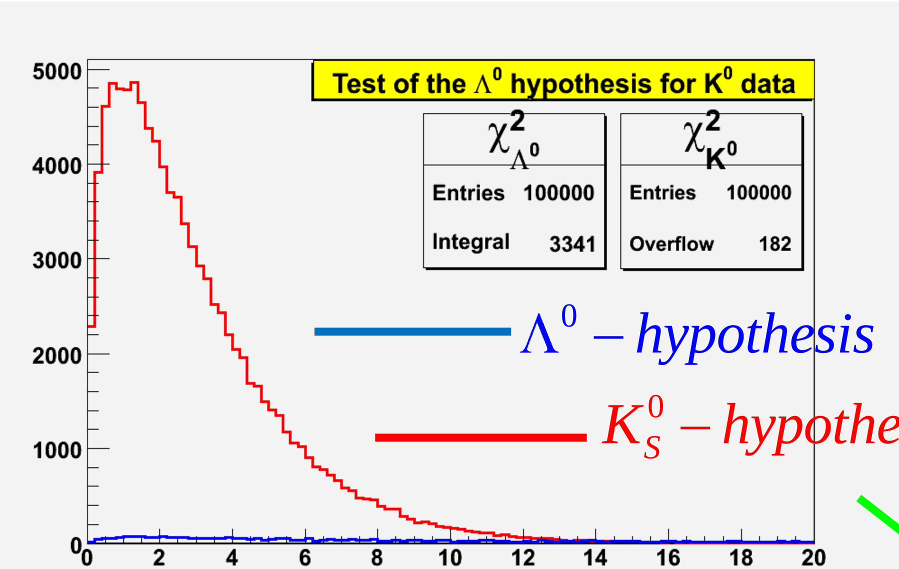
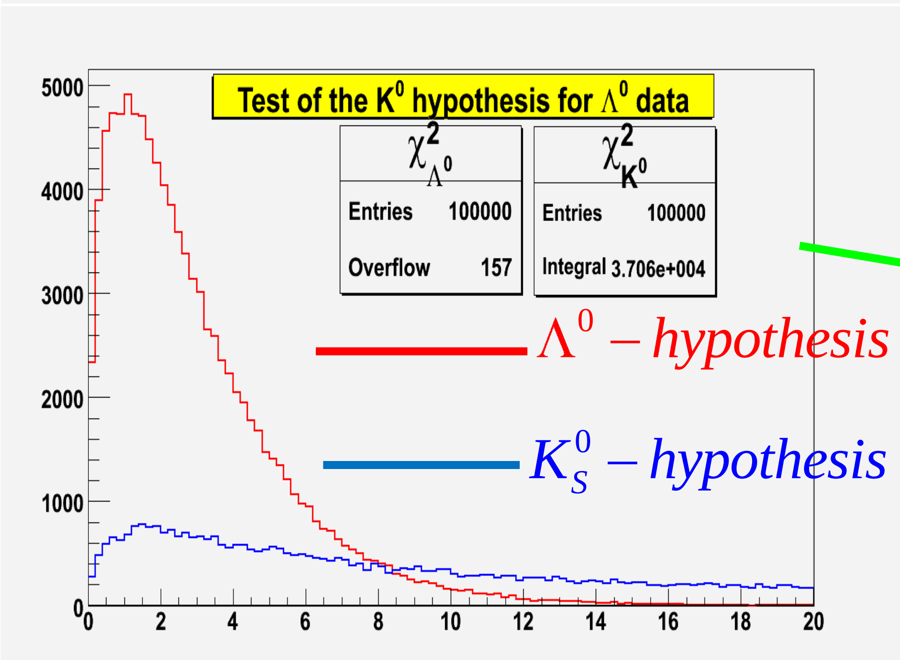
# Program testing




## Hypotheses division

Fitting of kinematic parameters for $\Lambda^0 \to p^+\pi^-$ and $K_S^0 \to \pi^+\pi^-$ for $K_S^0$ and $\Lambda^0$ hypotheses

Impurity of $\Lambda^0$ in $K_S^0$ decay with selection criteria $\chi^2 < 10$ is equal to 2.3% (*)

Impurity of $K_S^0$ in $\Lambda^0$ decay with selection criteria $\chi^2 < 10$ is equal to 26.4%

(*)It was supposed that we haven't any information about tracks besides their kinematic parameters. But in STAR experiment we can define particle type by ionization loses. That's why our hypothesis division will be much better.


30.09.2008