TOTAL CROSS SECTIONS OF HADRON-HADRON INTERACTIONS AND MULTIPLICITY DISTRIBUTION OF SECONDARY HADRONS AT LHC ENERGY

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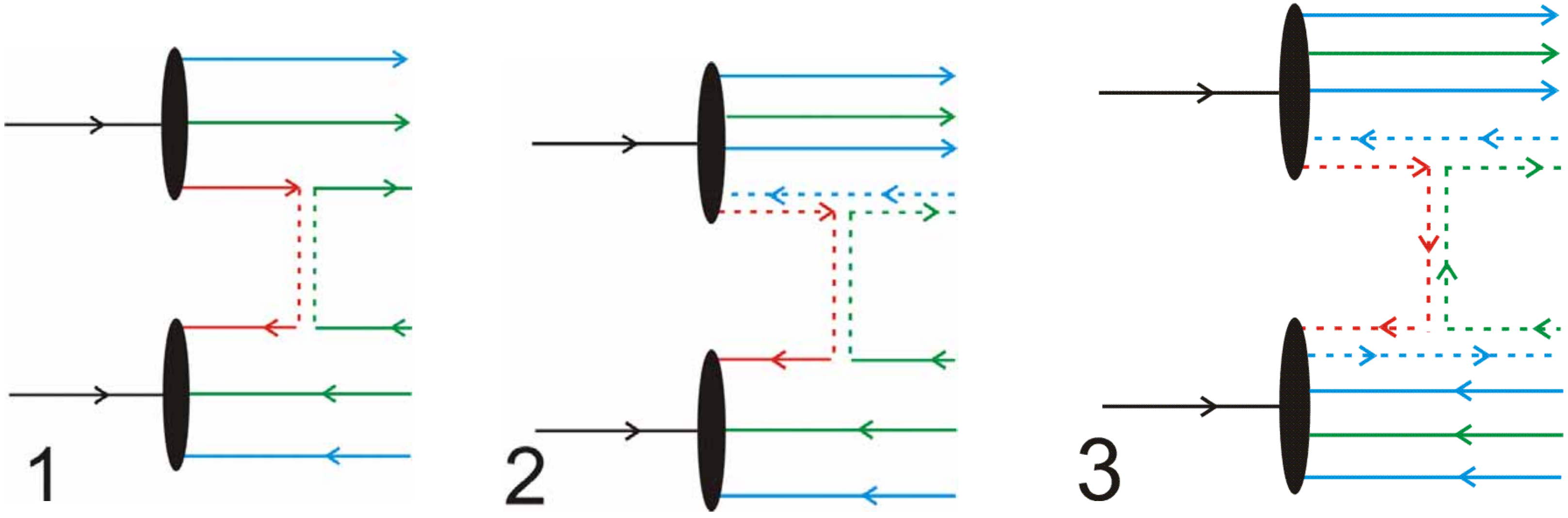
We suppose that contributions to total cross sections for proton-proton and proton-antiproton scattering are the following:

1. constant contribution corresponding to hadron configuration with only valent quarks;
2. rising logarithmic with energy contribution corresponding to configuration with one additional bremsstrahlung gluon;
3. rising as square of energy logarithm contribution corresponding to configuration with two additional bremsstrahlung gluons.

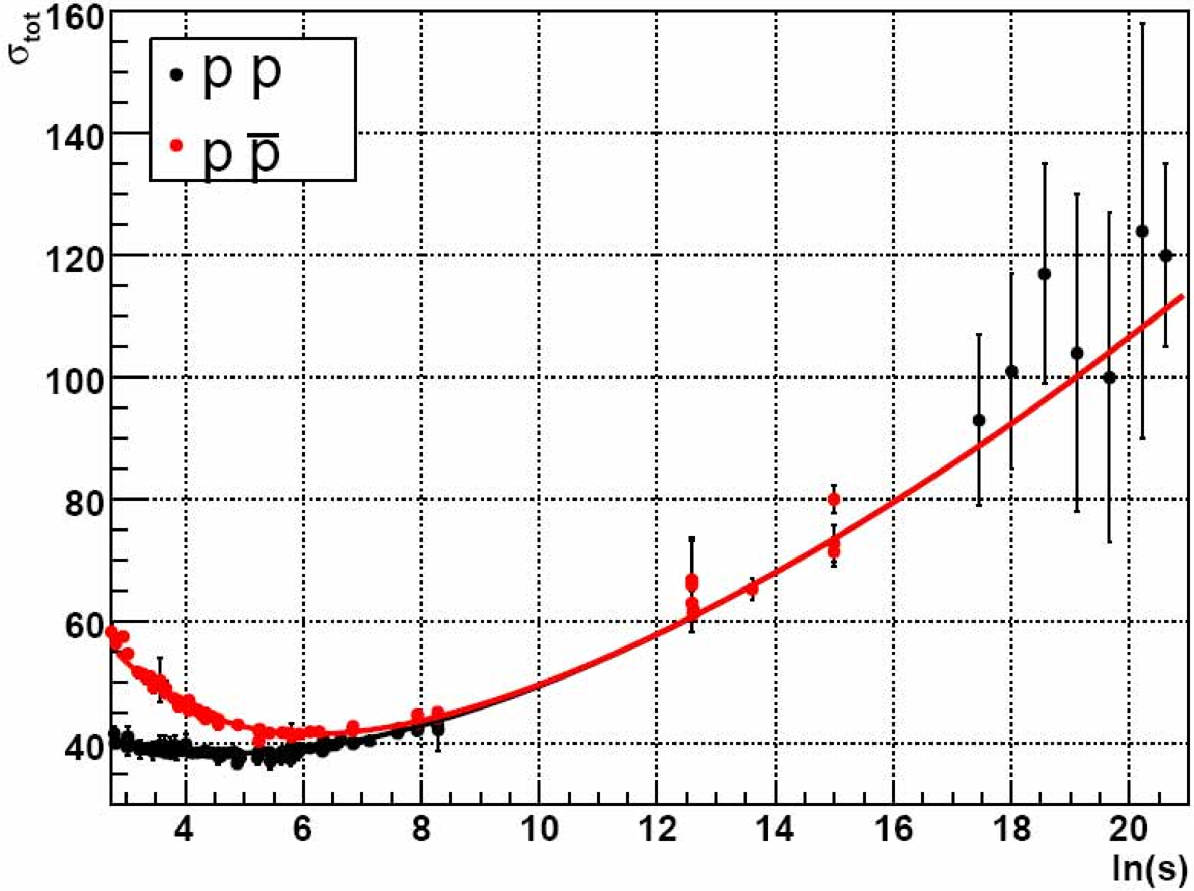


$$\sigma_{tot}^{pp} = 63.58\, e^{-0.358Y} - 35.46\, e^{-0.56Y} + 19.91(1 + 0.0622Y + 0.0078Y^2) \quad (1)$$

with $\chi^2/ndf = 1.2$

$$\sigma_{tot}^{p\bar{p}} = 63.58\, e^{-0.358Y} + 35.46\, e^{-0.56Y} + 19.91(1 + 0.0622Y + 0.0078Y^2) \quad (2)$$

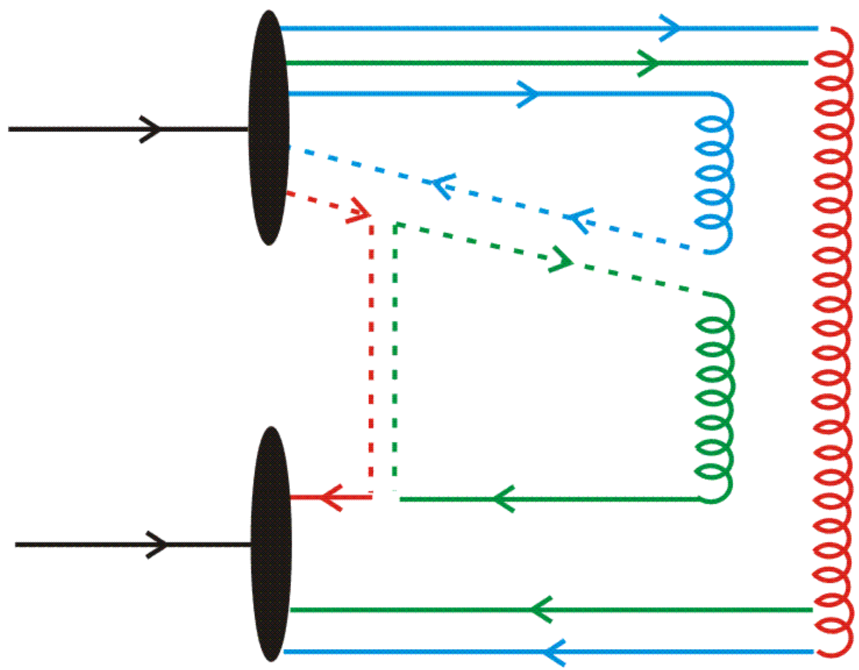
with $\chi^2/ndf = 1.5$



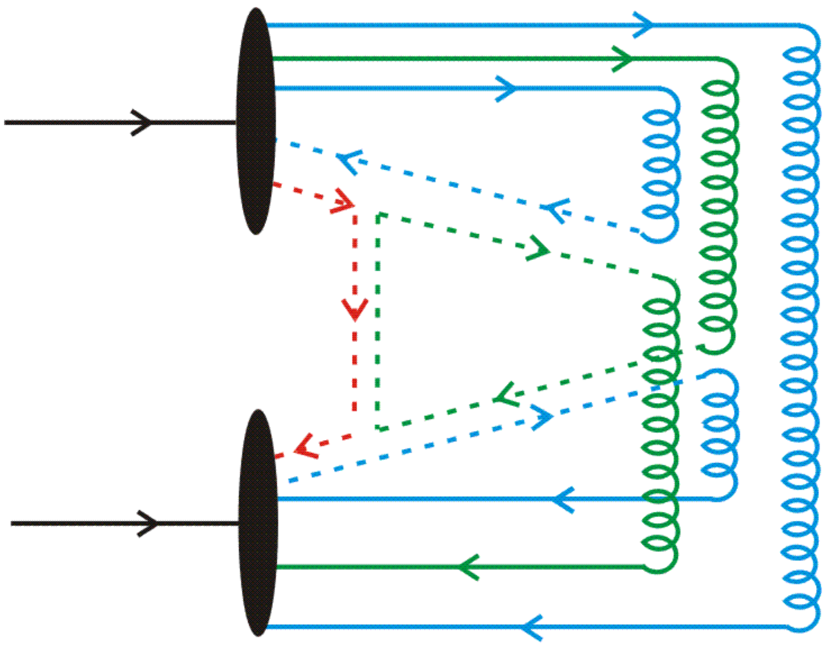
- So we can consider that there are three types of events:

1. hadron production in gluon string which appears in processes described by the diagram 1, this process corresponds to constant contribution in total cross sections;
2. hadron production in two divided quark strings;
3. hadron production in three divided quark strings.

Generally speaking, transverse sizes of quark string must be in order of confinement radius, i. e. in order of hadron size. But since transverse impulse of additional bremsstrahlung gluon is rather large, approximately 2 GeV, than compton wave length of the gluon is small. This gluon must be absorbed by one of the quark strings. Thus corresponding quark strings must have transverse sizes comparable with gluon compton wave length.



From the same reasons it follows that configuration with three divided quark strings is possible only when there are two additional gluons.

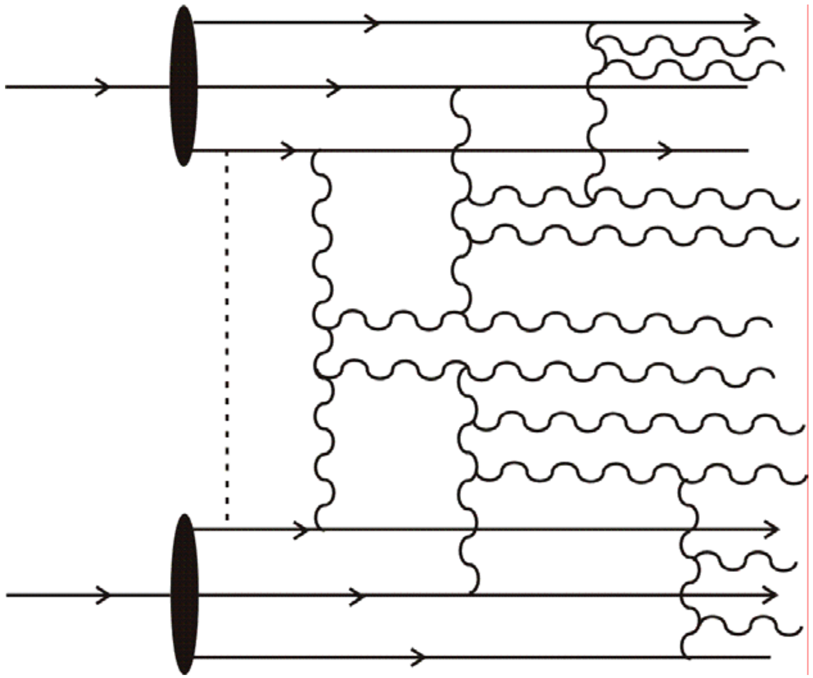


But two additional gluons may also produce configuration with two quark string.

So rising logarithmic contribution corresponds to configuration with two quark strings. Rising as square of energy logarithm contribution corresponds to both three quark strings configuration and also two strings configuration.

At that, in case with two additional gluons the probabilities of configurations with two strings or three strings do not depend on energy of colliding particles.

Hadron production in gluon string is connected with diagrams like this one. Here dotted line represents gluon exchange for the initial state interaction and waved lines represent gluons produced when color objects fly apart. The last ones pass into hadrons by one or another law. This diagram gives the certain value of secondary hadrons multiplicity. Since there is a large number (practically infinite) of alike diagrams and since they all have approximately the same order, than the random value - secondary hadrons multiplicity - must obey normal distribution due to the probability theory central limit theorem. Thus we suppose that multiplicity distribution in the gluon string is normal distribution.



By analogy with the electron-positron annihilation we suppose that multiplicity distribution in quark string fulfills negative binomial distribution.

$$P_n(k) = \frac{k(k+1)\dots(k+n-1)}{n!}\left(\frac{<n>}{<n> + k}\right)^n \left(\frac{k}{<n> + k}\right)^k$$

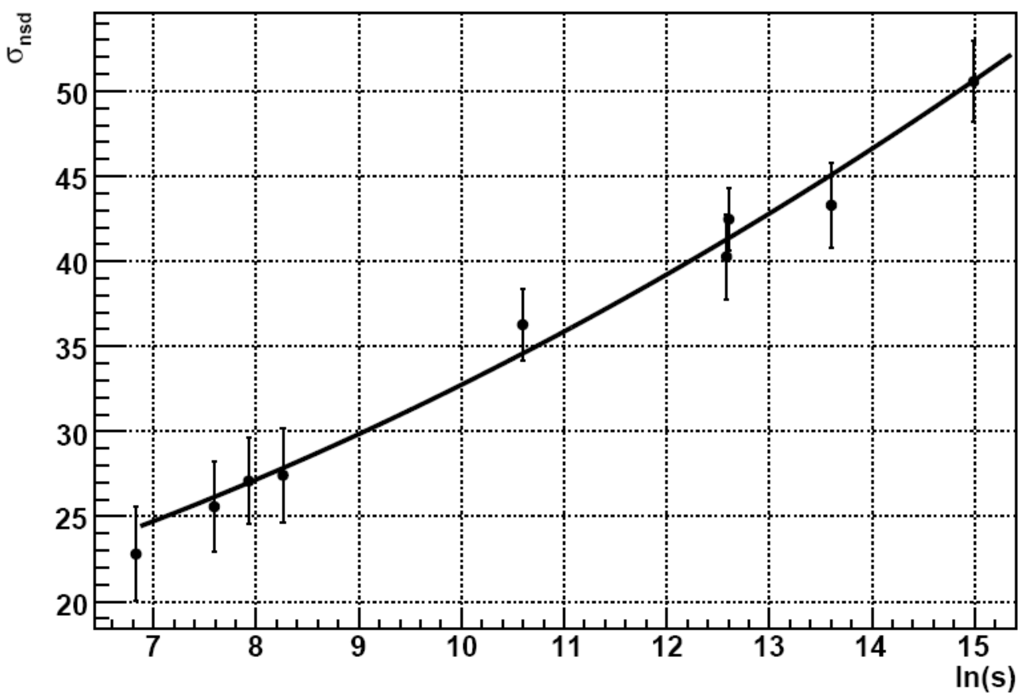
This distribution has two parameters: shape parameter k and mathematical expectation <n> - mean multiplicity. It is easy to show that convolution of two negative binomial distributions with the same <n> and k is again negative binomial distribution with parameters twice the old ones (2<n> and 2k). And convolution of three negative binomial distributions with the same <n> and k is again negative binomial distribution with parameters three times the old ones (3<n> and 3k).

The experimental data on multiplicity distributions for proton-proton and proton-antiproton scattering are given in form of non single diffraction distributions. Three process under consideration contribute to non single diffraction cross section.

$$\sigma_{nsd} = \sigma_{tot} - \sigma_{el} - \sigma_{sd}$$

We have fitted the data on non single diffraction cross sections similarly like total cross sections and obtained the following parameter values.

$$14.00(1 + 0.0526Y + 0.0081Y^2), \quad \chi^2/ndf = 0.3$$



When we have obtained the coefficients for non single diffraction cross sections we therefore defined the weights of the processes (only valent quarks, one additional gluon, two additional gluons) and then we fitted the weights of corresponding distributions – normal from gluon string, negative binomial for two quark strings and negative binomial for three quark strings.

Having these weights, we have also determined the parameters of all three distributions. The values are given in table.

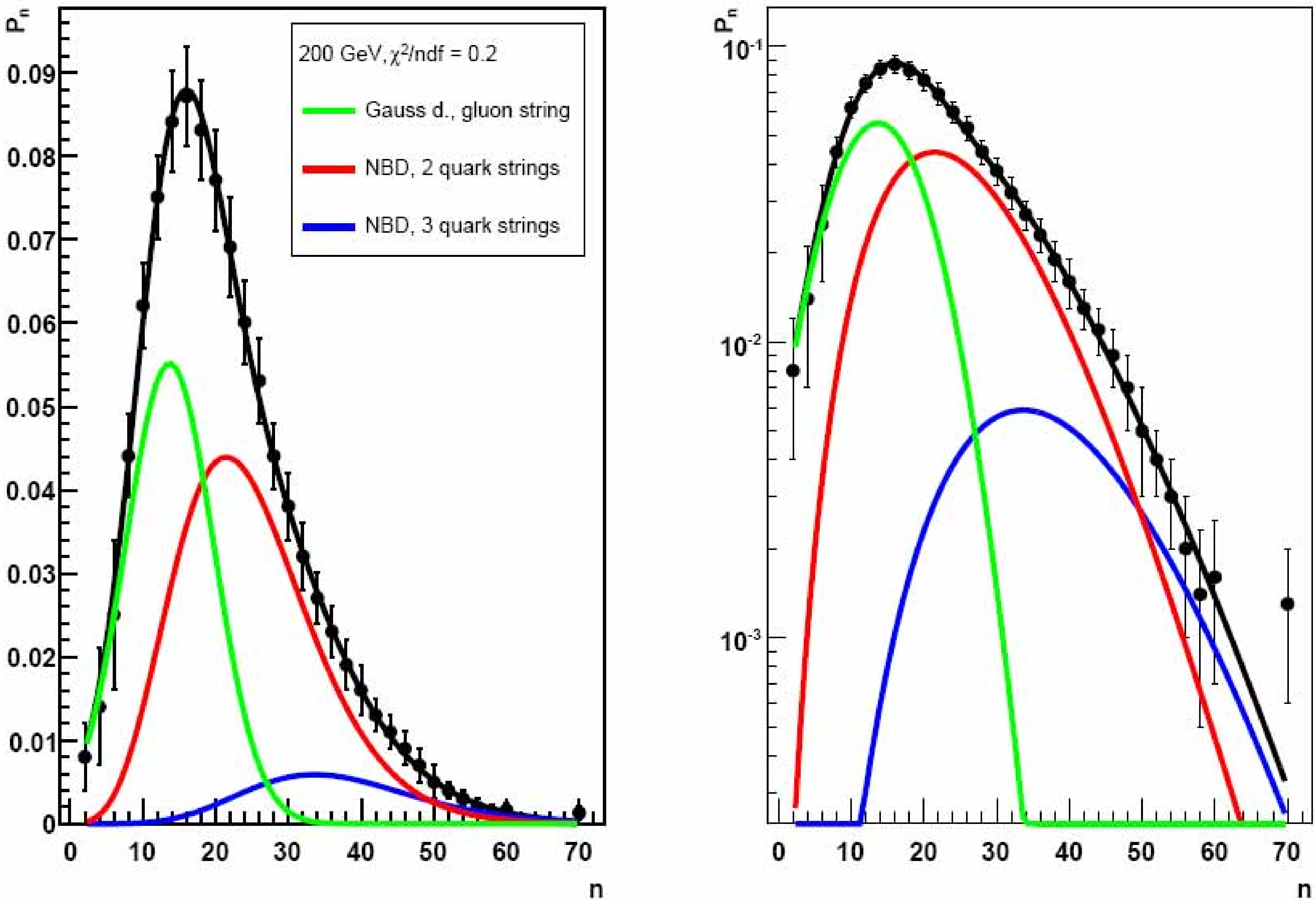
$$W_{norm.d.} = \frac{1}{1 + \delta_1^{nsd} Y + \delta_2^{nsd} Y^2}$$

$$W_{2NBD} = \frac{\delta_1^{nsd} Y + 0.77 \delta_2^{nsd} Y^2}{1 + \delta_1^{nsd} Y + \delta_2^{nsd} Y^2}, \quad W_{3NBD} = \frac{0.22 \delta_2^{nsd} Y^2}{1 + \delta_1^{nsd} Y + \delta_2^{nsd} Y^2}$$

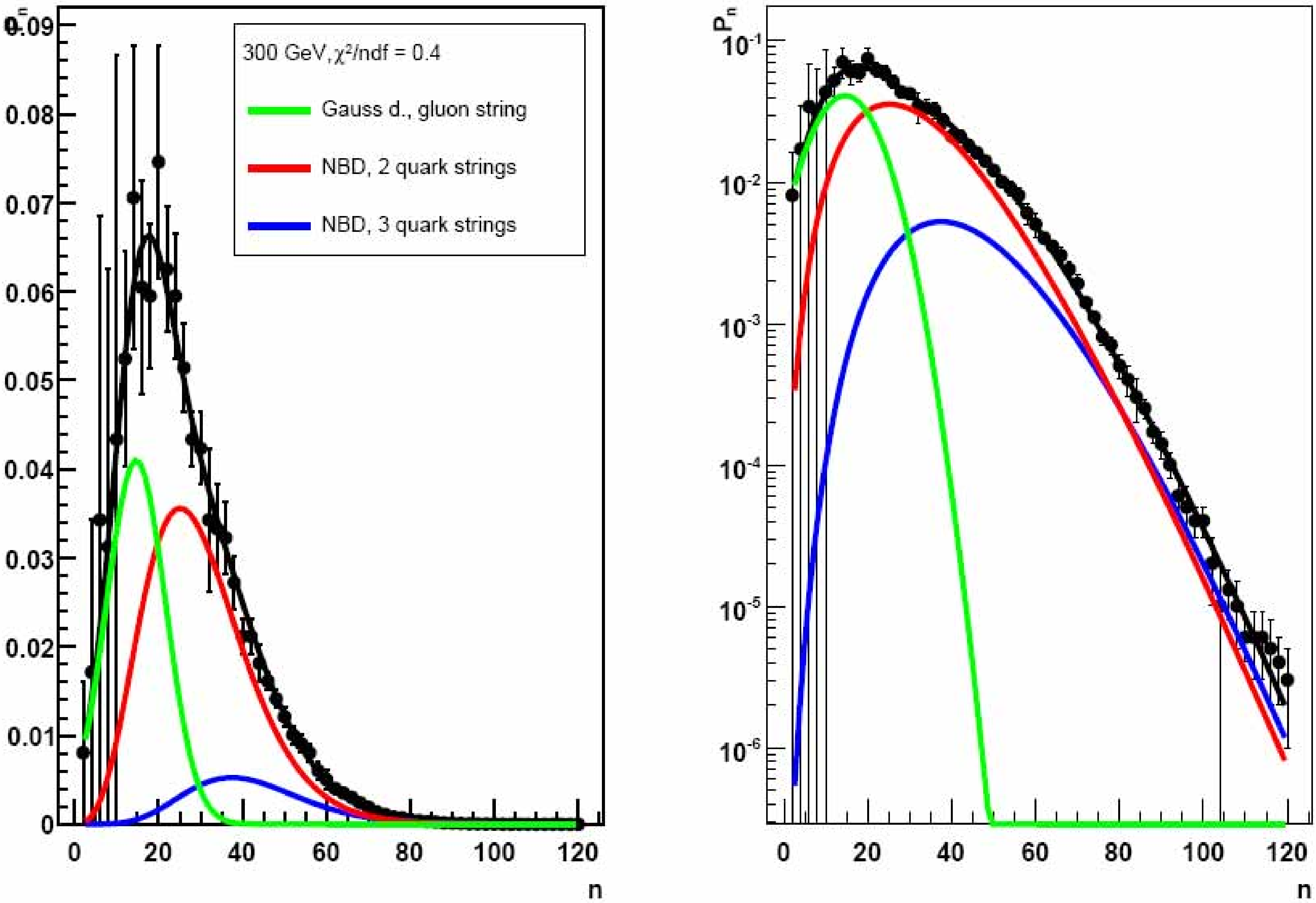
$$\delta_1^{nsd} = 0.0526, \quad \delta_2^{nsd} = 0.0081$$

$\sqrt{s}$ GeV	Gluon string		Two quark strings		Three quark strings		χ^2/ndf
	$<n>_g$	σ	$<n>_2$	k_2	$<n>_3$	k_3	
200	13.67	6.07	24.78	8.77	36.99	13.09	0.2
300	14.64	7.05	30.09	6.76	41.80	10.90	0.4
546 UA5	16.74	6.90	33.07	7.43	49.35	11.09	0.8
546 E735	19.22	5.68	36.61	5.47	52.72	8.83	0.4
900	21.73	8.14	39.75	4.44	55.21	7.15	0.2
1000	23.45	7.01	45.65	4.68	63.41	7.54	0.8
1800	21.85	9.80	53.52	4.23	76.22	6.82	1.1

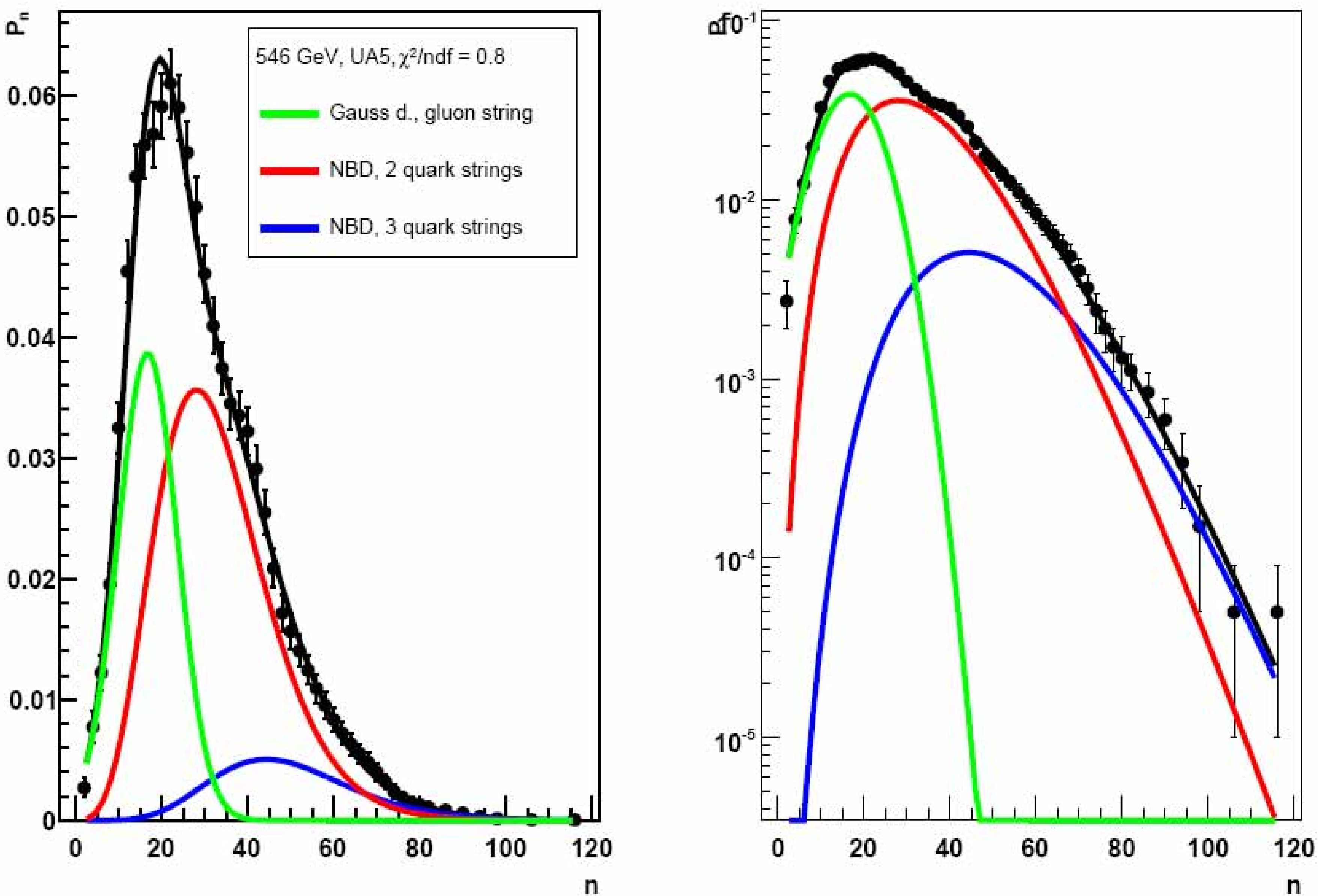
The results of fitting data on proton-antiproton scattering of UA5 and E735 experiments with all three processes are given in the following figures.



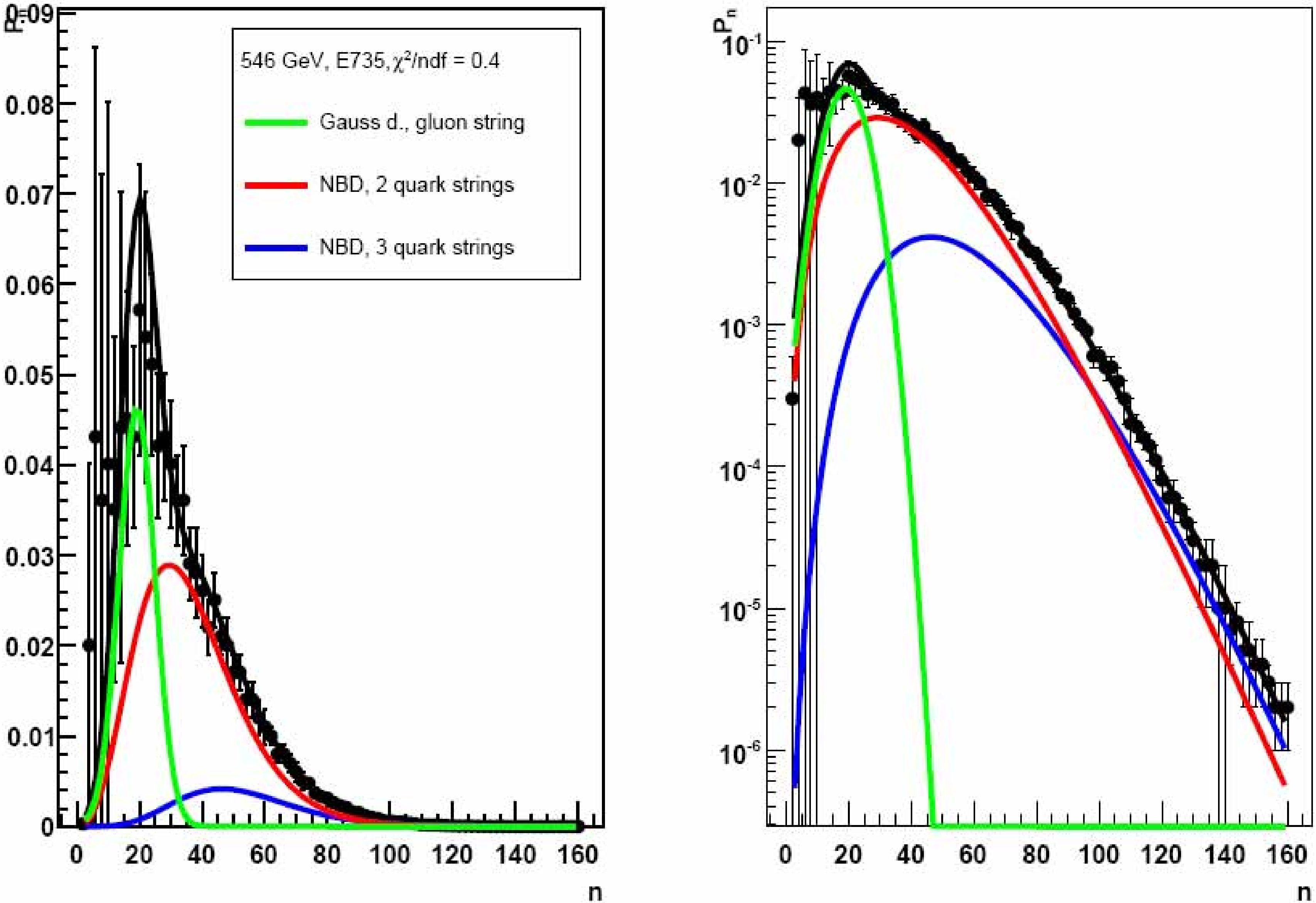
200 GeV, χ^2/ndf = 0.2
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
P_n
n
0
10
20
30
40
50
60
70
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
10^{-1}
10^{-2}
10^{-3}



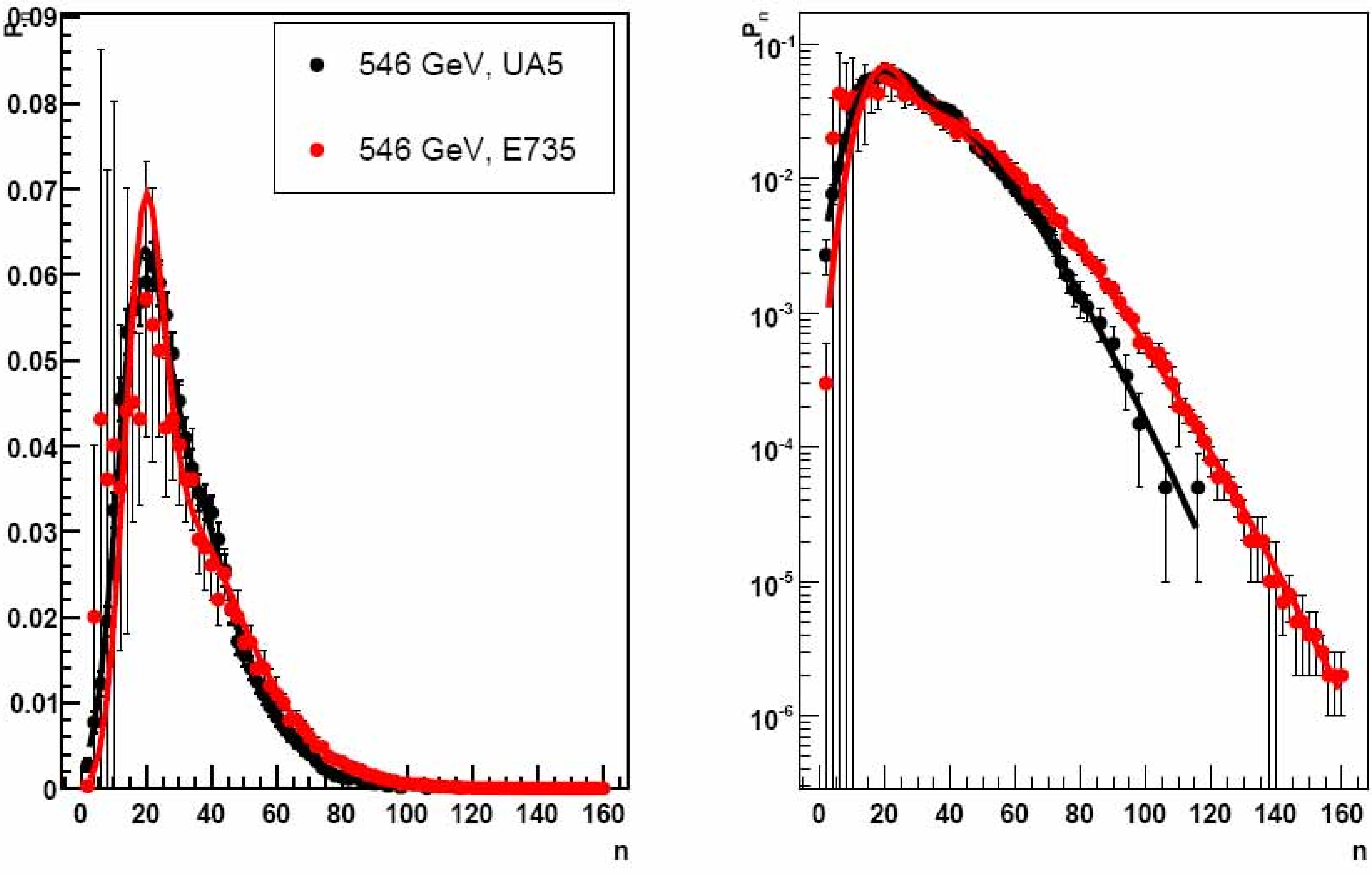
P_n
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
20
40
60
80
100
120
n
300 GeV, χ²/ndf = 0.4
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
P_n
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0
20
40
60
80
100
120
n



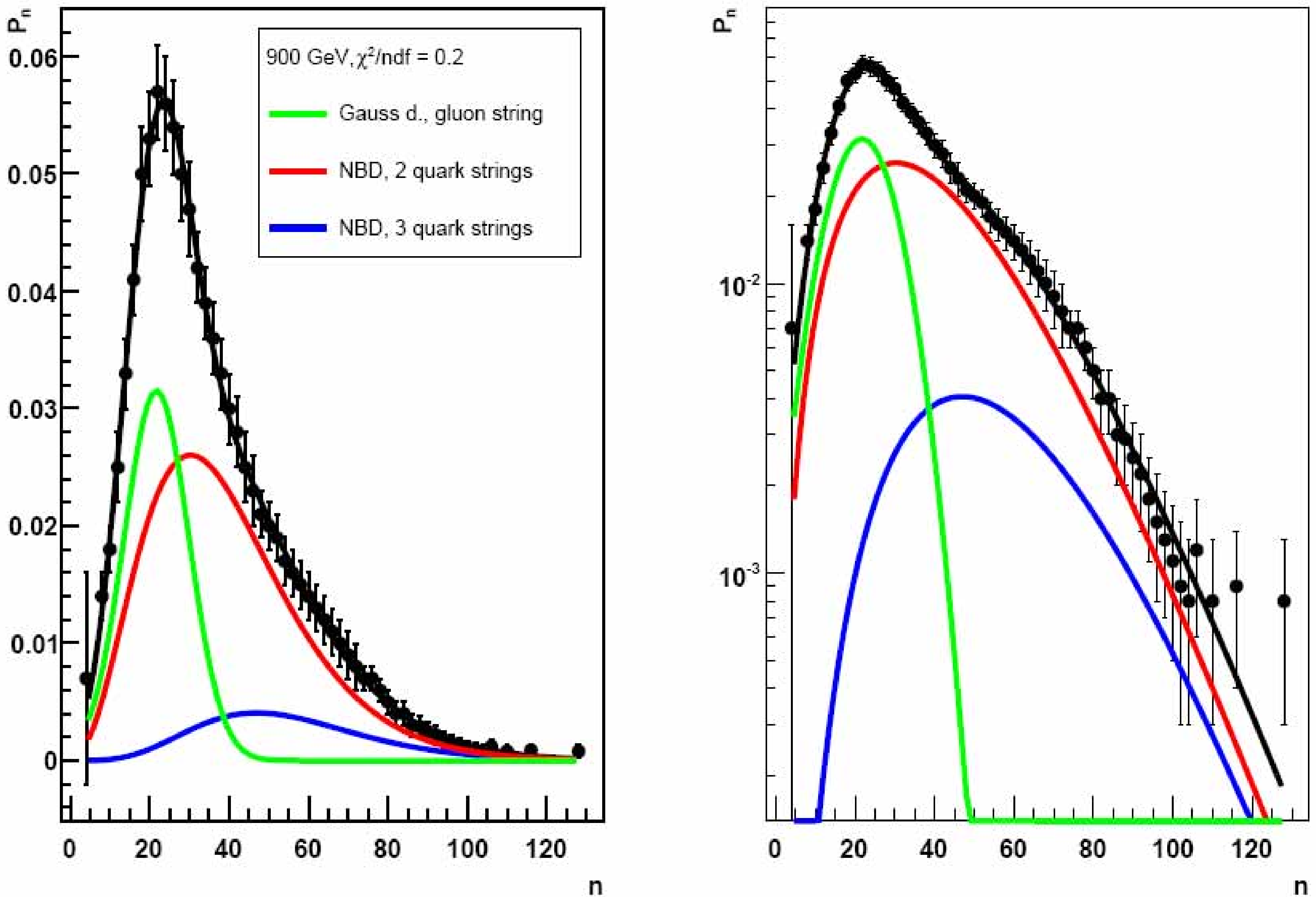
546 GeV, UA5, χ²/ndf = 0.8
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
P_n
0.06
0.05
0.04
0.03
0.02
0.01
0
0
20
40
60
80
100
120
n
10^-1
10^-2
10^-3
10^-4
10^-5
n



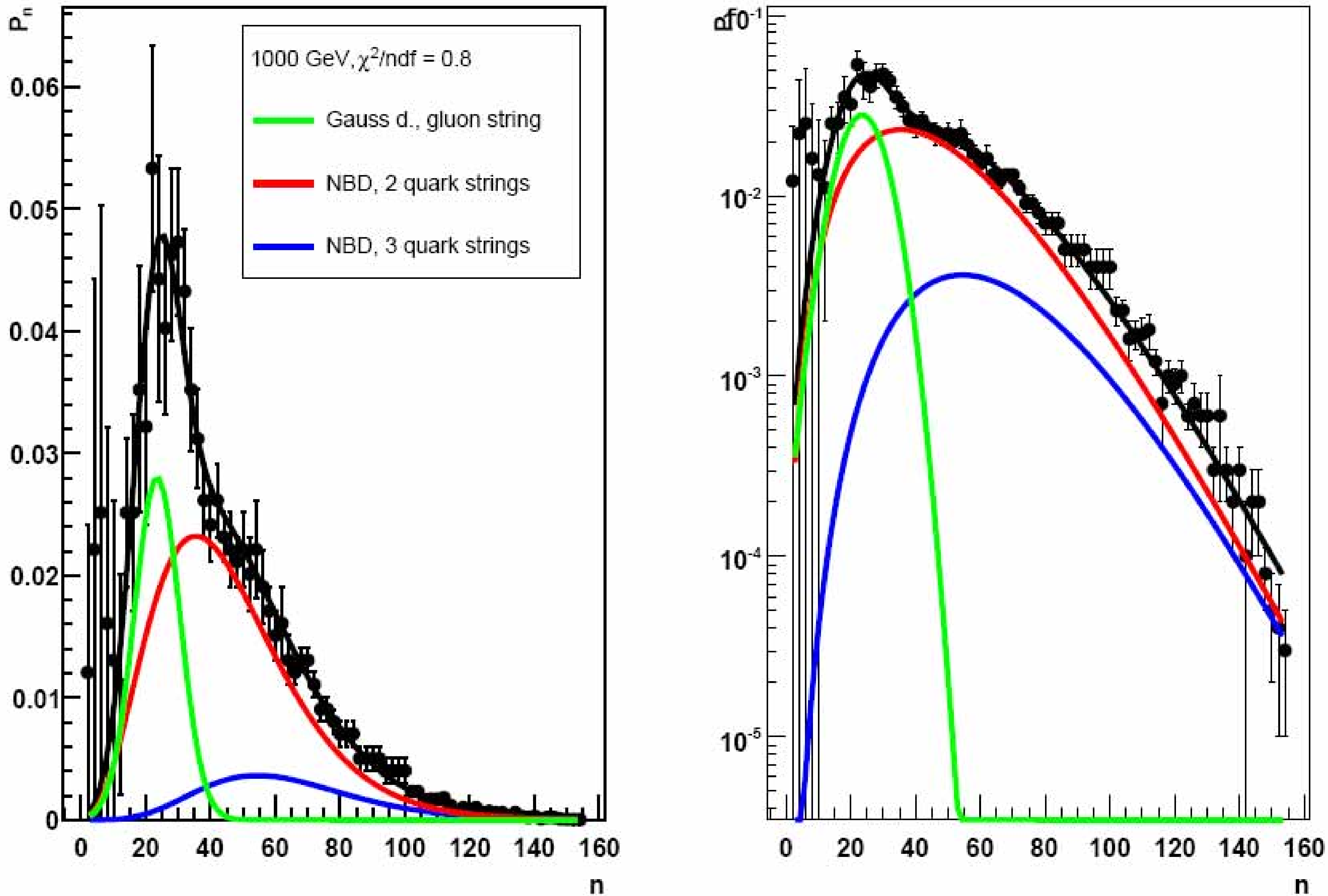
P_n
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
546 GeV, E735, χ^2/ndf = 0.4
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
0 20 40 60 80 100 120 140 160
n
P_n
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 20 40 60 80 100 120 140 160
n



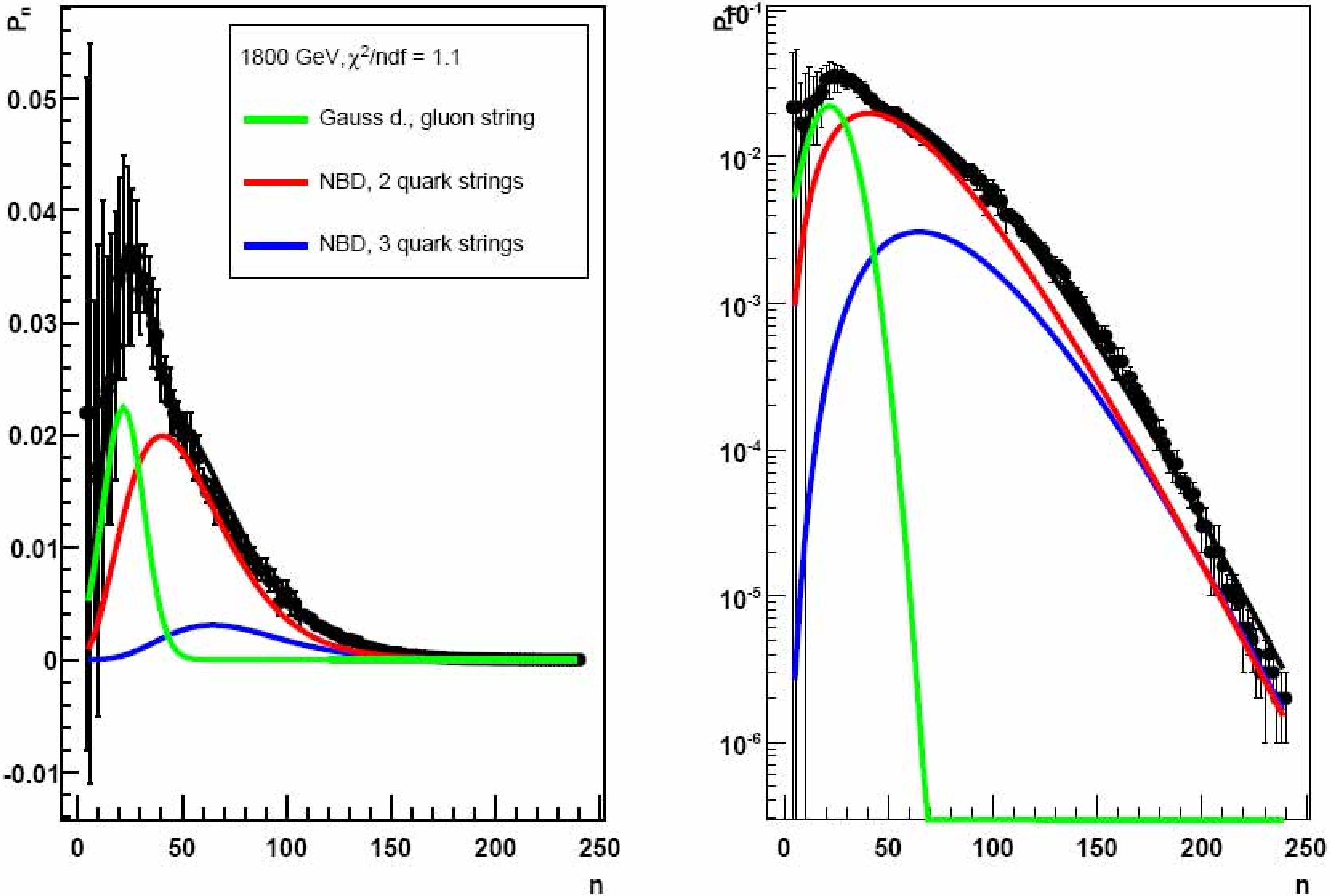
546 GeV, UA5
546 GeV, E735
P_n
n
P_n
n



900 GeV, χ²/ndf = 0.2
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
P_n
n
0.06
0.05
0.04
0.03
0.02
0.01
0
20
40
60
80
100
120
10^-2
10^-3

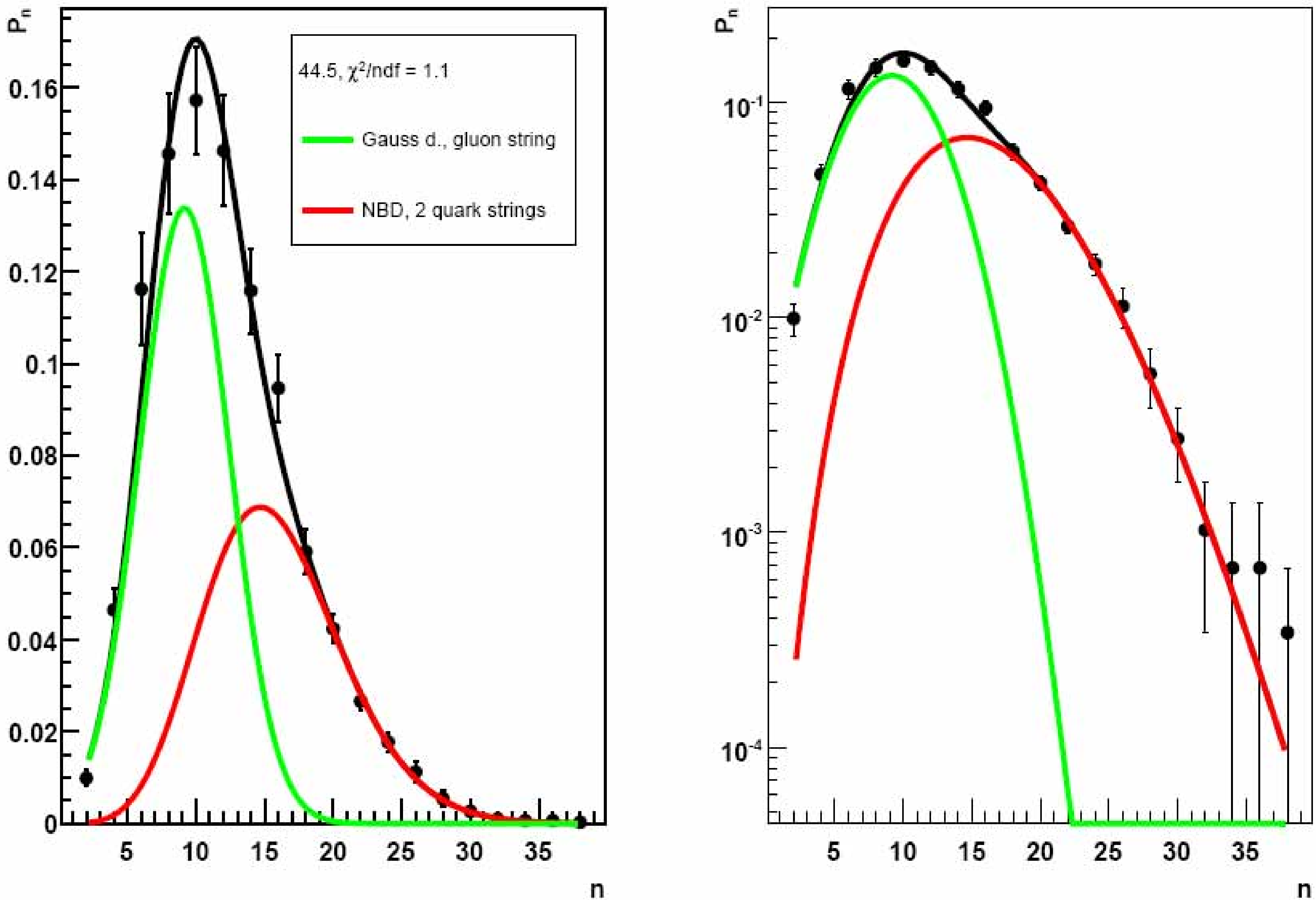


P_n
1000 GeV, χ²/ndf = 0.8
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
0.06
0.05
0.04
0.03
0.02
0.01
0
0 20 40 60 80 100 120 140 160
n
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
0 20 40 60 80 100 120 140 160
n

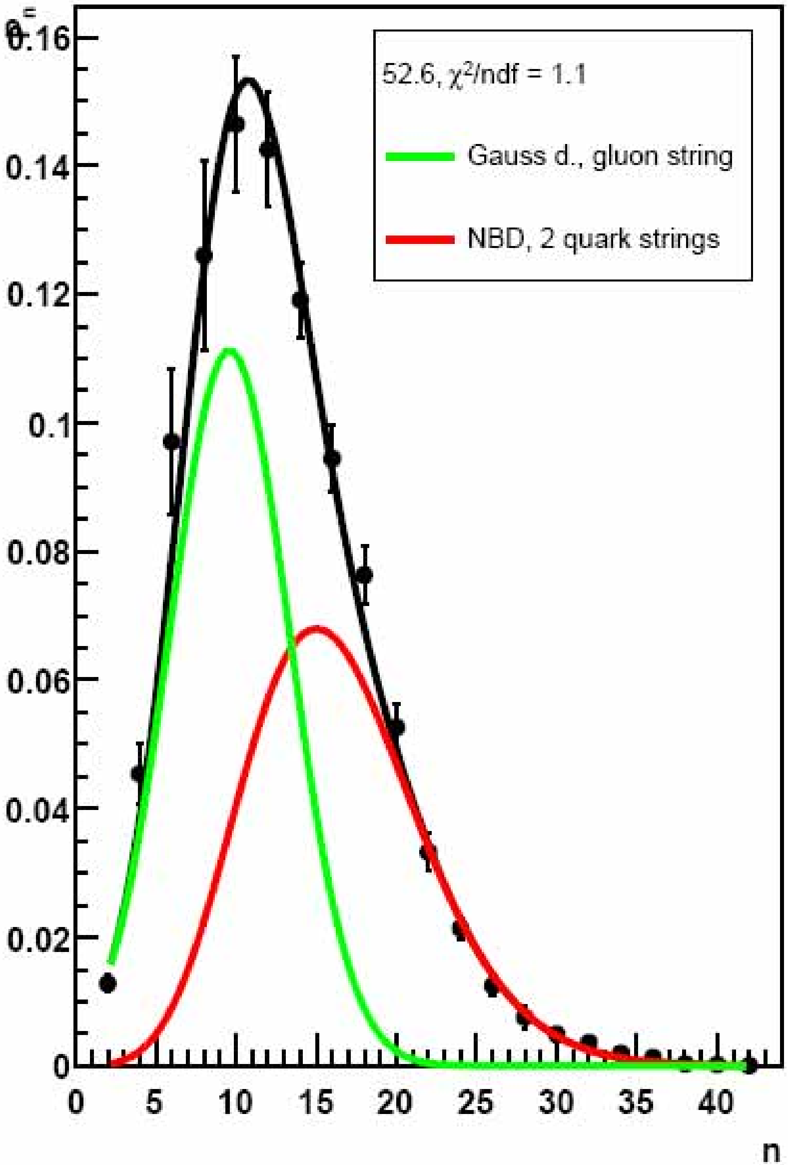


P_n
1800 GeV, χ^2/ndf = 1.1
Gauss d., gluon string
NBD, 2 quark strings
NBD, 3 quark strings
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
50
100
150
200
250
n
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}

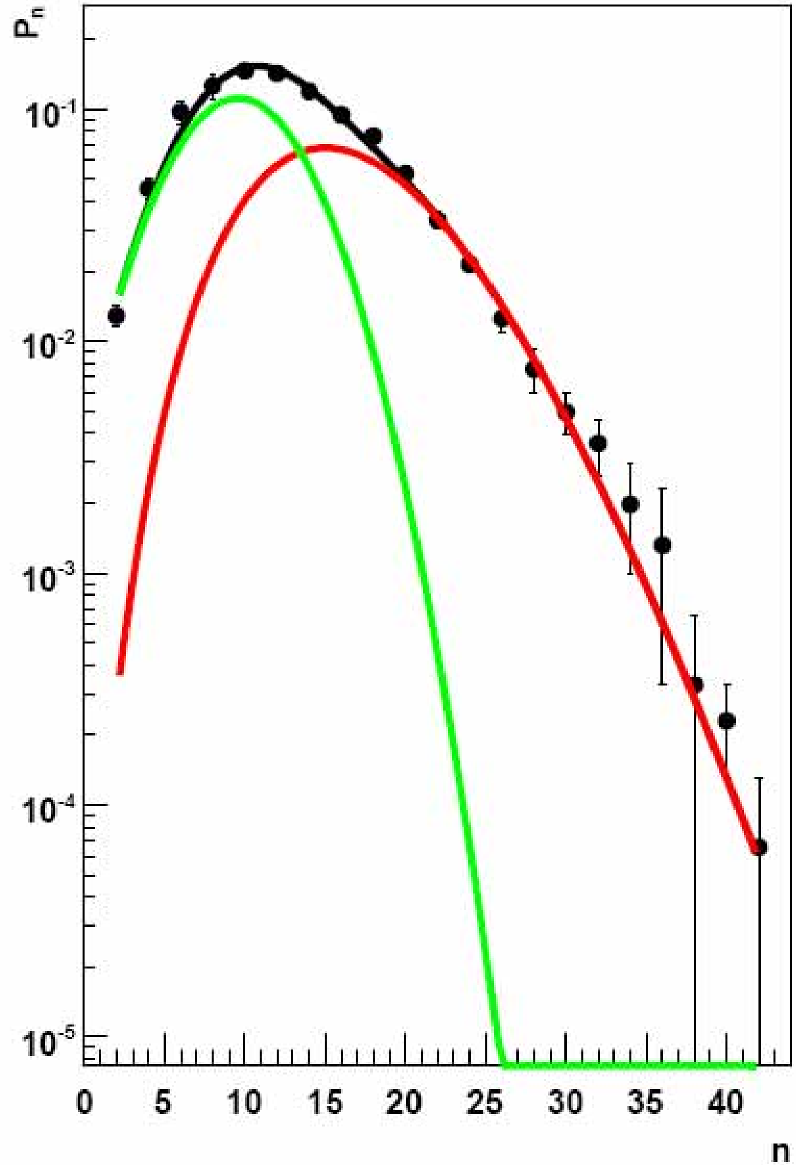
It should be noted that for proton-proton scattering there are only processes with gluon string and two quark strings. The configuration with three quark strings is not possible. We have fitted data of ISR experiment with two distributions – normal and negative binomial for two quark strings.



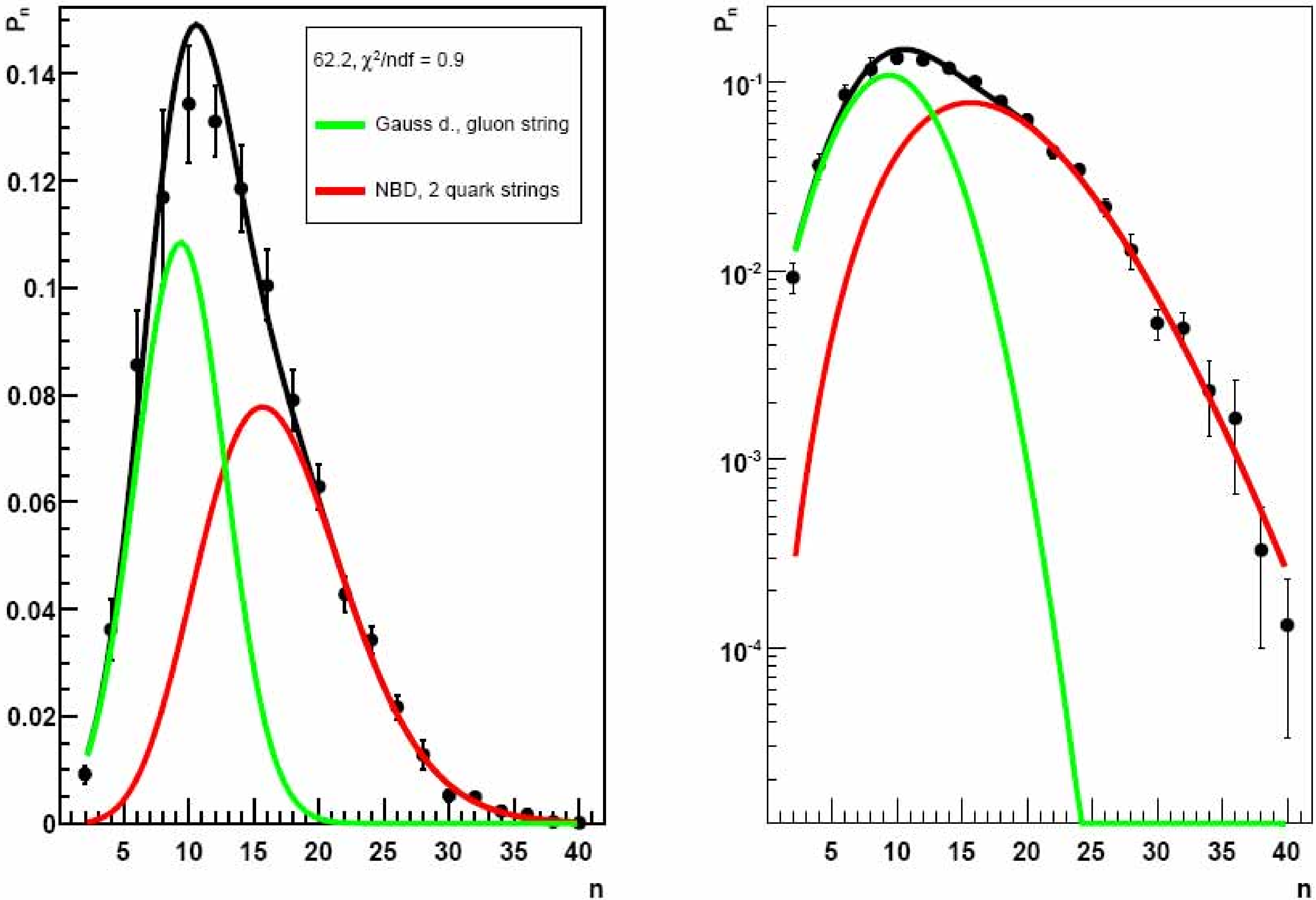
P_n
44.5, χ²/ndf = 1.1
Gauss d., gluon string
NBD, 2 quark strings
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30
35
n
P_n
10^-1
10^-2
10^-3
10^-4
5
10
15
20
25
30
35
n



P_n
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
52.6, χ^2/ndf = 1.1
Gauss d., gluon string
NBD, 2 quark strings
0 5 10 15 20 25 30 35 40
n



P_n
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0 5 10 15 20 25 30 35 40
n



62.2, χ²/ndf = 0.9
Gauss d., gluon string
NBD, 2 quark strings
P_n
n
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30
35
40
10^-1
10^-2
10^-3
10^-4

Predictions for LHC

The obtained parameter values we use to predict the charged particle multiplicity distribution in proton-proton scattering at LHC. The results are:

$$\begin{aligned} <n>_g &= -5.00 + 3.60 \ln \sqrt{s} \quad \text{with } \chi^2/ndf = 1.7, \\ \sigma &= -2.20 + 1.42 \ln \sqrt{s} \quad \text{with } \chi^2/ndf = 0.7. \end{aligned}$$

The corresponding values at $\sqrt{s} = 14$ TeV are $<n>_g = 29.35$, $\sigma = 11.40$. The dependence of $<n>$ from string energy E^{tot} is the following:

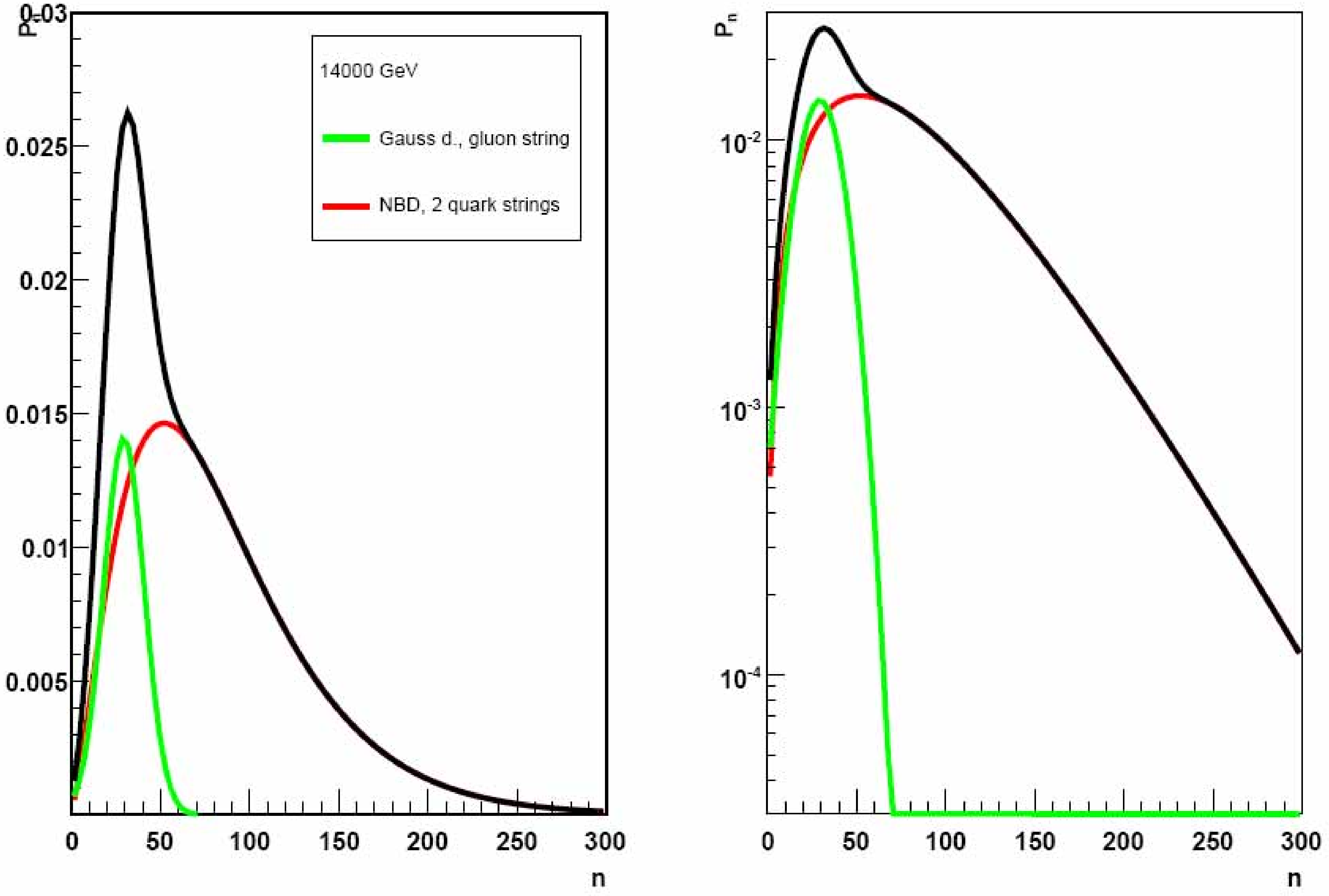
$$<n> = 3.23 + 0.50 \cdot (\ln E^{tot})^2 \quad \text{with } \chi^2/ndf = 2.7.$$

The corresponding value at $E^{tot} = \sqrt{s}/2 = 7000$ GeV is $<n>_2 = 84.13$.

$$k^{-1} = -0.29 + 0.12 \ln \sqrt{s} \quad \text{with } \chi^2/ndf = 1.4$$

The corresponding value at $E^{tot} = \sqrt{s}/2 = 7000$ GeV is $k_2 = 2.66$.

The charged particles multiplicity distribution for LHC



Thus our prediction for the value of total cross section at the LHC energy.

$$\sigma_{tot}^{pp} \simeq 100.05 \begin{array}{cc} + & 4.26 \\ - & 4.21 \end{array}$$

And we can also calculate the value of mean charged multiplicity for LHC

$$<n> \simeq 73.11 \pm 4.5.$$